

5) c) let us evaluate

$$\int_0^1 (x+1)^2 \ln(x+1) dx$$

put $u = x+1$, $du = dx$

$$\int (x+1)^2 \ln(x+1) dx = \int u^2 \ln u du. \text{ Integrate by parts}$$

$$\int u^2 \ln u du = \frac{u^3}{3} \ln u - \int \frac{u^3}{3} \frac{1}{u} du$$

$$= \frac{u^3}{3} \ln u - \frac{1}{3} \frac{u^3}{3} + C$$

we finally get

$$\int_0^1 (x+1)^2 \ln(x+1) dx = \left[\frac{(x+1)^3}{3} \ln(x+1) - \frac{1}{9} (x+1)^3 \right]_0^1$$

$$= \frac{8}{3} \ln 2 - \frac{8}{9} - \left(-\frac{1}{9} \right)$$

$$= \frac{8}{3} \ln 2 - \frac{7}{9}$$