King Saud University | Final Exam
Faculty of Science | Second Semester 1428/1429 H | Stat 491
Dept. of Statistics & O. R. | Time:3 hours

Answer the following questions

Question 1.

(a) Let X has a probability density function

$$f(x) = \frac{1}{2a}e^{-\frac{|X|}{2}}, -\infty < x < \infty.$$

Find the probability density function of the maximum observation $X_{(n)}$ and compute the $P(X_{(n)} \geq u)$ for $u \geq 0$.

(b) Prove that if $Y = F(X_{(k)})$ where $X_{(k)}$ has probability density function $f_n(X_{(k)})$, then Y has beta distribution with parameter $(k, n-k+1)$.

Question 2.

A random variable X is uniformly distributed over the range $0 < X < \theta$, where θ is known parameter A sample of size n is arranged in increasing order of magnitude are denoted as:

$$X_{(1)} \leq X_{(2)} \leq ... \leq X_{(n)}.$$

Derive the distribution of $X_{(k)}$ and then verify that:

(i) $E(X_{(k)}) = \frac{\theta k}{n+1}$ and $V(X_{(k)}) = \frac{\theta^2 k(n-k+1)}{(n+1)^2(n+2)}$.

(ii) For n odd, both the sample mean and sample median are unbiased estimate of $\frac{\theta}{2}$ and the ratio of variance is $\frac{n+2}{3n}$.

Question 3.

(a) If $f(x) = e^{-x}, x > 0$, and $W = X_{(k+1)} - X_{(k)}$. where $X_{(k)}$ is the kth order statistics. Find the distribution of W.

(b) Let $X_1, X_2, ..., X_n$ be $i.i.d.$ with common density

$$f(x) = \lambda e^{-\lambda x}, x > 0, \lambda > 0.$$

If $Y_i = (n-i+1)(X_{(i)} - X_{(i-1)})$ where $X_{(i)}$ is the i th order statistics and $X_{(0)} = 0$,Obtain the joint probability density function of the n order statistics of $Y_1, Y_2, ..., Y_n$.

Question 4.

Let $X_{(1)} \leq X_{(2)} \leq .... \leq X_{(n)}$ be the order statistics of random sample of size n from uniformly distributed over the range $0 < x < 1$.

(i) Find the distribution of $\frac{X_{(i)}}{X_{(i+1)}}$.

(ii) Prove that: $X_{(j)}$ and $\frac{X_{(i)}}{X_{(j)}}$ are independent.What about the distribution of two random variables $X_{(j)}$ and $\frac{X_{(i)}}{X_{(j)}}$.

Question 5.

Let X_1, $X_2, ..., X_n$ be a random sample from a population with probability density function

$$f(x) = \frac{\lambda e^{-\lambda x}}{1 - e^{-1.5\lambda}}, x < 1.5.$$

Use Newton method to find the M.L.E. of λ where $\overline{X} = 0.6914$, $\lambda_0 = 0.5727$ and $X^* = 0.0001$.

Good Luck

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