Lecture 4
Probability

Introduction

- Statistics
  - Descriptive Statistics
    - Graphics: bar plot, pie plot, histogram
    - Numerical: location (mean, median, mode)
      spread (range, variance, standard deviation)
      relative standing (rank, percentile)
  - Inferential Statistics
    - A general theoretical framework:
      - probability, random variables, and distributions
    - Confidence Intervals
    - Hypothesis Testing

Topics - Probability

- Lexicon of probability
- Law of large numbers
- Laws of probability

Section 4.2

The Language of Probability

Definitions

- *Probability* is a measure of how likely it is that something occurs.
- An *experiment* is any action with outcomes that are recordable data.
- The *sample space*, *S*, is the set of all possible outcomes of an experiment.
- An *event*, *A*, is a outcome or set of outcomes that are of interest to the experimenter.

Definitions (cont.)

- The *probability of an event A*, *P(A)*, is a measure of the likelihood that an event *A* will occur.
- The *complement* of an event A, denoted *A′*, is the set of all outcomes in the sample space, S, that are not in A.
- An *empirical probability* is one that is calculated from sample data and is an estimate for the true probability.

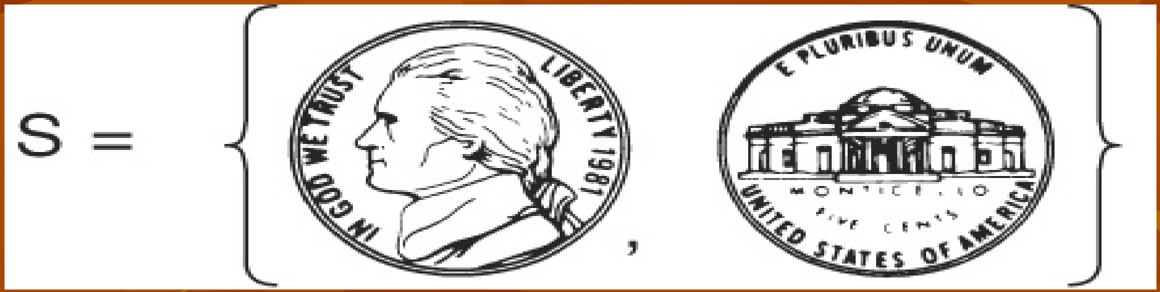
Definitions (cont.)

- The *law of large numbers* says that as the number of replications of an experiment increases, the estimate of the probability of an event gets closer to the true or real probability.

Goals

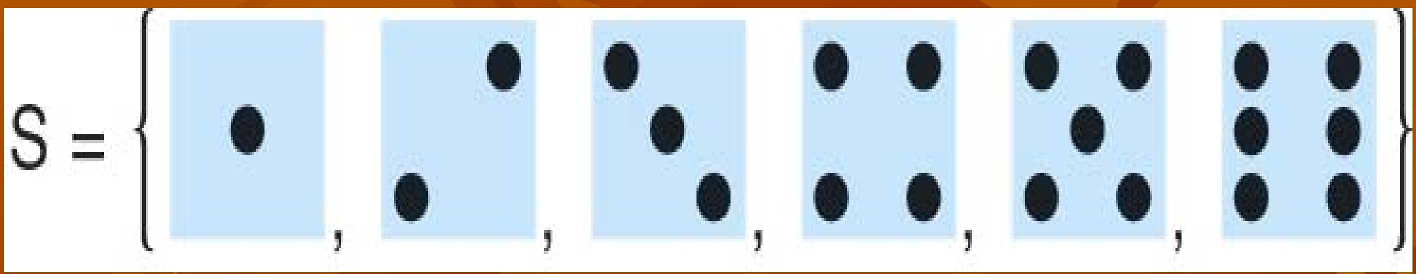
- To learn some basic terms associated with the theory of probability.
- To learn some basic probability calculations.

Example – Tossing a "Fair" Coin

- Experiment: toss a "fair" coin and observe the up face.
- 

- S = {H, T}

Example – Tossing a "Fair" Die

- Experiment: toss a "fair" six-sided die and observe the up face.
- 

- S = {1, 2, 3, 4, 5, 6}
- A = {roll an even number}
- A = {2, 4, 6}

Example – Toss a "Fair" Coin Twice

- Experiment: flip a "fair' coin twice and observe the sequence (keeping track of order) of up faces.
- S = {HH, HT, TH, TT}
- A = {toss at least one head}
  = {HH, HT, TH}

Example – Florida Lottery

- Experiment: randomly select 6 numbers, without replacement and ignoring order of selection, from 1, 2, . . . , 53.
- S = {(1,2,3,4,5,6), (1,2,3,4,5,7), . . . ,
  (48,49,50,51,52,53)}
- # of outcomes in S = 22,957,480

Finite Versus Infinite Sample Spaces

- In all previous examples the sample space was finite, i.e., had a finite number of outcomes.
- This usually happens when the outcomes are qualitative values or quantitative and integer values.
- Experiment that result in outcomes that are quantitative and continuous values usually have *infinite* sample spaces.

Example – Weights of UCF Students

- Experiment: randomly select a student from UCF and record their (actual) weight.
- S = {all numbers greater than 0 and less than 300 lbs}
- S = $\{x: 0 < x < 300\}$

Probability & Equally Likely Outcomes

- In many experiments the outcomes in S are "equally likely".
- In these cases, if there are *N* outcomes in S then the probability of any one outcome is 1/*N*.
- If A is any event and n_A is the number of outcomes in A, then

$$P(A) = \frac{n_A}{N}$$

Example – Tossing a "Fair" Die

- Experiment: toss a "fair" die and observe the up face.
- S = {1, 2, 3, 4, 5, 6}.
- Since the die is "fair" then the 6 outcomes are equally likely, i.e., P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6.
- A = {roll an even number} = {2, 4, 6}.

$$P(A) = \frac{3}{6} = 0.5$$

Some Probability Rules

- $0 \leq P(A) \leq 1$
- $P(S) = 1$

Complements

- $P(A) + P(A') = 1$.
- $P(A) = 1 - P(A')$.

Example – Toss a "Fair" Coin Twice

- Experiment: flip a "fair' coin twice and observe the sequence of up faces.
- $S = \{HH, HT, TH, TT\}$
- Since the coin is "fair" the outcomes are equally likely.
- A = {toss *at least* one head} = $\{HH, HT, TH\}$
- $P(A) = \frac{3}{4}$

Example – Toss a "Fair" Coin Twice

- $A' = \{TT\}$
- $P(A) = 1 - P(A') = 1 - \frac{1}{4} = \frac{3}{4}$

Random Sampling & Equally Likely Outcomes

- In this class of 254 students there are 8 student-athletes.
- Suppose that I plan to select a student at random from the 254.
- What is the probability that the selected student is a student-athlete?

Random Sampling & Equally Likely Outcomes

- S = {A1, A2, . . ., A8, NA1, NA2, . . ., NA246}
- Since the sampling is random, the 254 outcomes are equally likely.
- A = {selected student is a student-athlete}
- A = {A1, A2, . . . , A8}
- $P(A) = \frac{8}{254}$

Estimating Probabilities

- If the sample is a good representation of the of the population then the relative frequencies in the sample can serve as estimates of the true probabilities for the population.
- These are known as *empirical probabilities*.

Example – Election Poll

- In a USA TODAY CNN Gallup Poll (10/10/04) of 793 likely voters, 389 would vote for Kerry, 381 would vote for Bush, 8 would vote for Nader and 15 were undecided.

Example – Election Poll

Choice	Number of people	Relative frequency (%)
Bush	381	48
Kerry	389	49
Nader	8	1
Undecided	15	2
Total	793	100

Example – Election Poll

- Assume the sample of 793 likely voters is representative of all likely voters (approximately 160,000,000).
- Define A = {person selected at random from all likely voters votes for Kerry}
- Then $P(A) \cong 0.49$. Here "$\cong$" means "approximately estimated as".

The Law of Large Numbers

- As the sample size, *n*, becomes larger and larger the empirical probabilities computed from the sample become more precise estimates of the true probabilities.

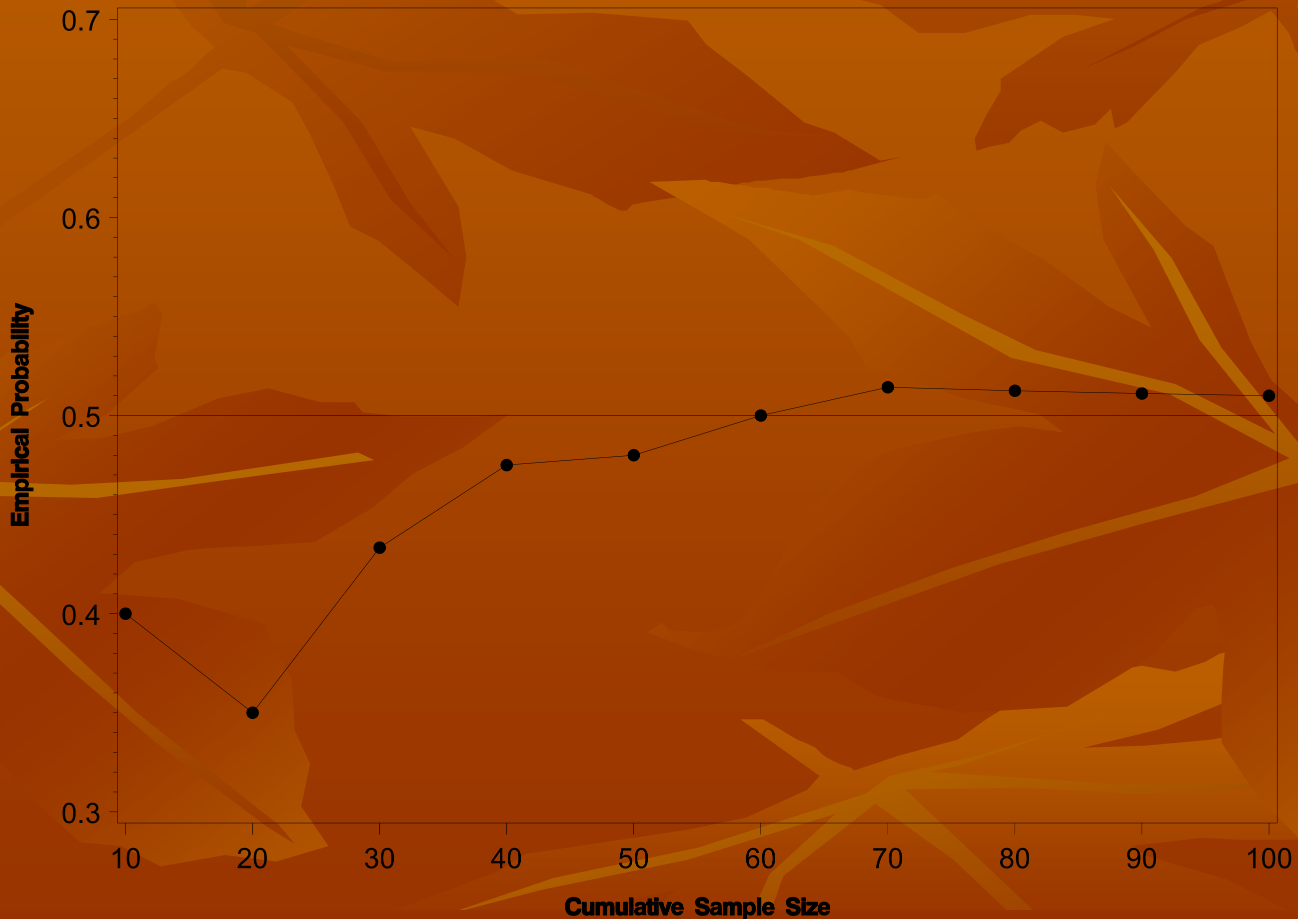
Example – Estimating the Probability of “Heads”

- Suppose you have a “fair” coin and you wish to determine the true probability of “heads”.

Example – Estimating the Probability of "Heads"

Sample size	# of heads	Cum. sample size	Cum. # of heads	Empirical Probability
10	4	10	4	0.400
10	3	20	7	0.350
10	6	30	13	0.433
10	6	40	19	0.475
10	5	50	24	0.480
10	6	60	30	0.500
10	6	70	36	0.514
10	5	80	41	0.513
10	5	90	46	0.511
10	5	100	51	0.510

Example – Estimating the Probability of "Heads"



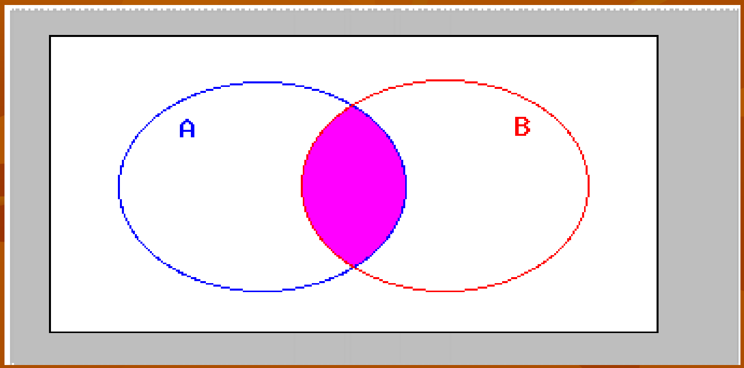
Section 4.3

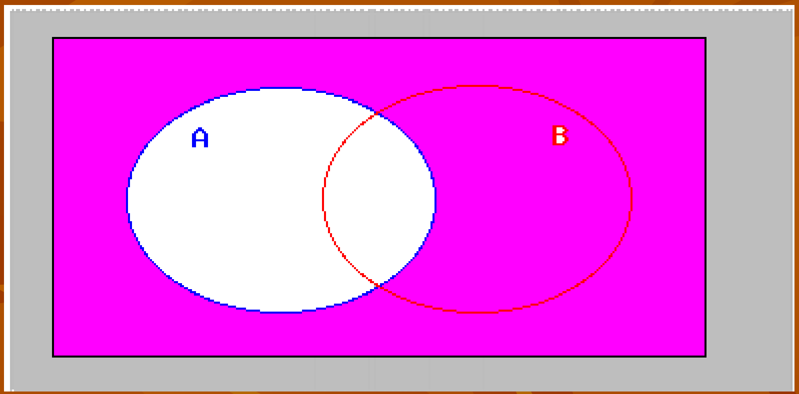
Laws of Probability: OR and AND

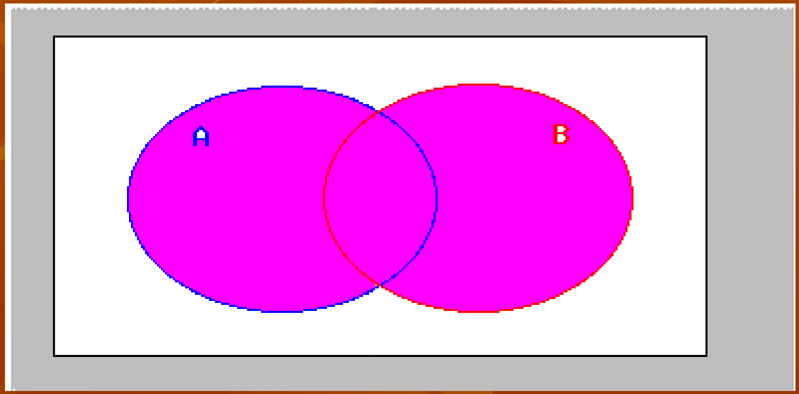
Definitions

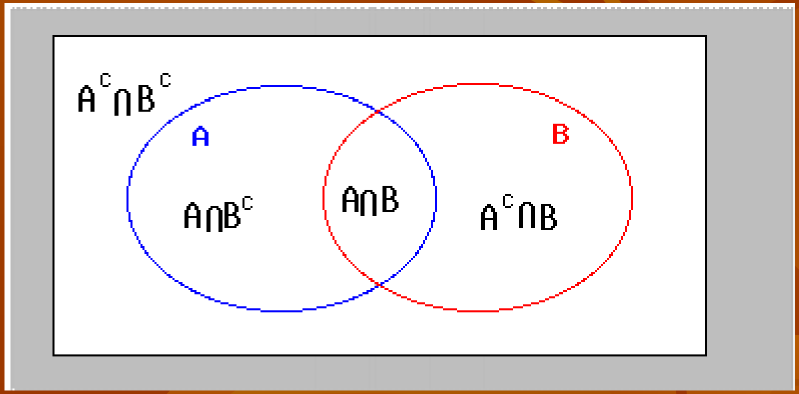
- The event *A OR B* ($A \cup B$) is the event that either A happens or B happens or they both happen. This is referred to as the union of A and B.
- The event *A AND B* ($A \cap B$) is the event that A and B both occur. This is refereed to as the intersection of A and B.
- Two events, A and B, are said to be *mutually exclusive* if they have no outcomes in common.

Venn Diagram









Goals

- To introduce compound events.
- To compute probabilities associated with compound events.

Probability of a Union

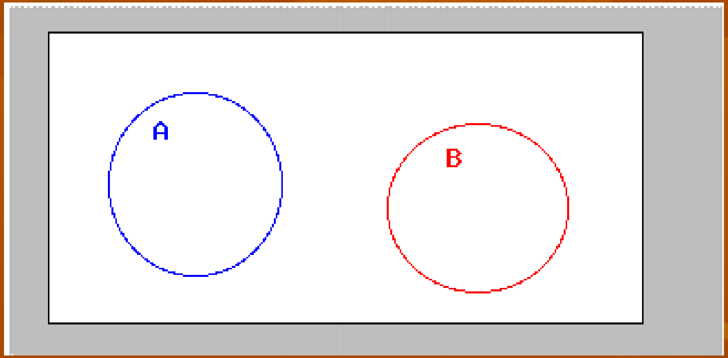
- Experiment: toss a “fair” coin 3 times and observe the sequence of up faces.
- S = {HHH, THH, HTH, HHT, TTH, THT,
  HTT, TTT}
- Since the coin is fair then the 8 possible outcomes are equally likely.
- A = {head on the last toss}
  = {HHH, THH, HTH, TTH}

Probability of a Union

- $B = \{\text{no heads}\} = \{TTT\}$
- {head on last toss <u>or</u> no heads} = $A \cup B$
- $A \cup B = \{HHH, THH, HTH, TTH, TTT\}$
- $P(A \cup B) = 5/8$
- Notice that $P(A) = 4/8$, $P(B) = 1/8$ and $P(A \cup B) = P(A) + P(B)$.
- This was no coincidence since A and B we *mutually exclusive* events.

Simple Addition Rule

- If A and B are mutually exclusive events then $P(A \cup B) = P(A) + P(B)$.



- If A, B and C are mutually exclusive events then $P(A \cup B \cup C) = P(A) + P(B) + P(C)$.

Probability of a Union

- Experiment: toss a "fair" coin 3 times and observe the sequence of up faces.
- A = {head on the last toss}

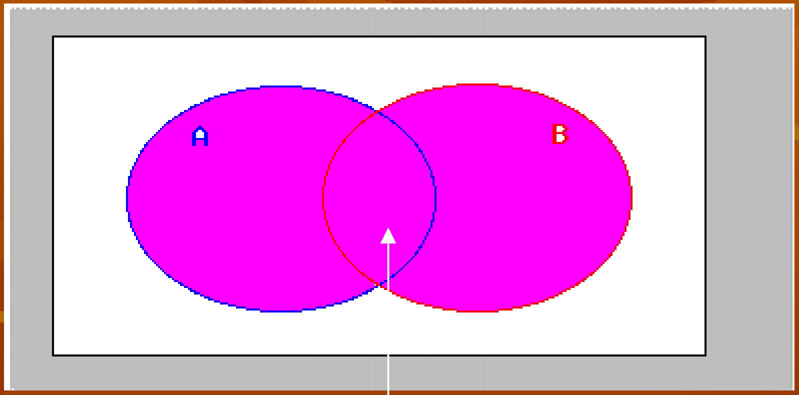
  = {HHH, THH, HTH, TTH}
- B = {no tails} = {HHH}
- P(A) + P(B) = 4/8 + 1/8 = 5/8

Probability of a Union

- $A \cup B = \{HHH, THH, THT, TTH\}$
- $P(A \cup B) = 4/8 \neq 5/8 = P(A) + P(B)$
- Problem: A and B have an outcome in common, i.e., $A \cap B = \{HHH\}$.
- A and B are not mutually exclusive.
- Simple addition rule does not apply.

General Addition Rule

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$A \cap B$

Probability of a Union

- Experiment: toss a "fair" coin 3 times and observe the sequence of up faces.
- A = {HHH, THH, HTH, TTH}
- B = {HHH}
- $A \cap B$ = {HHH}
- $P(A \cup B) = 4/8 + 1/8 - 1/8 = 4/8$

Contingency Tables & Probability

- Data that involve two qualitative variables can be summarized in a contingency table.
- A r by c contingency table is a two-dimensional version of a frequency table that has r rows and c columns.
- The entries in the contingency table are useful for computing the probabilities of events.
- Concepts: Cell counts, Grand total, Marginal (row/column) totals.

Example – Attendance & Class Standing

- In Sections 31 to 41 consider the two qualitative variables class standing (freshman, sophomore, junior, senior) and absence from Wednesday's class on 2/16/05 (yes, no).
- A = {student was a Freshman}
- B = {student was absent}
- $P(A \cup B) = ?$

Example – Attendance & Class Standing

	Absent		
Class	Yes	No	Total
Freshman	37	82	119
Sophomore	32	55	87
Junior	10	21	31
Senior	11	6	17
Total	90	164	254

Example – Attendance & Class Standing

- $P(A) = 119/254$
- $P(B) = 90/254$
- $P(A \cap B) = 37/254$
- $P(A \cup B) = 119/254 + 90/254 - 37/254$

  $= 172/254$

Example – Attendance & Class Standing

- C = {student was a sophomore}
- $P(A \cup C) = ?$
- $P(C) = 87/254$
- $P(A \cap C) = 0$
- $P(A \cup C) = 119/254 + 87/254 = 206/254$