

Probabilities and its Applications

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Randomized Experiment

•

T " " H " " :
{H , T } :

Sample Space

•

S

: $n(S)$

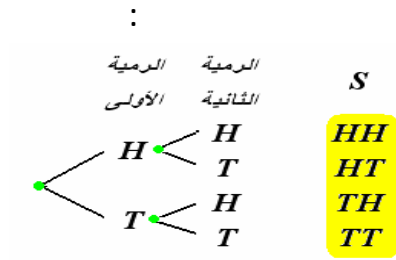
S: {H , T } :

-

. $n(S) = 2$:

()

-



$n(S) = 4$

$n(S) = 6 : S: \{1, 2, 3, 4, 5, 6\} :$

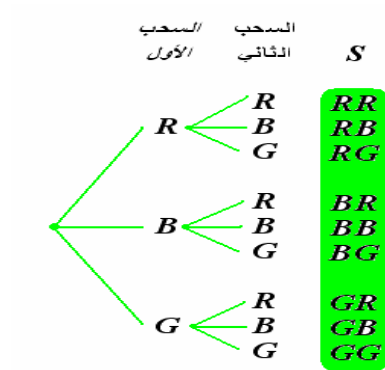
$n(S) = \infty : S: \{H, TH, TTH, TTTH, \dots\}$

(red)

:

(green)

(blue)



$n(S) = (10 \times 9) = 90 :$

.....

Event

[...,C

:

,B ,A]

:Simple Event -

:Component Event -

...,n(B) ,n(A)
A

S: {HH, HT, TH,

A: {HH}

B: {HT, TH, HH}

B

A

TT}

B

n(A)=1

n(B)=3

Union (\cup) •

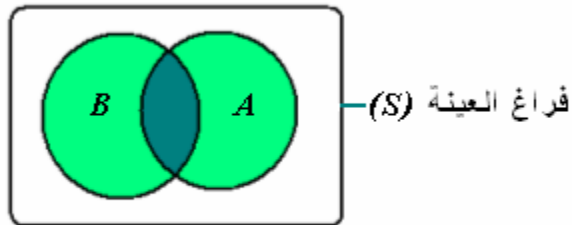
B , A

Ven. " "

(A or B) (A \cup B)

: Diagram

(-)



الجزء المظلل يعبر عن الاتحاد (A \cup B)

A

:

B

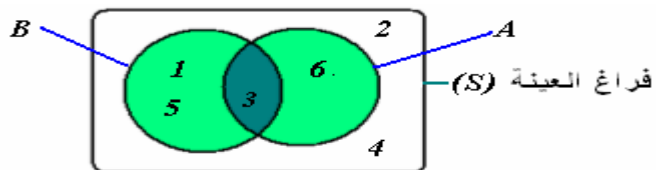
3

: B , A

B: {1,3,5}, A: {3,6}, S: {1,2,3,4,5,6}

: Ven

(A \cup B): {1,3,5,6}



(A \cup B): {1,3,5,6}

Intersection (\cap)

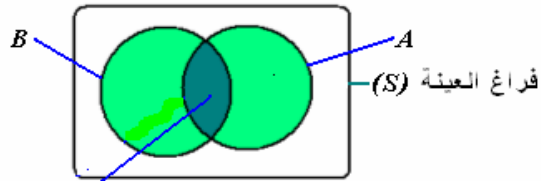
B, A

" "

(A and B) ($A \cap B$)

:

(-)



الجزء المشترك يعبر عن ($A \cap B$)

. ($A \cap B$): {3}

Mutually Exclusive events

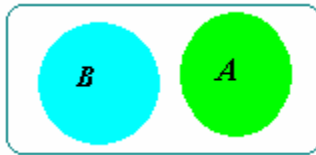
B, A

:

" "

$A \cap B = \phi$ ϕ

(-)



$(A \cap B) = \phi$

Compliment Event

A

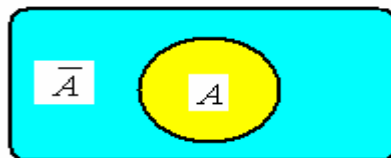
\bar{A}

A

:

$(A \cup \bar{A}) = S$, $(A \cap \bar{A}) = \phi$:

(-)



(-)

:

A
B
C

:

:

-

$$A \cup B, A \cup C, B \cup C, A \cup B \cup C$$

:

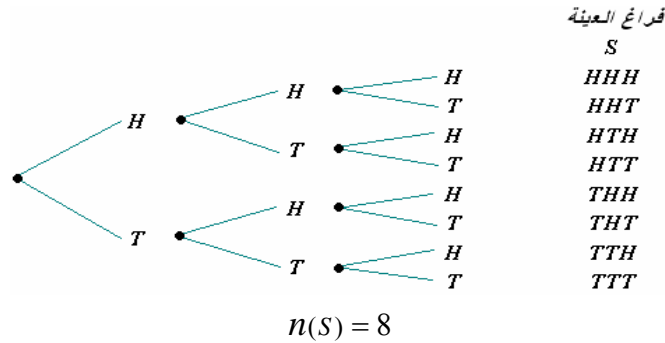
-

$$A \cap B, A \cap C, B \cap C, A \cap B \cap C$$

\bar{B}

-

:



:

$$A: \{HHT, HTH, THH\}, B: \{HTT, THT, TTH\}, C: \{HHH, HHT, HTH, HTT\}$$

$$n(A) = 3 \qquad n(B) = 3 \qquad n(C) = 4$$

:

-

$$(A \cup B): \{HHT, HTH, THH, HTT, THT, TTH\}, n(A \cup B) = 6$$

$$(A \cup C): \{HHT, HTH, THH, HHH, HTT\}, n(A \cup C) = 5$$

$$(B \cup C): \{HHH, HHT, HTH, HTT, THT, TTH\}, n(B \cup C) = 6$$

$$(A \cup B \cup C): \{HHH, HHT, HTH, HTT, THT, TTH, THH\}, n(A \cup B \cup C) = 7$$

$(A \cap B): \phi, n(A \cap B) = 0$

$(A \cap C): \{HHT, HTH\}, n(A \cap C) = 2$

$(B \cap C): \{HTT\}, n(B \cap C) = 1$

$(A \cap B \cap C): \phi, n(A \cap B \cap C) = 0$

$(\bar{B}): \{HHH, HHT, HTH, THH, TTT\}, n(\bar{B}) = 5$

: \bar{B} -
/

:

$P(A)$ A

: Empirical probability

:

$$P(A) = \frac{f(A)}{n}$$

(1-۷)

A

: $f(A)$ ()

n :

500

:

(Face)	H	T	SUM
	260	240	500

(-)

H

:

$$P(H) = \frac{f(H)}{n} = \frac{260}{500} = 0.52$$

:Theoretical Probability

:

$$P(A) = \frac{n(A)}{n(S)}$$

(۳-۷)

$n(A)$

$n(S)$:

$S: \{H, T\}$:

A

$A: \{H\}$

A

$n(S) = (2)^1 = 2$:

: A

$n(A) = 1$: A

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2} = 0.5$$

n

:

:

$$\lim_{n \rightarrow \infty} \frac{f(A)}{n} = \frac{n(A)}{n(S)}$$

(۳-۷)

(0.5)

:

$(1/n(S))$

(1/6)

$S: \{1,2,3,4,5,6\}$

:

$n(S) = 6^2 = 36$:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

.(1/36)

:

(R)

3/5

(W)

2/5



(1/4)

Probability Laws

:

Addition Law

:

$$\begin{aligned}
 P(A \cup B) &= \frac{n(A \cup B)}{n(S)} \\
 &= \frac{n(A) + n(B) - n(A \cap B)}{n(S)} \\
 &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} \\
 &= P(A) + P(B) - P(A \cap B)
 \end{aligned}$$

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:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(۳-۷)

: $P(A \cup B \cup C)$

C, B, A

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

(۴-۷)

:

$$P(A \cup B) = P(A) + P(B) ,$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

(۵-۷)

(-)

:

.10
.10
.10 7

-
-
-
-

:

:

	S					
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$n(S)=36$

:

A

-

$A: \{(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)\}, n(A)=6$

:

۱۳۴

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

∴ 10

B

-

$$B: \{(4,6) (5,5) (6,4)\}, n(B)=3$$

:

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(-)

10

(or)

-

:

$$P(A) = \frac{1}{6} \quad , \quad P(B) = \frac{1}{12}$$

:

10

(A ∩ B)

$$(A \cap B): \{(5,5)\}, n(A \cap B) = 1$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{6} + \frac{1}{12} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

B

7

C

-

∴ 10

$$B: \{(4,6) (5,5) (6,4)\}, n(B)=3 \quad , \quad C: \{(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)\}$$

$$n(B)=3$$

$$n(C)=6$$

$$P(B) = 3/36$$

$$P(C) = 6/36$$

(-)

C, B

:

$$P(B \cup C) = P(B) + P(C) = \frac{3}{36} + \frac{6}{36}$$

$$= \frac{9}{36} = \frac{1}{4}$$

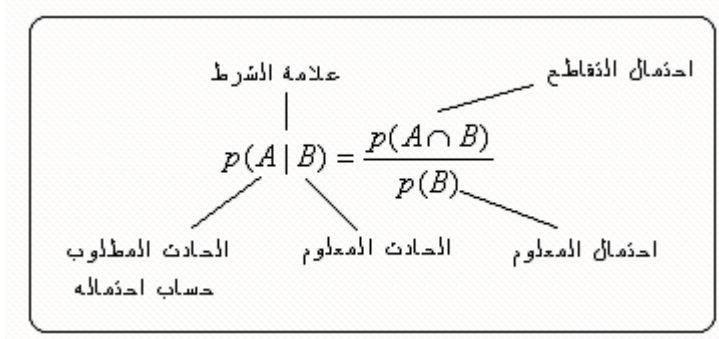
Conditional probability

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$$p(A|B) = \frac{p(A \cap B)}{p(B)} \quad (٦-٧)$$

$$p(B|A) = \frac{p(A \cap B)}{p(A)} \quad (٧-٧)$$

(-) (-)



(-)

				Sum
	15	5	10	30
	8	17	10	35
	12	10	13	35
Sum	35	32	33	100

	B		A		
	A_1	A_2	A_3	<i>Sum</i>	
B_1	15	5	10	30	
B_2	8	17	10	35	
B_3	12	10	13	35	
<i>Sum</i>	35	32	33	100	

$A \cap B$

$$P(B_1 \cap A_2) = \frac{f(B_1 \cap A_2)}{n} = \frac{5}{100} = 0.05$$

$$\begin{aligned} P(A_1 \cup B_2) &= p(A_1) + P(B_2) - P(A_1 \cap B_2) \\ &= \frac{35}{100} + \frac{35}{100} - \frac{8}{100} = \frac{62}{100} = 0.62 \end{aligned}$$

$$\begin{aligned} P(B_2 \cup B_3) &= p(B_2) + P(B_3) \\ &= \frac{35}{100} + \frac{35}{100} = \frac{70}{100} = 0.70 \end{aligned}$$

A_3

"

:

B_2

$$p(A_3 | B_2) = \frac{p(A_3 \cap B_2)}{p(B_2)} = \frac{\left(\frac{10}{100}\right)}{\left(\frac{35}{100}\right)} = \frac{10}{35}$$

:

. (C, B, A) :

60	24	36	A
123	63	60	B
87	33	54	C
270	120	150	

:

B

-

-

C

-

A

-

A

-

Probability Multiplying Law

•

B, A

:

$$P(A \cap B)$$

or

$$P(A \cap B) = P(B) P(A|B)$$

$$P(A \cap B) = P(A) P(B|A)$$

(A-V)

(-)

60%

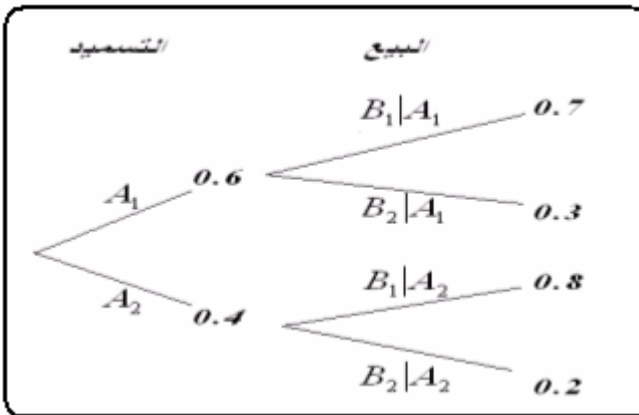
80%

70%

:

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-
-
-

{ (A₂) (A₁) } :
 { (B₂) (B₁) } :



:

$$P(A_1) = 0.6$$

$$P(B_1|A_1) = 0.7$$

: (-) (B₁ and A₁)

$$\begin{aligned} P(A_1 \cap B_1) &= P(A_1) P(B_1|A_1) \\ &= (0.6)(0.7) = 0.42 \end{aligned}$$

$$\begin{aligned}
 & : \\
 P(A_2 \cap B_1) &= P(A_2) P(B_1|A_2) \\
 &= (0.4)(0.8) = 0.32
 \end{aligned}$$

Independent Events

B, A

$$: P(A \cap B)$$

$$P(A \cap B) = P(A) P(B)$$

(۹-۷)

B, A

(-)

75%

60%

: 50%

"

B "

"

A

:

"

$$P(A) = 0.6, P(B) = 0.75, P(A \cap B) = 0.5$$

:

$$\begin{aligned}
 P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 &= (0.6) + (0.75) - 0.5 = 0.85
 \end{aligned}$$

:

$$P(\bar{B}) = 1 - P(B) = 1 - 0.75 = 0.25$$

(-)

$$P(A \cap B) = 0.5, P(A) P(B) = (0.6)(0.75) = 0.45$$

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$$(A) \quad P(A \cap B) \neq P(A) P(B) :$$

.(B)

(-)

$$P(B) = 0.5, P(A) = 0.6$$

B, A

. $P(A \cup B)$

:

: B, A

$$\begin{aligned} P(A \cap B) &= P(A) P(B) \\ &= (0.6)(0.5) = 0.3 \end{aligned}$$

: $P(A \cup B)$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.6 + 0.5 - 0.3 = 0.8 \end{aligned}$$