MA20033 - Question Sheet Four

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Attempt questions 1-4; questions 5-6 are for tutorial discussion. Hand in by 5.00pm Friday 12 November 2004 either to me, or the envelope on my door, 1W4.8. If you wish to also attempt the tutorial questions, I'm happy to mark them.

1. Calculate the mean, median and once trimmed mean for the following data:

   $$3, 12, 13, 14, 15, 15, 17, 19, 20, 24$$

   Now suppose that the observation with value 3 is replaced by a new observation with value x, where x takes some value between 0 and 30. Derive expressions for the three estimates calculated above as a function of x, and comment on their robustness.

2. Use your statistical tables to find the following quantities.

   (a) The values which enclose the central 90% of a $N(0, 1)$ distribution.

   (b) The values which enclose the central 90% of a $N(10, 5)$ distribution.

   (c) The values which enclose the central 95% of a χ^2_{15} distribution.

3. To estimate the mean gestation period of domestic dogs, 15 randomly selected dogs are observed during pregnancy. Their gestation periods, in days, are:

   62.0, 61.4, 59.8, 62.2, 60.3,
   60.4, 59.4, 60.2, 60.4, 60.8,
   61.8, 59.2, 61.1, 60.4, 60.9.

   We will make the assumption that these 15 observations are realisations from a population which may be modelled by a $N(\mu, \sigma^2)$ distribution.

   (a) Evaluate a 95% confidence interval for μ when σ^2 is known to be 1.

   (b) Evaluate a 95% confidence interval for σ^2 (assumed unknown).

4. Sometimes it is the case that a one-sided rather than a two-sided confidence interval is required, which means that we want the realisation of a random (half) interval of the form either

   $$P\{\theta > g_1(X_1, \ldots, X_n)|\theta\} \quad = \quad 1 - \alpha,$$

(a one-sided lower $(1-\alpha)100\%$ random interval for θ), or

$$P\{\theta < g_2(X_1,\ldots,X_n)|\theta\} \quad = \quad 1-\alpha,$$

(a one-sided upper $(1-\alpha)100\%$ random interval for θ). Derive and evaluate a 95% upper confidence interval for σ^2 using the dog data set of question 3.

5. Suppose we wish to find an interval estimator for the parameter p when we have a random variable $X \sim Bin(n,p)$.

   (a) By considering the Normal approximation to the Binomial distribution, show that $\frac{X/n-p}{\sqrt{p(1-p)/n}}$ is an approximate pivot for p, and state its sampling distribution.

   (b) Write down a random interval which contains p with probability (approximately) 0.95 (to do this you will need to make a further approximation of the variance of the Normal using a point estimator of p).

6. Suppose $X_1,\ldots,X_n$ are iid $U(0,\theta)$ random quantities, and that we wish to find an interval estimator for θ.

   (a) Recalling the cumulative distribution function of $M = \max\{X_1,\ldots,X_n\}$ (see question 3 on Question Sheet Two), find a pivot for θ by considering a particular linear transformation of M.

   (b) Now using the cumulative distribution function of the pivot you have derived, find a random interval which contains θ with probability 0.95.

   (c) Does this interval contain the maximum likelihood estimator of θ?