

(6) لتكن $z(x, y)$ دالة في $C^2(\mathbb{R}^2)$ تحقق

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} - 3 \frac{\partial^2 z}{\partial y^2} = 0$$

ضع $z(x, y) = f(u, v)$ بحيث $\begin{cases} u = x - y \\ v = 3x + y \end{cases}$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 f}{\partial u^2} + 6 \frac{\partial^2 f}{\partial u \partial v} + 9 \frac{\partial^2 f}{\partial v^2} \quad \text{(i) بين أن}$$

$$\frac{\partial^2 z}{\partial y^2} \quad \text{و} \quad \frac{\partial^2 z}{\partial x \partial y} \quad \text{(ii) احسب}$$

$$\frac{\partial^2 f}{\partial u \partial v} = 0 \quad \text{و أن} \quad \text{(iii) استنتج أن}$$

$$z(x, y) = \psi(x - y) + \psi(3x + y) \quad \text{حيث } \psi \text{ و } \psi \text{ في } C^2(\mathbb{R})$$