THE EBU AND EWU CLASSES OF LIFE DISTRIBUTIONS

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Abstract

A new class of life distributions named exponential better than used(EBU)and its dual class exponential worse than used(EWU) is introduced.Thier relations to other classes of life distributions ,closure properties under reliability operations,moment inequalities,and heritage property under shock model are investigated.

1 Introduction

In reliability theory,various concepts of aging and wear have been proposed to study lifetimes of systems,or components in terms of conditional distributions of lifetimes failure rate,and renewal failure rate.The classes IFR, IFRA, DMRL, NBU, NBUE, HNBUE, GHNBUE, NBUFR, NBAFR, NBURFR, IFR(2), NBU(2) are examples of these distributions.For definitions of these classes and dual their classes ,see Bryson and Siddiqui (1969),Barlow and Proschan (1981), and Loh (1984).Let X be anonnegative random variable representing equipment life with distributions $F(t)$.The residual life X_t of the equipment of age$t$ has survival function $\overline{F_t}\,(x)$ given by

$$\overline{F}_t(x) = \frac{\overline{F}(t+x)}{\overline{F}(t)}\ ,\ \ \overline{F}(t) > 0.$$

Obviously ,any study of the phenomenon of aging has to be based $\overline{F}_t(x)$and functions related to it. It is well known that F belongs to the IFR (DFR) class if and only if X_t is decreasing (increasing) in $t \geq 0$ in stochastic ordering. F belongs to theNBU (NWU) class if and only if X_t is smaller (larger)than X for any $t \geq 0$ in stochastic ordering.And F belongs to the $NBUC$ if and only if X_t is smaller than X for any $t \geq 0$ in convex ordering.In the context of reliability

" no aging" is equivalent to the phenomenon that age has no effect on the residual survival function of a unit

$$i.e.\overline{F}(x \mid t) = \overline{F}(x) \qquad \text{for all } t, x > 0.$$

The last equation is satisfied only by the exponential survival function $\overline{F}(x) = e^{-\lambda x}$, $x > 0$, $\lambda > 0$,among continuous survival functions.

The following definitions willbe used in the sequel.

Let X and Y be two non-negative random variables,then X is said to be less than Y .

(i)in the stochastic order(denoted by $X \underset{st}{\leq} Y$) if and only if $P(X > x) \leq P(Y > x)$ for all x.

(ii) in the increasing convex order (denoted by $X \underset{icx}{\leq} Y$) if

$$\int_x^\infty \overline{F}(u)du \leq \int_x^\infty \overline{G}(u)du \quad \text{for all } x$$

(iii)in the increasing convcave order (denoted by $X \underset{icv}{\leq} Y$) if

$$\int_0^x \overline{F}(u)du \leq \int_0^x \overline{G}(u)du \quad \text{for all } x$$

(iv) in the laplace transform order(denoted by $X \underset{lt}{\leq} Y$) if

$$\int_0^\infty e^{-su}\overline{F}(u)du \leq \int_0^\infty e^{-su}\overline{G}(u)du \quad \text{for all } s \geq 0.$$

In this paper,we introduce a new class of life distributions in which we compare the survival function of a component of age t to a new component having the exponential distribution as its survival function.In section 2 ,we give the relationship between EBU and other well-known classes of life distributions.In section3,we discuss whether the EBU property is preserved under common reliability operations.Finally,the preservation of the property of EBU under shock models,and moment inequalities are established in section4.

2 Basic Properties of EBU(EWU)

We begin with the following:

Definition2.1

A non-negative random variable X with distribution F and finite mean μ is said to be exponentially better than used(EBU) if

$$\overline{F}(t + x) \leq \overline{F}(t)e^{-\frac{x}{\mu}}, \text{for all } x, t > 0$$

Note that ,the above definition is motivated by comparing the life lengh X_t of a component of age t with another new component of life lengh Y wich is exponential with the same mean as$X$ ..In this regard,we note that X is EBU if and only if $X_t \underset{st}{\leq} Y$ for all $t \geq 0$, where Y is an exponential random variable with the same mean as X .

In the next two results we shall show that

$$EBU \Longrightarrow NBUE \Longrightarrow HNBUE$$

Theorem2.2

If X is $EBU(EWU)$ then X is $NBUE(NWUE)$

Proof:

We shall prove the statement for the EBU case.Similar arguments hold for the EWU case.

If X is EBU,then integrating both sides of (2.1) with respect to x over $(0, \infty)$ implies that

$$\begin{aligned} \int_0^\infty \overline{F}(t+x)dx &\leq (\geq)\overline{F}(t)\int_0^\infty e^{-\frac{x}{\mu}}dx = \mu\overline{F}(t) \\ &\Longleftrightarrow \frac{\int_0^\infty \overline{F}(t+x)dx}{\overline{F}(t)} \leq (\geq)\mu \\ &\Longleftrightarrow \frac{\int_t^\infty \overline{F}(u)du}{\overline{F}(t)} \leq (\geq)\mu. \end{aligned}$$

Then X is $NBUE$

Theorem2.3

If X is $EBU(EWU)$ then X is $HNBUE$

Proof:

X is EBU means that

$$\overline{F}(t+x) \leq \overline{F}(t)e^{-\frac{x}{\mu}}, \text{for all } x, t > 0.$$

Integrating both sides of (2.1) with respect to t over $(0, \infty)$, we get

$$\begin{aligned} \int_0^\infty \overline{F}(t+x)dt &\leq (\geq)e^{-\frac{x}{\mu}}\int_0^\infty \overline{F}(t)dt = \mu e^{-\frac{x}{\mu}} \\ &\Longleftrightarrow \int_x^\infty \overline{F}(u)du \leq \mu e^{-\frac{x}{\mu}}, \text{ for all } x \geq 0. \end{aligned}$$

Hence X is $HNBUE$

Another interesting property of EBU relates its failure rate function to its mean.This is seen from the following.

If t=0 in (2.1),then

$$\overline{F}(x) \leq e^{-\frac{x}{\mu}}, \text{for all } x \geq 0.$$

Let $G(x)=1-e^{-x}$ for $x\geq 0$, it follows that $G^{-1}F(x)=-\ln\overline{F}(x)$.Observe that

$$\begin{aligned}
F \quad\in\quad & EBU \Longleftrightarrow G^{-1}F(x+t)\leq G^{-1}F(t)+\frac{x}{\mu} \text{ for all } x,t\geq 0\\
\Longleftrightarrow\quad & \frac{G^{-1}F(x+t)-G^{-1}F(t)}{x}\leq\frac{1}{\mu}\\
\Longrightarrow\quad & \lim_{x\longrightarrow 0^{+}}\frac{G^{-1}F(x+t)-G^{-1}F(t)}{x}\leq\frac{1}{\mu}\\
\Longrightarrow\quad & \frac{dG^{-1}F(t)}{dt}\leq\frac{1}{\mu}\\
\Longleftrightarrow\quad & \frac{f(t)}{gG^{-1}F(t)}=\frac{f(t)}{\overline{F}(t)}\Longleftrightarrow r_F(t)\leq\frac{1}{\mu} \quad \text{for all } t\geq 0.
\end{aligned}$$

The next result shows that the EBU class is closed under convolution.

Theorem2.4

Suppose that F_1 and F_2 are two independent EBU life distributions,then their convolution is also EBU

Proof

$$\begin{aligned}
\overline{F}(t+y) &= \int_0^\infty \overline{F_1}(t+y-z)dF_2(z)\\
&\leq \int_0^\infty e^{-\frac{t}{\mu_1}}\overline{F_1}(y-z)dF_2(z)\\
&= e^{-\frac{t}{\mu_1}}\overline{F}(y)\\
&\leq e^{-\frac{t}{\mu}}\overline{F}(y)
\end{aligned}$$

The first inequality follows sinceF_1 is EBU while the second inequality follows since $\mu_1\leq\mu$. Thus ,EBU is closed under convolution.

3 Stochastic comparisons of excess life times of renewal processes.

Let us concider a renewal process with independent and identically distributed non-negative inter-arrival times X_i with common distribution F and $F(0)=0$. Let $S_0=0$ and $S_k=\sum_{i=1}^{k}X_i$ and consider the renewal counting process $N(t)$ =sup$\{n: S_n\leq t\}$.

Several papers have investigated some characteristics of the renewal process related to ageing properties of F .See for example,Brown(1980, 1981),Barlow and Proschan (1981),and Shaked and Zhu(1992).Chen (1994) investigated the relationship between the ageing property of$F$.

Some other results are given for the remaining life time variable defined by $\gamma(t) = S_{N(t)+1} - t$, where $\gamma(t)$ is the remaining life of the unit in use at time t .Note that

$$\begin{aligned} P\left[\delta(t) \le t\right] &= 1 \text{ and } \quad P\left[\delta(t) = t\right] = \overline{F}(t) \\ and\ \ P\left[\delta(t) > u\right] &= \overline{F}(t+u) + \textstyle\sum_{n=1}^{\infty} \int_0^t \overline{F}(t-x+u) dF_{(x)}^{(n)} \end{aligned}$$

Where $F_{(x)}^{(n)}$ is the n-fold convolution of F,so that

$$P\left[\delta(t) > u\right] = \overline{F}(t+u) + \textstyle\int_0^t \overline{F}(t-x+u) dM(x).$$

From the above equation we may obtain a lower bounded for $P\left[\gamma(t) > u\right]$.

Some examples of such results are the following:

(i)$Chen$(1994) showed that:If $\gamma(t)$ is stochastically decreasing in $t \ge 0$,then $F \in NBU$ and if $E\gamma(t)$ is decreasing in $t \ge 0$,then $F \in NBUE$.

(ii)Li et al(2000) showed that ,if $\gamma(t)$ is stochastically decreasing in $t \ge 0$ in the increasing convex order then F

$\in NBUC$

(iii)Li and Kochar (2001) showed that if $\gamma(t)$ $\downarrow$in $t \ge 0$ in the increasing concave order then F

$\in NBU(2)$

(iv) Belzunce et al (2001)showed that if $\gamma(t)$ $\downarrow$in $t \ge 0$ in the Laplace order then $F \in NBU_{Lt}$.

Next we show a similar result for EBU class.

Theorem3.1

If $F \in EBU$ then $\gamma(t) \underset{st}{\le} Y$,where Y has the exponential distribution with mean $=E(x)$.

Proof:

$$\begin{aligned} P\left[\delta(t) > u\right] &= \overline{F}(t+u) + \textstyle\int_0^t \overline{F}(t-x+u) dM(x). \\ &= \overline{F}(t) e^{-\frac{u}{\mu}} + e^{-\frac{u}{\mu}} \textstyle\int_0^t \overline{F}(t-x) dM(x) \\ &= e^{-\frac{u}{\mu}}. \left[\overline{F}(t) + \textstyle\int_0^t \overline{F}(t-x) dM(x)\right] \\ &= e^{-\frac{u}{\mu}} P\left[\gamma(t) > 0\right] = e^{-\frac{u}{\mu}} \\ & \quad \text{Therfore } \gamma(t) \underset{st}{\le} Y \end{aligned}$$

4 Shock Models leading to EBU(EWU) Survivals

Suppose that a device is subjected to shocks occurring randomly as events in a Poisson process with constant intensity λ . Suppose further that the device has probability

$\overline{P}_K$ of surviving the first K shocks.Then the survival function of the device is given by

$$\overline{H}(t) = \sum_{K=0}^{\infty} \overline{P_K} \frac{(\lambda t)^K}{K!} e^{-\lambda t} (4.1)$$

For the discrete distribution$\left\{\overline{P_K}, K \in N\right\}$,it is well known that properties of $\overline{P_K}$ are reflected in the corresponding properties of the continuous life distribution H(t).This is shown by Esary et al (1973) for $IFR, IFRA, DMRL, NBU$ and $NBUE$ classes. Klefsjo(1981) for $HNBUE$ and Abouammoh and Ahmed (1988) for $NBUFR$..

Definition 4.1

A discrete distribution $\overline{P}_K, K = 0, 1, ...$ or its survival propapility function $\left\{\overline{P}_K\right\}_{k=0}^{\infty}$ with finite mean m=$\sum_{K=0}^{\infty} \overline{P}_K$ is called discrete EBU if

$$\overline{P_{j+1}} \leq \overline{P_j} \left(1 - \frac{1}{m}\right)^l \quad \text{for all } j = 0, 1, ...$$

Theorem 4.2

The survival function $\overline{H}(t)$ in (4.1) is EBU if and only if $\left\{\overline{P}_K\right\}_{k=0}^{\infty}$ has the discrete EBU property.

Proof

We first note that

$$\begin{aligned} \mu &= \int_0^{\infty} \overline{H}(t)dt = \frac{1}{\lambda} \sum_{K=0}^{\infty} \overline{P}_K \int_0^{\infty} \frac{(\lambda t)^K}{K!} e^{-\lambda t} d(\lambda t) \\ &= \frac{1}{\lambda} \sum_{K=0}^{\infty} \overline{P}_K = \frac{m}{\lambda} \end{aligned}$$

Let $\overline{P_K}$ be the probability that the device survives the first K shocks ,where 1 =

$\overline{P_0} \geq \overline{P_1} \geq ....$ The survival function is

$$\begin{aligned}
\overline{H}(t+x) &= \textstyle\sum_{K=0}^{\infty} \overline{P}_K \frac{[\lambda(t+x)]^K}{K!} e^{-\lambda(t+x)} \\
&= \textstyle\sum_{K=0}^{\infty} \frac{\overline{P}_K}{K!} \sum_{j=0}^{K} \begin{pmatrix} K \\ J \end{pmatrix} (\lambda t)^j (\lambda x)^{K-j} \quad e^{-\lambda(t+x)} \\
&= \textstyle\sum_{K=0}^{\infty} \overline{P}_K \sum_{j=0}^{K} \frac{(\lambda t)^j}{j!} . \frac{(\lambda x)^{K-j}}{(k-j)!} e^{-\lambda x} . e^{-\lambda t} \\
&= \textstyle\sum_{j=0}^{\infty} \frac{(\lambda t)^j}{j!} \sum_{K=j}^{\infty} \overline{P}_K \frac{(\lambda x)^{K-j}}{(k-j)!} e^{-\lambda x} . e^{-\lambda t} \\
&= \textstyle\sum_{j=0}^{\infty} \frac{(\lambda t)^j}{j!} \sum_{l=0}^{\infty} \overline{P}_{l+j} \frac{(\lambda x)^l}{(l)!} e^{-\lambda x} . e^{-\lambda t} \\
&\leq \textstyle\sum_{j=0}^{\infty} \frac{(\lambda t)^j}{j!} e^{-\lambda t} . \overline{P}_j \sum_{l=0}^{\infty} \frac{(1-\frac{1}{m})^l (\lambda x)^l}{l!} e^{-\lambda x} \\
&\leq \textstyle\sum_{j=0}^{\infty} \overline{P}_j . \frac{(\lambda t)^j}{j!} e^{-\lambda t} . \sum_{l=0}^{\infty} \frac{\left[\lambda x (1-\frac{1}{m})\right]^l}{l!} e^{-\lambda x} \\
&\leq \textstyle\sum_{j=0}^{\infty} \overline{P}_j \frac{(\lambda t)^j}{j!} e^{-\lambda t} . e^{-\lambda x} . e^{-\lambda x (1-\frac{1}{m})} \\
&= \overline{H}(t) e^{-\frac{\lambda x}{m}} = \overline{H}(t) . e^{-\frac{x}{\mu}}
\end{aligned}$$

which implies that $\overline{H}$ has the EBU property.This completes the proof.

Remark : A similar result can be written for the EWU class.

5 Moment Inequalities for EBU(EWU)

In this section we establish useful moment inequalities for the EBU(EWU) classes.These inequalities are interest for engineers and field reliability.

Let s be a non negative integer.We use λ_s denote $\frac{E(X^s)}{\Gamma(s+1)}$.

Theorem5.1

Let F be a life distribution which is EBU(EWU) with mean μ, then

$$\lambda_{s+t} \leq (\geq) \lambda_s . \mu^t \quad \text{for all } s \geq 0, t \geq 0.$$

Proof:

We shall consider only the EBU case.The EWU follows by reversing all inequalities.

$$\overline{F}(x+y) \leq \overline{F}(y) . e^{-\frac{x}{\mu}} \qquad \text{for all } x \geq 0, y \geq 0. (5.1)$$

Multiplying both sides of (5.1) by $\frac{x^{t-1}.y^{s-1}}{\Gamma s.\Gamma t}$ and integrating we get,L.H.S

$$\begin{aligned}
\int_0^\infty\int_0^\infty \frac{x^{t-1}.y^{s-1}}{\Gamma s.\Gamma t}\overline{F}(x+y)dxdy &= \int_0^\infty\int_0^\infty \frac{x^{t-1}.y^{s-1}}{\Gamma s.\Gamma t}\overline{F}(y).e^{-\frac{x}{\mu}}dxdy \\
&= \left(\int_0^\infty \frac{x^{t-1}.e^{-\frac{x}{\mu}}}{\Gamma t}dx\right)\left(\int_0^\infty \frac{y^{s-1}\overline{F}(y)}{\Gamma s}dy\right) \\
&= \mu^t.\lambda_s
\end{aligned}$$

On the other hand the left hand side is equal to

$$\begin{aligned}
&\int_0^\infty\int_0^\infty \frac{x^{t-1}.y^{s-1}}{\Gamma s.\Gamma t}\int_{x+y}^\infty dF(t)dxdy \\
&= \int_0^\infty dF(t)\int_0^z \frac{x^{t-1.}}{\Gamma t}dx.\int_0^{z-x}\frac{y^{s-1}}{\Gamma s}dy \\
&= \int_0^\infty\int_0^z \frac{x^{t-1.}}{\Gamma t}\frac{(z-x)^s}{\Gamma(s+1)}dx.dF(t) \\
&= \int_0^\infty\int_0^1 \frac{z^{t-1}.z^s.z}{\Gamma(t)\Gamma(s+1)}.u^{t-1}(1-u)^s dudF(t) \\
&= \int_0^\infty \frac{z^{t+s}}{\Gamma(t)\Gamma(s+1)}\left(\int_0^1 u^{t-1}(1-u)^s du\right)dF(z) \\
&= \int_0^\infty \frac{z^{t+s}}{\Gamma(t)\Gamma(s+1)}.\frac{\Gamma(t)\Gamma(s+1)}{\Gamma(t+s+1)}dF(z) \\
&= \int_0^\infty \frac{z^{t+s}}{\Gamma(t+s+1)}dF(z) = \frac{E(z^{t+s})}{\Gamma(t+s+1)} = \frac{\mu^{t+s}}{\Gamma(t+s+1)} \\
&= \lambda_{s+t}
\end{aligned}$$

combining the two sides,we obtain $\lambda_{s+t} \leq (\geq)\lambda_s.\mu^t$, s,t$\in \{0,1,2,.....\}$,which is the desired result.

To establish our next result,we shall need the following Lemma.

Lemma5.2

Let Xbe a continuous nonnegative random variable with mean μ .Then

$$E(X^2) = 2\int_0^\infty\int_0^\infty \overline{F}(u)dudx$$

Proof:

The proof follows easily using integration by parts and Fubini theorem.

Corollary5.3

The coefficient of variation η of X is given by

$$\eta^2 = \frac{2}{\mu^2}\int_0^\infty\int_0^\infty \overline{F}(u)dudx - 1$$

Proof:
Follows easily from 5.2.
Theorem5.4:
If F is $EBU(EWU)$ with finite mean μ ,then $\eta \leq (\geq)1$.
Proof :
Let F be EBU

$$\begin{aligned}
&\Longrightarrow \quad \overline{F}(x+y) \leq \overline{F}(x).e^{-\frac{y}{\mu}} \qquad \text{for} \quad x \geq 0,\ y \geq 0. \\
&\Longrightarrow \quad \int_0^\infty \overline{F}(x+y)dx \leq \int_0^\infty \overline{F}(x).e^{-\frac{y}{\mu}}dx \\
&\Longleftrightarrow \quad \int_y^\infty \overline{F}(x)dx \leq \mu e^{-\frac{y}{\mu}} \\
&\Longrightarrow \quad \eta^2 \leq \frac{2}{\mu^2}\int_0^\infty \mu e^{-\frac{y}{\mu}}dy - 1 \qquad \text{(using corollary5.3)} \\
&\Longrightarrow \quad \eta^2 \leq 1.
\end{aligned}$$

Reversing all inequalities establishes the result for the EBU class.
Remark:
Theorem 5.4 says that if F is $EBU(EWU)$ then F is more(less) peaked than the exponential distribution for which $\eta = 1$.
Lemma5.5(Kitchen and Proschan,1981):
Let X and Y be continuous nonnegative random variables(possibly depended)with $E(X) \leq E(Y), \eta(X) \leq 1, \eta(Y) \leq 1, \eta(X+Y) = 1$.Then ,$X = \alpha Y$ as $\alpha = \left\{ \begin{array}{ll} \frac{E(X)}{E(Y)}, & E(Y) > 0 \\ 0 \quad , & E(Y) > 0 \end{array} \right\}$
We now present the following;
Theorem5.6:
If the convolution of n EBU distributions is exponential,then (n-1) of the distributions are degenerate at zero and the other distributions is exponential.
Proof:
Follow word for word the arguments used in Kitchen and Proschan,(1981),using Theorem 5.4 and Lemma5.5.

6 Sharp bounds for the EBU(EWU) classes

Next we present bounds on the survival function assuming one moment is known,and the underlying distribution is EBU or EWU .such bounds are usefule in reliability applications,since in a typical situation the only facts known as a prior,may be for example that the component is EBU due to wear and that its mean life is μ ,say
Theorem6.1
Let F be EBU.Then

$$\overline{F}(t) \leq \left\{ \begin{array}{ll} 1 & \text{t} \leq \mu \\ e^{1-\frac{t}{\mu}} & \text{t} \geq \mu \end{array} \right.$$

Proof:

$$\begin{aligned}
\overline{F}(x+t) &\leq \overline{F}(t).e^{-\frac{x}{\mu}} \\
&\Longrightarrow \overline{F}(x) \leq e^{-\frac{x}{\mu}} \\
\int_s^t \overline{F}(x)dx &\leq \int_s^\infty \overline{F}(x)dx \leq \int_s^\infty e^{-\frac{x}{\mu}}dx \\
&= \mu e^{-\frac{s}{\mu}} \\
\int_s^t \overline{F}(x)dx &\geq (t-s)\overline{F}(t) \\
&\Longrightarrow \overline{F}(t) \leq \frac{\int_s^t \overline{F}(x)dx}{(t-s)} \leq \frac{\mu e^{-\frac{s}{\mu}}}{(t-s)} \text{ for all } s,t \geq 0. \\
\overline{F}(t) &\leq \inf_{0<s<t} \frac{\mu e^{-\frac{s}{\mu}}}{(t-s)}.
\end{aligned}$$

Theorem 6.2:
Suppose that F is a life distribution which is EBU with mean μ .Then

$$\overline{F}(t) \geq \left\{ \begin{array}{ll} e^{-\frac{\alpha}{\mu}} & \text{for } 0 \leq \text{ t} \leq \mu \\ 0 & \text{for } t \geq \mu \end{array} \right\}$$

where $\alpha = \alpha(t)$ is the largest non-negative number for which

$$(\alpha - t + \mu)e^{-\frac{\alpha}{\mu}} - \mu + t = 0$$

Proof :

$$\begin{aligned}
\int_0^s \overline{F}(x)dx &= \int_0^t \overline{F}(x)dx + \int_t^s \overline{F}(x)dx \\
&\leq t + \overline{F}(t)\int_t^s dx \leq t + \overline{F}(t)(s-t) \quad \text{for all } s > t \\
\int_0^s \overline{F}(x)dx - t &\leq \overline{F}(t)(s-t) \quad \text{and} \quad \overline{F}(x) \leq e^{-\frac{x}{\mu}} \\
\text{but } \mu &= \int_0^\infty \overline{F}(x)dx = \int_0^s \overline{F}(x)dx + \int_s^\infty \overline{F}(x)dx \\
&\Longrightarrow \int_0^s \overline{F}(x)dx \geq \mu - \mu e^{-\frac{s}{\mu}}; \quad \text{accordingly} \\
\overline{F}(t) &\geq \frac{\int_0^s \overline{F}(x)dx - t}{(s-t)} \\
&\geq \frac{\mu - \mu e^{-\frac{s}{\mu}} - t}{(s-t)} \quad \text{for all s>t} \\
\text{Thus}\overline{F}(t) &= \left\{ \begin{array}{ll} e^{-\frac{\alpha}{\mu}} & \text{for } 0 \leq \text{ t} \leq \mu \\ 0 & \text{for } t \geq \mu \end{array} \right\}
\end{aligned}$$

where $\alpha = \alpha(t)$ is the largest non-negative number for which

$$(\alpha - t + \mu)e^{-\frac{\alpha}{\mu}} - \mu + t = 0$$

standard calculus then gives that for $t < \mu$,the supremum is attained for $s = \alpha$ given by

$$\overline{F}(t) = \sup_{s>t} \frac{\mu - \mu e^{-\frac{s}{\mu}} - t}{(s-t)}$$

Remark:

The bounds obtained in theorem 6.1 *and* 6.2 are all sharp.To see it,choose F exponential and note that the exponential distributions are the boundary members of the EBU and EWU classes.

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