Central Tendency

/

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.

/

Arithmetic Mean / /

:

:

n .

$x_1, x_2, ..., x_n$: .

$\overline{x}$:

$$\text{الوسط الحسابي} = \frac{\text{مجموع القيم}}{\text{عدد القيم}}$$

(٣-١)

$$\overline{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\sum_{i-1}^{n} x_i}{n}$$

Σ

(-)

8

34   32   42   37   35   40   36   40

(-) :

$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}$$

$$= \frac{34 + 32 + 42 + 37 + 35 + 40 + 36 + 40}{8} = \frac{296}{8} = 37$$

37

:

k $x_1, x_2, ..., x_k$

$f_1, f_2, ..., f_k$:

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + ... + x_k f_k}{f_1 + f_2 + ... + f_k} = \frac{\sum_{i=1}^{k} x_i f_i}{\sum_{i=1}^{k} f_i} \qquad (\text{٣-٢})$$

(-)

40

	32-34	34-36	36-38	38-40	40-42	42-44
	4	7	13	10	5	1

.

(-)   :

-                          $\sum f$.   -   . x

-                                    (xf) $\sum xf$

-                                    . (-)

(C)	f	x	x f
32-34	4	(32+34)÷2=33	4×33=132
34-36	7	35	7×35=245
36-38	13	37	13×37=481
38-40	10	39	10×39=390
40-42	5	41	5×41=205
42-44	1	43	1×43=43
	40		1496

:

$$\overline{x} = \frac{\sum\limits_{i=1}^{6} x_i f_i}{\sum\limits_{i=1}^{6} f_i} = \frac{1496}{40} = 37.4 \ k.g$$

37.4   k.g

:

-                                    x :

$x : a , a ,..., a$:

$$\overline{x} = \frac{a + a + ... + a}{n} = \frac{na}{n} = a$$ (٣−٣)

63   5

:

$$\bar{x} = \frac{63+63+63+63+63}{5} = \frac{315}{5} = 63 \ k.g$$

.   -

$$\boxed{\sum (x - \bar{x}) = 0}$$ (٣-٤)

:   (-)

: $\bar{x} = 37$ 34, 32, 42, 37, 35, 40, 36, 40

x	34	32	42	37	35	40	36	40	296
$(x - \bar{x})$	34-37	32-37	42-37	37-37	35-37	40-37	36-37	40-37	0
$(x - 37)$	-3	-5	5	0	-2	3	-1	3	

$\sum (x - 37) = 0$:

(   )   -

:   .   (   )

(a) $x_1, x_2, ..., x_n$

: (   ) y : $y = x + a$ y

$$\boxed{\bar{y} = \bar{x} + a}$$ (٣-٥)

$\bar{y}$

. (-)

5

{(37+5)=42}   .

x	34	32	42	37	35	40	36	40	296
$y = (x + 5)$	34+5	32+5	42+5	37+5	35+5	40+5	36+5	40+5	336
	39	37	47	42	40	45	41	45	

$$\sum y = 336 :$$

:

$$\bar{y} = \frac{\sum y}{n} = \frac{336}{8} = 42 \quad \rightarrow (\bar{x} + 5 = 37 + 5 = 42)$$

- (a)   )

(   )   .

$$y = a \; x :$$ y :

$$\boxed{\bar{y} = a \; \bar{x}}$$ (٣-٦)

.

50   100

(a=2) $$\bar{y} = a \; \bar{x} = 2(37) = 74 :$$

-   :

$$\boxed{\sum (x - \bar{x})^2 \langle \sum (x - a)^2 \; if \; a \neq \bar{x}}$$ (٣-٧)

$$\sum (x - 37)^2 < \sum (x - a)^2 :$$ $a \neq 37$

:

.

	1	2	3	4	5	sum
( ) x	23	40	36	28	46	173
( ) w	1	3	3	2	4	

:

$$\bar{x} = \frac{\sum x}{n} = \frac{23+40+36+28+46}{5} = \frac{173}{5} = 346$$

x

w

:

$$(\bar{w}) = \frac{\sum xw}{\sum w} = \frac{23 \times 1 + 40 \times 3 + 36 \times 3 + 28 \times 2 + 46 \times 4}{1+3+3+2+4}$$

$$= \frac{23+120+108+56+184}{13} = \frac{491}{13} = 37.769$$

.

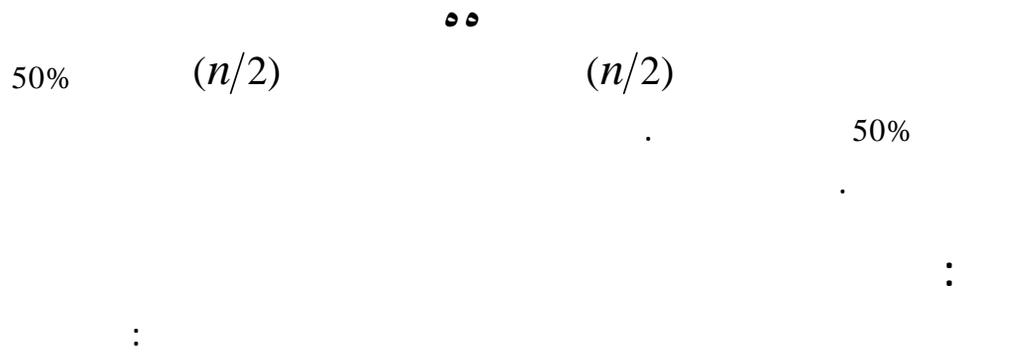$(\bar{w})$

:

$$\boxed{(\bar{w}) = \frac{\sum xw}{\sum w}}$$ (٣−٨)

:

- .
- .
- .
- .
- .
- .
- .

/ /   Median

٥٥

50%　　　　　$(n/2)$　　　　　　　　$(n/2)$

50%　　　　　　　　　　　．

．

:

:

．　　　　　●

$$\left(\frac{n+1}{2}\right) =$$ 　　　: 　　　　●

:　　　　　　　(n)　　　●

$$\boxed{\text{الوسيط} = \text{القيمة رقم} \left(\frac{n+1}{2}\right)}$$ 　　　(٣-٩)

$(n/2)$　　　　　　　　　　(n)　　　●

:　　　　　　　　　$((n/2)+1)$
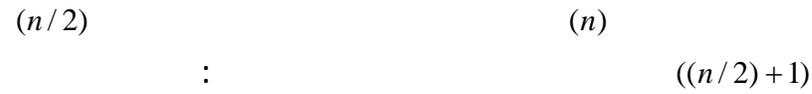
$$\boxed{\text{الوسيط} = \frac{\text{القيمة رقم}\left(\frac{n}{2}\right) + \text{القيمة رقم}\left(\frac{n}{2}+1\right)}{2}}$$ 　　　(٣-١٠)

(-)

17

(b)　　　　　　　7　　　　(a)　　:

/　　　　　　　　　　　　　　　　　　10

:

| (a) | 1.2 | 2.75 | 3.25 | 2 | 3 | 2.3 | 1.5 | | | |
| (b) | 4.5 | 1.8 | 3.5 | 3.75 | 2 | 2.5 | 1.5 | 4 | 2.5 | 3 |

．

: _____

• :

(a)

قيمة الوسيط

الإنتاج	1.2	1.5	2	2.3	2.75	3	3.25
الرتبة	1	2	2	4	5	6	7

رتبة الوسيط

• $(n = 7)$

• $\big((n+1)/2 = (7+1)/2 = 4\big)$.

• 4 a :

/ $Med_a = 2.3$

: _____

• .

(b) :

قيمة الوسيط $= \dfrac{2.5 + 3}{2}$

2.75

الإنتاج	1.5	1.8	2	2.5	2.5	3	3.5	3.75	4	4.5
الرتبة	1	2	3	4	5	5.5	6	7	8	9	10

رتبة الوسيط

• $(n = 10)$

• $\big((n+1)/2 = (10+1)/2 = 5.5\big)$.

• $=$ (5 6) .

/ $Med_b = \dfrac{2.5 + 3}{2} = 2.75$

(b) (a)

: $Med_b > Med_a$.

:

.

.

•
•
•

$$\left(\frac{n}{2}\right) = \left(\frac{\sum f}{2}\right) :$$

:

f_1 (A)

$(n/2)$ $\longleftarrow$ Med

f_2

•   .

$$\boxed{Med \; = \; A + \frac{\frac{n}{2} - f_1}{f_2 - f_1} \times L}$$ (٣-١١)

:
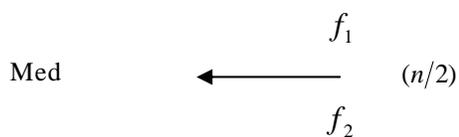
L :

=   −

$L = \; Upper \text{ - } Lower$

(-)

50

	1.5 -	4.5 -	7.5 -	10.5 -	13.5 – 16.5
f	4	12	19	10	5

:   :   -   -

:

•   : $$\frac{n}{2} = \frac{\sum f}{2} = \frac{50}{2} = 25$$

•   :

أقل من	تكرار متجمع صاعد
1.5	0
4.5	4
A 7.5	f_1 16
Med (الوسيط)	25
10.5	f_2 35
13.5	45
16.5	50

——— رتبة الوسيط

•   : $(n/2)$

$(n/2)$

(25) $(35 , 16)$

7.5

10.5   .   :

$(7.5-10.5)$.

•   (-)   :

$$A = 7.5 \ , \ f_1 = 16 \ , \ f_2 = 35 \ , \ L = 10.5 - 7.5 = 3$$

:

$$Med = A + \frac{\frac{n}{2} - f_1}{f_2 - f_1} \times L = 7.5 + \frac{25 - 16}{35 - 16} \times 3$$

$$= 7.5 + \frac{9}{19} \times 3 = 7.5 + \frac{27}{19} = 7.5 + 1.421 = 8.921 \ k.g$$

:

•   .
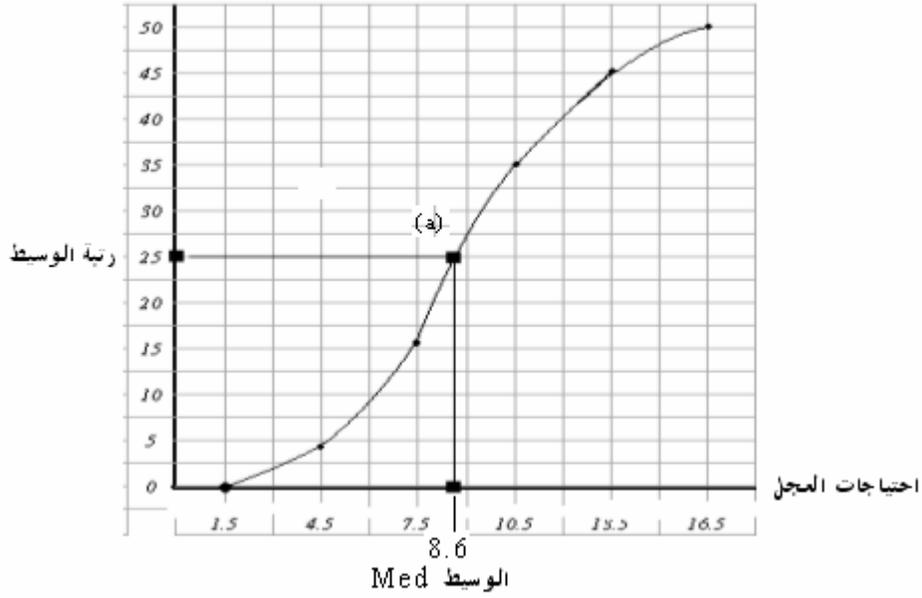
- (25) .

(a) .

- (a) .
- .
- Med = 8.6 .

- .
- .
- .

$$: \qquad \sum | x - Med | \le \sum | x - a | \ , \ a \ne Med$$

- .

- nominal

Mode / /

( ) :

$$(٣-١٢) \quad \boxed{\text{المنوال (Mod) = القيمة (المستوى) الأكثر تكرارا}}$$

: ( )

$$\boxed{Mod = A + \frac{d_1}{d_1 + d_2} \times L}$$

(٣-١٣)

:

A : ( ) .

d_1 : = (–)

d_2 : = (–)

L : .

=

d_1 = (تكرار فئة المنوال – تكرار سابق) ← تكرار سابق ← تكرار فئة المنوال

d_2 = (تكرار فئة المنوال – تكرار سابق) ← تكرار لاحق

(-)

:

	80	77	75	77	77	77	65	70	58	67
	88	68	60	75	93	65	77	85	95	90
	80	65	69	80	65	88	76	65	86	80
	85	73	69	85	73	69	69	73	72	85

٦١

:

:

=

.

	77 4	= 77
	65 3 80 3	: 65 = 80 =
	69 3 73 3 85 3	: 69 = 73 = 85 =

(-)

30   .

	2 -	5 -	8 -	11 -	14 - 17
f	4	7	10	5	4

.

(-)   :   •

(8-11) :

التكرارات	الفئات
4	2 -
7	5 -	$d_1 = 10 - 7 = 3$
10	8 -	فئة المنوال ، $A = 8$ ، أكبر تكرار
5	11 -	$d_2 = 10 - 5 = 5$
4	14 - 17

d   :   •

$$d_1 = (10 - 7) = 3 \quad d_2 = (10 - 5) = 5$$

$(A = 8)$ •

$(L = 3)$ •

: .

$$Mod = A + \frac{d_1}{d_1 + d_2} \times L$$

$$= 8 + \frac{3}{3 + 5} \times 3 = 8 + 1.125 = 9.125$$

/

:

(-)

منحنى سالب الالتواء	منحنى موجب الالتواء	منحنى متماثل
الوسط > الوسيط > المنوال	الوسط < الوسيط < المنوال	الوسط = الوسيط = المنوال

- :
  = = .
- ( ) :
  < <
- ( ) :
  > >

(-)
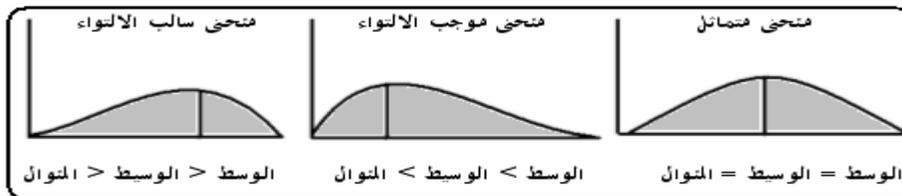
5 10

:

115 123 119 123 124 119 123 121 123 121

_____ : .

:

$$\overline{x} = \frac{\sum x}{n} = \frac{1211}{10} = 121.1$$

- :

  : $(n+1)/2 = (10+1)/2 = 5.5$

					قيمة الوسيط 122				
الطاقة	115	119	119	121	121 123	123	123	123	124
الرتبة	1	2	3	4	5 5.5 6	7	8	9	10
					رتبة الوسيط				

10 = . = (5 , 6)

$$Med = \frac{121+123}{2} = \frac{244}{2} = 122$$

- :

  : 123

$$Mod = 123$$

:

121.1   122   123
المنوال الوسيط الوسط

:   >   >   .

(-)
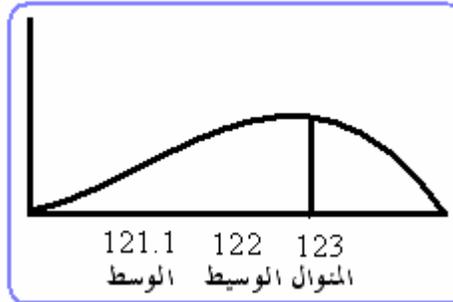
100 .

	50 -	70 -	90 -	110 -	130 -	150 -	170 - 190
	8	15	28	20	15	8	6

:

• .
• .

• .

: _____ $\overline{x}$

	(f)	(x)	f x
50 – 70	8	60	480
70 – 90	15	80	1200
90 – 110	28	100	2800
110 - 130	20	120	2400
130 - 150	15	140	2100
150 – 170	8	160	1280
170 - 190	6	180	1080
	100		11340

$$\overline{x} = \frac{\sum fx}{\sum f} = \frac{11340}{100} = 113.4 \ R.S$$

$\underline{\quad Med}$:

$(n/2 = 100/2 = 50)$:

.

50	0
70	8
90	$23 \leftarrow f_1$
110	$51 \leftarrow f_1$
130	71
150	86
170	94
190	100

$(50 \ )$

:

$$\frac{n}{2} = 50 \ , f_1 = 23 \ , f_2 = 51 \ , A = 90 \ , L = 110 - 90 = 20$$

:

$$Med = A + \frac{\dfrac{n}{2} - f_1}{f_2 - f_1} \times L = 90 + \frac{50 - 23}{51 - 23} \times 20$$

$$= 90 + \frac{27}{28} \times 20 = 90 + \frac{540}{28} = 90 + 19.286 = 109.3 \ R.S$$
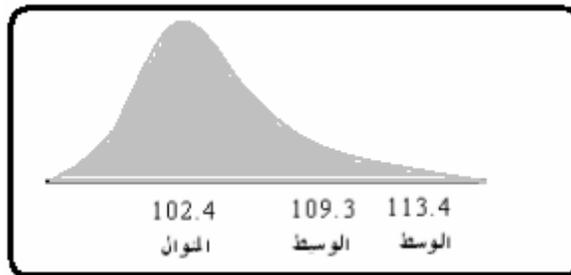
$\underline{\quad Mod}$:

$28 =$ $(90 - 110)$.

: $d_1 = 28 - 15 = 13$, $d_2 = 28 - 20 = 8$

$A = 90$: $L = 110 - 90 = 20$:

:

$$Mod = A + \frac{d_1}{d_1 + d_2} \times L = 90 + \frac{13}{13+8} \times 20 = 90 + \frac{260}{21} = 1024 \ R.S$$

● .

:

$Mod = 1024$: $Med = 109.3$: $\overline{x} = 113.4$:

: <   <   ._____

:

Quartiles   /

:

● :   25%

Q_1.

● :   50%

Q_2

.

● :   75%

Q_3.

(-) .

(-)

:

• n : :

$$X_{(1)} < X_{(2)} < X_{(3)} \qquad < X_{(n)}$$
$$1 \qquad 2 \qquad 3 \qquad n$$

• i (Q_i) : $R = (n+1) \times \left(\dfrac{i}{4}\right)$

• R : $Q_i = X_{(R)}$.

• R (Q_i) : $X_{(l)} < Q_i < X_{(u)}$

(Q_i) :

$$\boxed{Q_i = x_{(l)} + (R - l)(x_{(u)} - x_{(l)})} \tag{٣-١٤}$$

(-)

10

:

25 23 29 32 34 29 20 18 27 30

:

:

• :

	22.25				28				30.5	
	18	20	23	25	27	29	29	30	32	34
	1	2	3	4	5	6	7	8	9	10
	2.75			5.5				8.25		

• (Q_1) :

$$R = (n+1) \times \left(\frac{i}{4}\right) = (10+1) \times \left(\frac{1}{4}\right) = 2.75 \quad :$$

: (-) $(20 < Q_1 < 23)$:

$$l = 2, \ R = 2.75 \ , x_{(l)} = 20 \ . x_{(u)} = 23$$

:

$$Q_1 = x_{(l)} + (R - l) \times (x_{(u)} - x_{(l)}) = 20 + 0.75(23 - 20) = 22.25$$

• ( ) Q_2

$$R = (n + 1) \times \left(\frac{i}{4}\right) = (10 + 1) \times \left(\frac{2}{4}\right) = 5.5 \quad :$$

: (-) $(27 < Q_2 < 29)$:

$$l = 5, \quad R = 5.5 \quad , x_{(l)} = 27 \quad . x_{(u)} = 29$$

:

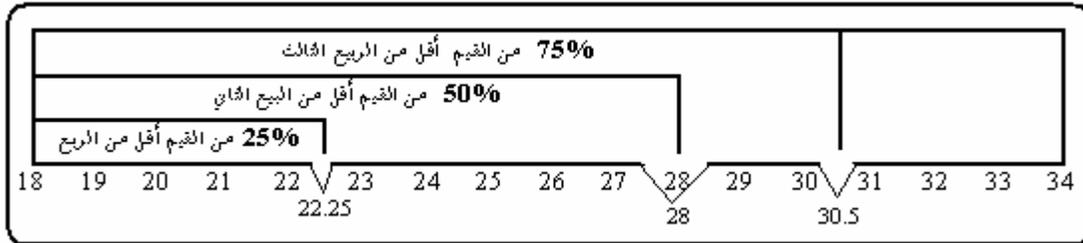$$Q_2 = x_{(l)} + (R - l) \times (x_{(u)} - x_{(l)}) = 27 + 0.5(29 - 27) = 28$$

• Q_3

$$R = (n + 1) \times \left(\frac{i}{4}\right) = (10 + 1) \times \left(\frac{3}{4}\right) = 8.25 \quad :$$

: (-) $(30 < Q_3 < 32)$:

$$l = 8, \quad R = 8.25 \quad , x_{(l)} = 30 \quad . x_{(u)} = 32$$

:

$$Q_3 = x_{(l)} + (R - l) \times (x_{(u)} - x_{(l)}) = 30 + 0.25(32 - 30) = 30.5$$

:

• 25% 22.25 .

• 50% 28 .

• 75% 30.5 .

:

: (x) 10

x: 342   216   105   291   107   216 210   165 90 216

-   :

(a)   (b)   (c)   (d)

-    $\sum x$:

(a) 1000   (b) 1958   (c) 195.8   (d) 216

-                              50%   :

(a)   (b)   (c)   (d)

-                              :

(a)   (b)   (c)   (d)

-                                   :

(a) 216   (b) 1958   (c) 195.8   (d) 213

-

(a) 216   (b) 1958   (c) 195.8   (d) 347

-

(a) 213   (b) 1958   (c) 195.8   (d) 216

-

(a)   (b)   (c)
                                                      (d)   .

-   50

.

(a) 216   (b) 1958   (c) 195.8   (d) 245.8

-                    $y = 0.5x$ y :

(a) 216   (b) 97.9   (c) 195.8   (d) 245.8

: 50

.

	4.5 –	7.5 –	10.5 -	13.5 -	16.5-	19.5 – 22.5
	3	8	12	15	10	2

(-)

-

(a) 1   (b) 2   (c) 3   (d) 5

-

(a) 14.5   (b) 16   (c) 15   (d) 13.5

-

(a) 9   (b) 8   (c) 10   (d) 3

- :

(a) 0.30   (b) 0.20   (c) 1   (d) 1.50

- x   f   ∑ fx

(a) 225   (b) 225   (c) 50   (d) 681

-

(a) 8.33   (b) 13.5   (c) 13.62   (d) 681

- :

(a) 13.5 –   (b) 16.5- 19.5   (c) 14 –   (d) 10.5 – 13.5
16.5   17

- :

(a) 50   (b) 10   (c) 25   (d) 1

- .

(a) 13.9   (b) 13.5   (c) 15   (d) 12.5

- :

14.625 (d)   13.5 (c)   15 (b)   14 (a)

-   .

(a)   (b)   (c)   (d)

:   :

:   :

:   (–)

	(a)	(b)	(c)	(d)