

Question 1:

(2 + 3 + 2)

(a) Find the derivative of the function $f(x) = (\tan x + \frac{1}{x})^2 \cos 2x$.

(b) Find an equation of the normal line to the graph of the function $f(x) = (4x - 1)^2(x + 1)^3$ at $x = 0$.

(c) Find the derivative y' , if $\sin(x + y) = y$.

Question 2:

(3 + 4)

(a) If $f(x) = \sin^2 x$, then show that $f'''(\frac{\pi}{4}) = -4$.

(b) Let $f(x) = 4x^5 - 5x^4$ be a function.

(i) Find all critical numbers of the function.

(ii) Find intervals over which the given function is increasing and decreasing.

(iii) Locate local extrema using the first derivative test.

(iv) Sketch the graph of the given function.

Question 3:

(3 + 3)

(a) A rectangle has a perimeter of 12 meters. Find the dimensions of the rectangle whose area is as large as possible.

(b) Evaluate the integral $\int \frac{\cos(\sqrt{x} + 1)}{\sqrt{x}} dx$.