2.4. Calculating Measures from an ungrouped (simple) frequency table:

For the general ungrouped (simple) frequency table of the data:

$x_1, x_2, \ldots, x_n$

Value m_i	Freq. f_i	$m_i f_i$	m_i^2	$m_i^2 f_i$
m_1	f_1	$m_1 f_1$	m_1^2	$m_1^2 f_1$
m_2	f_2	$m_2 1_2$	m_2^2	$m_2^2 f_2$
.	.	.	.	.
m_k	f_k	$m_k f_k$	m_k^2	$m_k^2 f_k$
	$n=\sum f_i$	$\sum m_i f_i$		$\sum m_i^2 f_i$

Note: $n = \sum_{i=1}^{k} f_i$ = no. of observations

$$\sum x = \sum_{i=1}^{k} m_i f_i \quad \text{= sum of the observations}$$

$$\sum x^2 = \sum_{i=1}^{k} m_i^2 f_i \quad \text{= sum of the squared observations}$$

For calculating $\overline{X}$ and S^2, we need:

$n =$ the sample size $= \sum_{i=1}^{k} f_i$

$\sum x =$ the sum of the values $= \sum_{i=1}^{k} m_i f_i$

Sample Mean:

$$\bar{x} = \frac{\sum_{i=1}^{k} m_i f_i}{\sum_{i=1}^{k} f_i} \quad \Leftrightarrow \quad \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Sample Variance:

$$S^2 = \frac{\sum_{i=1}^{k} m_i^2 f_i - \left(\sum_{i=1}^{k} f_i\right) \bar{x}^2}{\left(\sum_{i=1}^{k} f_i\right) - 1} \quad \Leftrightarrow \quad S^2 = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}$$

Example: (p.41)

Data $\left(x_i\right)$

1, 2, 1, 2, 2, 2, 3, 3, 4, 5, 1, 2, 2, 2, 2, 3, 3, 4, 6, 5,1,1, 2, 2, 2, 3, 3, 3, 4, 7

Value m_i	Freq. f_i	$m_i f_i$	m_i^2	$m_i^2 f_i$
1	5	5	1	5
2	11	22	4	44
3	7	21	9	63
4	3	12	16	48
5	2	10	25	50
6	1	6	36	36
7	1	7	49	49
	30	83		295
	$n=\sum f$	$\sum mf$		$\sum m^2 f$

•Mean:

$$\bar{x} = \frac{\sum mf}{\sum f} = \frac{83}{30} = 2.8$$ • **(unit)**

•• Variance:

$$s^2 = \frac{\sum m^2 f - \left(\sum f\right)\bar{x}^2}{\left(\sum f\right) - 1} = \frac{295 - (30)(2.8)^2}{30 - 1} = 2.3$$ • **(unit²)**

•Standard Deviation:

$$S = \sqrt{2.3} = 1.517$$ • **(unit)**

Coefficient of variation:

C.V. $$= \frac{S}{\bar{x}} * 100\% = \left(\frac{1.517}{2.8}\right) * 100\% = 54.16\%$$

Mode: mode = 2 (unit)

· Median: (**n** = 30 even)

$$\textbf{median} = \frac{\left(\frac{n}{2}\right)^{th} \textit{order observation} + \left(\frac{n}{2}+1\right)^{th} \textit{ordered obs.}}{2}$$

$$= \frac{15^{th} \textit{ ordered obs.} + 16^{th} \textit{ ordered obs.}}{2}$$

$$= \frac{2+2}{2} = 2 \quad \textbf{(unit)}$$

2.5. Approximating Measures From Grouped Data:-

For grouped data:

- We do not know the actual values.
- We know how many of the values in each class interval
- Thus, we cannot find the actual values for $\bar{x}$ and S^2.
- We assume that all values in a particular class interval are located at the mid point of that interval.

Recall that, for calculating $\bar{x}$ and S^2, we need:

$$n, \qquad \sum x, \qquad \text{and} \quad \sum x^2$$

Let **k**= the number of class intervals

m_i= the mid point of the i-th C.I.

f_i = the frequency of the i-th C.I.

C.I.	mid-pointm_i	Freq. f_i	$m_i f_i$	$m_i^2 f_i$
1st C.I.	m_1	f_1	$m_1 f_1$	$m_1^2 f_1$
2nd C.I.	m_2	f_2	$m_2 1_2$	$m_2^2 f_2$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
kth C.I	m_k	f_k	$m_k f_k$	$m_k^2 f_k$
		$n=\Sigma f_i$	$\Sigma x_i = \Sigma m_i f_i$	$\Sigma x_i^2 = \Sigma m_i^2 f_i$

Therefore, the approximation of $\bar{x}$ and s^2 are :

$$\bar{x} = \frac{\sum m_i f_i}{n}$$

$$n = \sum_{i=1}^{k} f_i$$

$$s^2 = \frac{\sum_{i=1}^{k} m_i^2 f_i - n\bar{x}^2}{n-1}$$

Example:

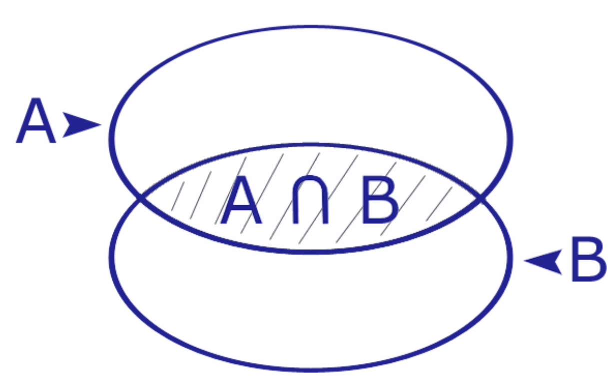
C.I.	mid-point m_i	Freq. f_i	$m_i f_i$	$m_i^2 f_i$
15 - 19	17	8	136	2312
20 - 24	22	16	352	7744
25 - 29	27	32	864	23328
30 - 34	32	28	896	28672
35 - 39	37	12	444	16428
40 - 44	42	4	168	7056
		$n=\Sigma f_i$ $=100$	$\Sigma m_i f_i$ 2860	$\Sigma m_i^2 f_i$ $=85540$

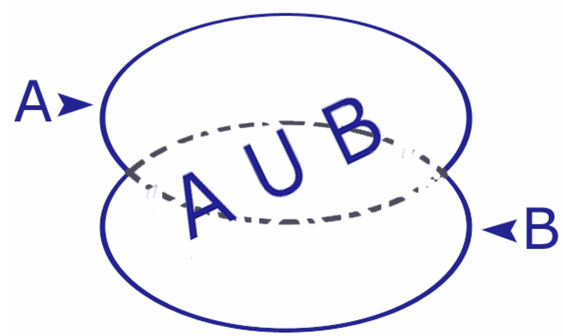
$$\bar{x} = \frac{\sum mf}{n} = \frac{2860}{100} = 28.6 \qquad \text{(year)}$$

$$S^2 = \frac{\sum_{i=1}^{k} m_i^2 f_i - n\bar{x}^2}{n-1} = \frac{85540 - (100)(28.6)^2}{100-1} = 37.8 \quad \text{(year)}^2$$

$$S = \sqrt{S^2} = \sqrt{37.8} = 6.1 \quad \text{(year)}$$

$$\text{C.V.} = \frac{S}{\bar{x}} * 100\% = \frac{6.1}{28.6} * 100\% = 21.5\%$$


A
A ∩ B
B


A
A U B
B