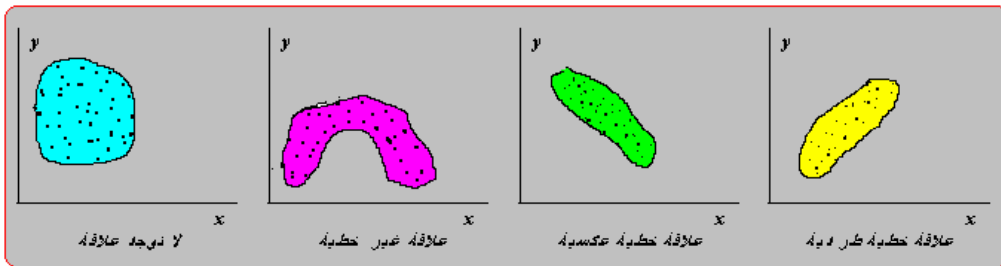


/

(y , x)

(-)

y , x



Simple Correlation

/

//

r () ρ
 :
 :
 $(r < 0)$ -
 $(r > 0)$ -
 $(r = 0)$ -
 (± 1) :
 $(-1 < r < 1)$:
 $(-)$

ارتباط عكسي					ارتباط طردي					
قوي جدا	قوي	متوسط	ضعيف	ضعيف جدا	ضعيف جدا	ضعيف	متوسط	قوي	قوي جدا	
-1	-0.9	-0.7	-0.5	-0.3	0	0.3	0.5	0.7	0.9	1
نام					عدم					نام

Pearson " " //

(y , x)

:

Pearson " "

: " "

$$r = \frac{S_{xy}}{S_x S_y} = \frac{\frac{\sum (x - \bar{x})(y - \bar{y})}{(n-1)}}{\sqrt{\frac{\sum (x - \bar{x})^2}{(n-1)}} \sqrt{\frac{\sum (y - \bar{y})^2}{(n-1)}}} \quad (1-6)$$

:

(y , x) : $S_{xy} = \sum (x - \bar{x})(y - \bar{y}) / (n-1)$

(x) : $S_x = \sqrt{\sum (x - \bar{x})^2 / (n-1)}$

(y) : $S_y = \sqrt{\sum (y - \bar{y})^2 / (n-1)}$

:

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2} \sqrt{\sum (y - \bar{y})^2}} \quad (2-6)$$

(-)

.2002 1995

	1995	1996	1997	1998	1999	2000	2001	2002
	305	313	297	289	233	214	240	217
	592	603	662	607	635	699	719	747

:

(y , x) (y) (x)
 : (-)
 .(ȳ , x̄)

$$\bar{x} = \frac{\sum x}{n} = \frac{2108}{8} = 263.5, \bar{y} = \frac{\sum y}{n} = \frac{5264}{8} = 658$$

x	y	x - x̄	(x - x̄) ²	y - ȳ	(y - ȳ) ²	(x - x̄)(y - ȳ)
305	592	41.5	1722.25	-66	4356	-2739
313	603	49.5	2450.25	-55	3025	-2722.5
297	662	33.5	1122.25	4	16	134
289	607	25.5	650.25	-51	2601	-1300.5
233	635	-30.5	930.25	-23	529	701.5
214	699	-49.5	2450.25	41	1681	-2029.5
240	719	-23.5	552.25	61	3721	-1433.5
217	747	-46.5	2162.25	89	7921	-4138.5
2108	5264	0	12040	0	23850	-13528

$$\sum(x - \bar{x})^2 = 12040, \sum(y - \bar{y})^2 = 23850,$$

$$\sum(x - \bar{x})(y - \bar{y}) = -13528$$

:

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2} \sqrt{\sum(y - \bar{y})^2}} = \frac{-13528}{\sqrt{12040} \sqrt{23850}}$$

$$= \frac{-13528}{(109.727)(154.434)} = \frac{-13528}{16945.619} = -0.798$$

.

:

(-)

(-)

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)\left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}}$$

(*)

x	y	xy	x ²	y ²
305	592	180560	93025	350464
313	603	188739	97969	363609
297	662	196614	88209	438244
289	607	175423	83521	368449
233	635	147955	54289	403225
214	699	149586	45796	488601
240	719	172560	57600	516961
217	747	162099	47089	558009
2108	5264	1373536	567498	3487562

$\sum x = 2108$, $\sum y = 5264$ $\sum xy = 1373536$ $\sum x^2 = 567498$ $\sum y^2 = 3487562$

(-)

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)\left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}}$$

$$= \frac{1373536 - \frac{(2108)(5264)}{8}}{\sqrt{\left(567498 - \frac{(2108)^2}{8}\right)\left(3487562 - \frac{(5264)^2}{8}\right)}}$$

$$= \frac{-13528}{\sqrt{(12040)(23850)}} = \frac{-13528}{16945.619} = -0.798$$

Spearman () //

Spearman "

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)} \quad (4-7)$$

y x d
 $d = R_x - R_y$:
 (-)

: 10

		+		+	+	+	+		+	+
	+						+			

:

y x -
 : (-)

الرتب	1	2	3	4	5	6	7	8	9	10
تقديرات احصاء	+i	i	+ب	+ب	+ب	ب	+ج	+ج	+د	د
رتب x	1	2	(3+4+5)/3=4			6	(7+8)/2=7.5		9	10
تقديرات اقتصاد	+i	i	+ب	ب	ب	ب	-ج	-ج	-ج	د
رتب y	1	2	3	(4+5+6)/3=5			(7+8+9)/3=8			10

: $\sum d^2$:

x	y	x	y	d	d^2
	+	2	1	1	1
+		7.5	10	-2.5	6.25
		10	8	2	4
+		9	8	1	1
+		4	2	2	1
+		7.5	5	2.5	6.25
+	+	1	3	-2	4
		6	5	1	1
+		4	8	-4	16
+		4	5	-1	1
					44.5

$$\sum d^2 = 44.5$$

$$r = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(44.5)}{10(10^2 - 1)} = 1 - \frac{267}{990}$$

$$= 1 - 0.2697 = 0.7303$$

$$r = 0.703$$

($\sum d^2 = 148$:) : (-)

Simple Regression

/

:
•
•
•
•

//

:

$$y = \beta_0 + \beta_1 x + e$$

(5-7)

:

()

: y

()

: x

y

: β_0

x = 0

x

x

y

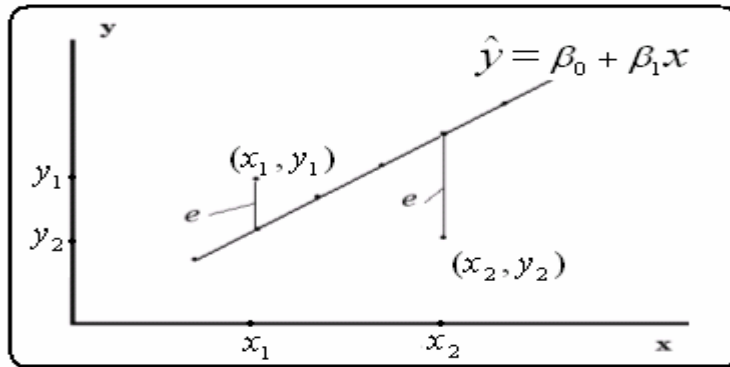
($\beta_0 + \beta_1 x$)

: β_1

y

: e

$$e = y - (\beta_0 + \beta_1 x) : \quad \hat{y} = \beta_0 + \beta_1 x$$



//

(-)

(β_1, β_0)

: $\sum e^2 = \sum (y - (\beta_0 + \beta_1 x))^2$

$$\hat{\beta}_1 = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2},$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

(7-7)

.x y " $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$:

(-)

.10

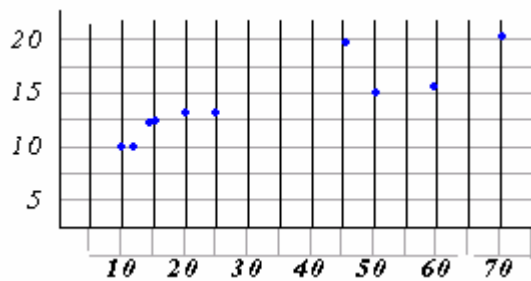
	10	11	14	15	20	25	46	50	59	70
	10	10	12	12	13	13	19	15	16	20

:
-
-
-
-
-

50

. ()

y



x

()

(-)

y x
:

x	y	x y	x ²
10	10	100	100
11	10	110	121
14	12	168	196
15	12	180	225
20	13	260	400
25	13	325	625
46	19	874	2116
50	15	750	2500
59	16	944	3481
70	20	1400	4900
320	140	5111	14664

$\sum x = 320$ $\sum y = 140$ $\sum xy = 5111$ $\sum x^2 = 14664$ $\bar{x} = \frac{\sum x}{n} = \frac{320}{10} = 32$ $\bar{y} = \frac{\sum y}{n} = \frac{140}{10} = 14$
--

: $\hat{\beta}_1$ (-) •

$$\hat{\beta}_1 = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} = \frac{(10)(5111) - (320)(140)}{(10)(14664) - (320)^2}$$

$$= \frac{6310}{44240} = 0.1426$$

: $\hat{\beta}_0$ (-) •

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 14 - (0.1426)(32) = 9.4368$$

:

$$\hat{y} = 9.44 + 0.143x$$

:

$$: \hat{\beta}_0 = 9.44$$

• 9.44

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$$: \hat{\beta}_1 = 0.143$$

143

0.143

$$: x = 50$$

$$\hat{y} = 9.44 + 0.143(50) = 16.59$$

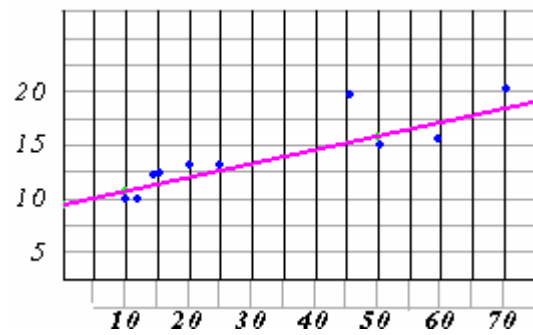
:

$$\hat{e}_{x=50} = y_{x=50} - \hat{y}_{x=50} = 15 - 16.59 = -1.59$$

x	50	10
\hat{y}	16.59	10.87

:

y



x