$$D^2 f_2(1,1,1)\big((-1,1,1),(1,1,-1)\big) = 4$$

$$D^2 f_3(1,1,1)\big((-1,1,1),(1,1,-1)\big) = -4$$

نستنتج أن

$$D^2 f(1,1,1)\big((-1,1,1),(1,1,-1)\big) = (2,4,-4)$$

السؤال ③

$f(x,y) = \sin xy$ $\quad a = (-2, \frac{\pi}{4})$

لدينا مستور تايلور من الدرجة الثانية للدالة f عند a

$$\frac{\partial f}{\partial x} = y\cos xy \qquad \frac{\partial f}{\partial x}(-2,\pi/4) = 0$$

$$\frac{\partial f}{\partial y} = x\cos xy \qquad \frac{\partial f}{\partial y}(-2,\pi/4) = 0$$

$$\frac{\partial^2 f}{\partial x^2} = -y^2 \sin xy \qquad \frac{\partial^2 f}{\partial x^2}(-2,\frac{\pi}{4}) = \frac{\pi^2}{16}$$

$$\frac{\partial^2 f}{\partial x \partial y} = -xy\sin xy + \cos xy \qquad \frac{\partial^2 f}{\partial x \partial y}(-2,\frac{\pi}{4}) = \frac{\pi}{2}$$

$$\frac{\partial^2 f}{\partial y^2} = -x^2 \sin xy \qquad \frac{\partial^2 f}{\partial y^2}(-2,\frac{\pi}{4}) = 4$$

ومنه

$$P_2(x,y) = f(-2,\tfrac{\pi}{4}) + \frac{\partial f}{\partial x}(-2,\tfrac{\pi}{4})(x+2) + \frac{\partial f}{\partial y}(-2,\tfrac{\pi}{4})(y-\tfrac{\pi}{4}) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(-2,\tfrac{\pi}{4})(x+2)^2 + \frac{\partial^2 f}{\partial x \partial y}(-2,\tfrac{\pi}{4})(x+2)(y-\tfrac{\pi}{4}) + \frac{1}{2}\frac{\partial^2 f}{\partial y^2}(y-\tfrac{\pi}{4})^2$$

$$P_2(x,y) = -1 + \frac{\pi^2}{32}(x+2)^2 + \frac{\pi}{2}(x+2)(y-\tfrac{\pi}{4}) + 2(y-\tfrac{\pi}{4})^2$$