MA20033 - Question Sheet One

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1. State the sample space $\mathcal{S}$ and the parameter space Ω for the following random quantities. In each case, what is the interpretation of the parameters? Based on this, try to suggest an intuitive estimator for each parameter.
   (a) $X \sim Bernoulli(p)$
   (b) $X \sim N(\mu, \sigma^2)$
   (c) $X \sim Bin(n, p)$
   (d) $X \sim \exp(\lambda)$
   (e) $X \sim Geo(p)$
   (f) $X \sim U(a, b)$
2. Suppose $X_1, X_2, \ldots, X_n$ are iid $N(\mu, \sigma^2)$ random quantities. Using the properties of independent Normals and expectation and variance operators, derive the sampling distribution of $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i$.
3. (a) Suppose that in an attempt to estimate the proportion of the population who are left-handed, 100 people are surveyed of whom 8 are left-handed. Assuming a Binomial model, evaluate the probability that $X = 8$ under the three values $p = 0.01$, $p = 0.1$ and $p = 0.5$. If these are the only possible values of p, so $\Omega = \{0.01, 0.1, 0.5\}$, what is the maximum likelihood estimate of p?
   (b) Now write down the probability that $X = 8$ under an arbitrary value of $p \in \Omega = (0, 1)$. What is the maximum likelihood estimate of p? Comment on this value.
4. Let $x_1, x_2, \ldots, x_n$ be a random sample (so the X_i are independent) from an exponential distribution with probability density function

$$f(x|\tau) \quad = \quad \frac{1}{\tau}\exp(-x/\tau) \quad 0 \leq x < \infty$$

   and zero otherwise, where $\tau > 0$.
   (a) Show that if the model is correct it is sufficient to know the sample size n and the sample mean $\overline{x}$ to evaluate the likelihood function for any value of τ.
   (b) Find the maximum likelihood estimator $\hat{\tau}$ of τ in terms of $\overline{X}$.
   (c) Show that $\hat{\tau}$ is an unbiased estimator of τ; that is, $E(\hat{\tau}|\tau) = \tau$ for all $\tau > 0$.