MA20033 - Question Sheet Six

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Attempt questions 1-3. Hand in by 5.00pm Friday 26 November 2004 either to me, or in the envelope on my door, 1W4.8. Questions 4 and 5 are for tutorial discussion, but I'm happy to also mark these if you wish to attempt them.

1. Suppose $X_1, \ldots, X_n$ are independent and identically distributed $N(\mu, \sigma^2 = 20)$ random quantities. Consider the hypothesis test

$$H_0 : \mu = 100 \quad \text{versus} \quad H_1 : \mu = 95.$$

   (a) Use the Neyman-Pearson lemma to find the critical region for a test with significance $\alpha = 0.05$. Your critical region should be expressed as simply as possible.

   (b) Find the corresponding probability of a Type II error (in terms of n and the Normal CDF Φ).

2. Suppose $X_1, \ldots, X_{100}$ are independent and identically distributed $Bin(n = 10, p)$ random quantities. We wish to test the hypothesis

$$H_0 : p = 0.5 \quad \text{versus} \quad H_1 : p = 0.75.$$

   (a) Use the Neyman-Pearson lemma to find the test of significance α with the largest power. Your critical region should be expressed as simply as possible.

   (b) What is the sampling distribution of your test statistic? Hence find the critical value for a test of significance $\alpha = 0.05$ (you do not need to evaluate this critical value, but express it in terms of a quantile of the distribution of your test statistic).

3. Suppose $X_1, \ldots, X_n$ are independent and identically distributed $N(\mu = 0, \sigma^2)$ random quantities and we wish to test the hypothesis

$$H_0 : \sigma^2 = \sigma_0^2 \quad \text{versus} \quad H_1 : \sigma^2 = \sigma_1^2$$

   where $\sigma_1^2 < \sigma_0^2$.

   (a) Use the Neyman-Pearson lemma to find the test of significance α which has the largest power. Try to ensure that your final test statistic is as simple as possible.

   (b) What is the sampling distribution of your test statistic? Hence find the critical value for a test of significance $\alpha = 0.05$ (you do not need to evaluate this critical value, but express it in terms of a quantile of the distribution of your test statistic).

4. Suppose $X_1, \ldots, X_{10}$ are independent and identically distributed $N(\mu, \sigma^2 = 20)$ random quantities, and we want to test the hypothesis

$$H_0 : \mu = 100 \quad \text{versus} \quad H_1 : \mu = \mu_1 < 100.$$

   (a) Find the critical value for a size $\alpha = 0.05$ test.

   (b) Calculate the corresponding probability of a Type II error as a function of μ_1, and roughly plot these probabilities against μ_1. Summarise what you deduce about the relative difficulties of testing for μ_1 close to 100 or μ_1 far from 100.

5. Suppose $X_1, \ldots, X_n$ are independent and identically distributed $Exp(\lambda)$ random quantities, so $f(x|\lambda) = \lambda \exp(-\lambda x)$. We wish to test the hypothesis

$$H_0 : \lambda = \lambda_0 \quad \text{versus} \quad H_1 : \lambda = \lambda_1$$

   where $\lambda_1 > \lambda_0$.

   (a) Use the Neyman-Pearson lemma to find the test of significance α with the largest power. Try to ensure that your final test statistic is as simple as possible.

   (b) If you knew the sampling distribution of your test statistic, how would you find the critical value for a test of significance $\alpha = 0.05$? You do not need to evaluate this critical value, but express it in terms of a quantile of the distribution of your test statistic.