

Question 1:

(a) Solve the inequality and express your answer in terms of inequality $\frac{x+1}{3} \geq 2 + \frac{2}{x}$. (3 + 4 + 3)

(b) Find the domain of $\frac{f(x)}{g(x)}$ and $(f \circ g)(1)$, where

$$f(x) = \frac{3x}{x-1} \quad \text{and} \quad g(x) = \frac{x}{x-3}$$

(c) Test the continuity of the function at $x = 1$.

$$f(x) = \begin{cases} \sqrt{\frac{x-1}{x^2-1}}, & \text{if } x \neq 1 \\ \frac{1}{2}\sqrt{2}, & \text{if } x = 1. \end{cases}$$

Question 2:

(a) Use the definition of the derivative to find $f'(x)$ of the function $f(x) = \frac{1}{2x+1}$. (3 + 4 + 3)

(b) Use implicit differentiation to find y' for the function $x + y^3 = y^5 - x^2 + 2y$ and find an equation of the tangent line to the graph of this equation at the point $(1, 1)$.

(c) Find the limit, if it exists

$$\lim_{x \rightarrow 0} \left(\frac{(1+x)\sin x - \cos x \sin x}{x^2} \right)$$

Question 3:

(a) Find $\frac{dy}{dx}$ if $y = \sin^3 3x \cos^2 2x$. (3 + 4 + 3)

(b) Let $f(x) = x^2 - 6x + 9$ be a function

(i) Find critical number of the function.

(ii) Find intervals over which the given function is increasing and decreasing.

(iii) Locate local extrema of the given function.

(iv) Sketch the graph of the given function.

(c) Find the dimensions of a rectangle with perimeter 100 meter whose area is as large as possible.

Question 1:

(2 + 3 + 3 + 3)

(a) Find the first derivative of the function $f(x) = (2x - 5)^4(8x^2 - 5)^{-3}$ at $x = 0$.

(b) Find the second derivative of the function $f(x) = x^2 \cos\left(\frac{1}{x}\right)$.

(c) Find the equation of the tangent line to the graph of the equation $x^3 + y^3 = 6xy$ at the point $(3, 3)$.

(d) Find the absolute extrema of $f(x) = x - 2 \sin x$ on $[0, 2\pi]$.

Question 2:

(4 + 2 + 3)

(a) Let $f(x) = x^3(1 - 12/x^2 + 1/x^3)$ be a function, then

(i) Find local extrema of $f(x)$ using second derivative test.

(ii) Find the intervals where the graph of $f(x)$ is concave upward and concave downward.

(iii) Find the point of inflection in the graph of $f(x)$.

(iv) Sketch the graph of the given function.

(b) A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has largest area.

(c) Evaluate the integral $\int \sqrt{1 + x^2} x^5 dx$.

Question 1: (4 + 3 + 3)

(a) Use implicit differentiation to find y' for the equation

$$x + y^3 = y^5 - x^2 + 2y$$

and find an equation of the tangent line to the graph of this equation at the point (1, 1).

(b) Find $\frac{dy}{dx}$ if $y = \sin^5 4x \cos^3 2x$.

(c) Find the third derivative of $f(x) = \sqrt{1 - x^2}$.

Question 2: (3 + 4 + 3)

(a) Find the absolute extrema of $f(x) = \sqrt{8 - 2x - x^2}$ on $[-3, 2]$.

(b) Let $f(x) = x^2 - 6x + 9$ be a function

(i) Find a critical number of the function.

(ii) Find local extrema of $f(x)$ using first derivative test.

(iii) Find the intervals where the function $f(x)$ is increasing and decreasing.

(iv) Sketch the graph of the given function.

(c) Find a positive number such that the sum of the number and its reciprocal is as small as possible.

Question 1: (4 + 3 + 3)

(a) Solve the inequality and express your answer in terms of interval $\frac{x+1}{3} \leq 2 + \frac{2}{x}$.

(b) Given $f(x) = \sqrt{x-4}$ and $g(x) = x+1$, find $(f \circ g)(4)$ and the domain of $(f \circ g)(x)$.

(c) Find the following limit, if it exists

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1} - 1}$$

Question 2: (4 + 3 + 3)

(a) Find the vertical and horizontal asymptotes to the graph of the function

$$f(x) = \frac{x^2 + 1}{x^2 - 4}$$

(b) For what value of k , the following function is continuous for all x ?

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x \leq 1 \\ k(x+1), & \text{if } x > 1. \end{cases}$$

(c) Apply the definition of the derivative to find $f'(x)$ where

$$f(x) = \frac{1}{2x+1}$$

Also, find the equation of the tangent line to the graph of this function at $x = -1$.

Question 1:

(3 + 4 + 3)

(a) Solve the inequality and express your answer in terms of intervals

$$\frac{2x + 3}{x^2 + 5x + 6} \geq 0.$$

(b) Find $(f \circ g)(1)$ and $(g \circ f)(1)$, where

$$f(x) = x + \frac{1}{x} \quad \text{and} \quad g(x) = \frac{x + 1}{x + 2}.$$

(c) Determine the value of k such that the given function is continuous on $(-\infty, \infty)$

$$f(x) = \begin{cases} 5x + 2, & \text{if } x \leq 1 \\ kx^2, & \text{if } x > 1. \end{cases}$$

Question 2:

(4 + 3 + 3)

(a) Find the following limits, if exist

$$(i) \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}, \quad (ii) \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + x + 4} - 2}{x^2 + x}.$$

(b) Find the horizontal asymptote and the vertical asymptote for the graph of the function

$$f(x) = \frac{x^2 + 2}{x^2 - 1}.$$

(c) Use the definition of the derivative to find $f'(x)$ of the function $f(x) = \frac{1}{x+1}$; and find the equation of the tangent line to the graph of this function at the point $P(1, \frac{1}{2})$.

Question 1:

(2 + 3 + 2 + 3)

(a) Find the equation of the tangent line to the graph of the function

$$f(x) = (x^2 + 2)(x^3 + 1) \text{ at } x = 1.$$

(b) Find the derivative of the function $f(x) = \sin^2(2x - 1)^{3/2}$.

(c) Find the second derivative of $f(x) = 5\sqrt{x^3} + \frac{2}{\sqrt[3]{x}}$.

(d) Use implicit differentiation to find y' at the point (1,0) if $y + \cos(xy^2) + 3x^2 = 4$.

Question 2:

(3 + 4 + 3)

(a) Find the absolute extrema of $f(x) = 6\sqrt{x} - 4x$ on $[1,4]$.

(b) Let $f(x) = x^3 - 3x + 1$ be a function

(i) Find local extrema of $f(x)$ using second derivative test.

(ii) Find the intervals where the graph of $f(x)$ is concave upward and concave downward.

(iii) Find the point of inflection on the graph of $f(x)$.

(iv) Sketch the graph of the given function.

(c) Find two numbers whose product is -16 and the sum of whose squares is a minimum.

Question 1:

(2 + 3 + 3 + 3)

(a) Solve the inequality and express your answer in terms of interval $|\sqrt{x-1} - 2| \leq 1$.

(b) Given $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{\sqrt{x+2}}$, find $(f \circ g)(-1)$ and the domain of $(f \circ g)(x)$.

(c) Find the following limit, if it exists

$$\lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1}$$

(d) Find the vertical and horizontal asymptotes to the graph of the function

$$f(x) = \frac{x^2 - 1}{x^2 - x - 2}.$$

Question 2:

(2 + 3 + 4)

(a) Use Squeeze theorem to evaluate the following limit:

$$\lim_{x \rightarrow 0} x^2 \cos \frac{1}{\sqrt[3]{x}}$$

(b) For what value of k , the following function is continuous for all x ?

$$f(x) = \begin{cases} 2x + k, & \text{if } x \leq 3 \\ 2k - x, & \text{if } x > 3. \end{cases}$$

(c) Apply the definition of the derivative to find $f'(x)$ where

$$f(x) = \sqrt{2x + 1}$$

Also, find the equation of the tangent line to the graph of this function at the point (4, 3).

Question 1:

(3 + 4 + 2)

(a) Solve the inequality and express your answer in terms of inequality

$$|4x - 3| \geq 1$$

(b) Find the domain of $f(x) = \frac{x^2}{\sqrt{x^2 - 2x - 3}}$.

(c) Find the limit, if it exists

$$\lim_{x \rightarrow 0} \frac{x + \tan x}{\sin x}$$

Question 2:

(3 + 4 + 4)

(a) Discuss for what value of α , the function

$$f(x) = \begin{cases} \cos 2x, & \text{if } x \leq 0 \\ \alpha(x + 1), & \text{if } x > 0 \end{cases}$$

is continuous at $x = 0$.

(b) Find horizontal and vertical asymptotes to the graph of the function

$$f(x) = \frac{x + 3}{x^2 - 9}.$$

(c) Use the definition of the derivative to find the derivative of the function

$f(x) = \sqrt{4 - x}$. Also, find an equation of the tangent line to the graph of this function at a point $(0, 2)$.

Question 3:

(4 + 6)

(a) If $y^3 + y + 1 = 2 \cos x + 1$, find the value of $\frac{d^2y}{dx^2}$ at the point $(0, 1)$.

(b) Let $f(x) = 2 - 15x + 9x^2 - x^3$ be a function

(i) Find all critical points of the function.

(ii) Find the intervals over which the given function is increasing and decreasing.

(iii) Locate all local maximum and local minimum using first derivative test.

(iv) Sketch the graph of the curve.

Question 1:

(3 + 3 + 3 + 3 + 4 + 4)

- (a) Solve the inequality and express your answer in terms of intervals $\frac{1}{x+1} \geq \frac{3}{x-3}$.
- (b) Find the domain of the function $f(x) = \frac{\sqrt{250 - 2x^3}}{x - 5}$.
- (c) Use Sandwich theorem to show that $\lim_{x \rightarrow 0} x \cos \frac{2}{x} = 0$.
- (d) Show that the following function is continuous at $x = 3$.

$$f(x) = \begin{cases} 1/(x+2), & \text{if } x \leq -2 \\ x^2 - 5, & \text{if } -2 < x \leq 3 \\ \sqrt{x^2 + 7}, & \text{if } x > 3. \end{cases}$$

- (e) Use logarithmic differentiation to find y' if

$$y = (x \cos x)^{\sin x}.$$

- (f) Let $f(x) = 3x^{5/3} - 15x^{2/3}$ be a function

- (i) Find local extrema of $f(x)$ using first derivative test.
(ii) Find the intervals where the $f(x)$ is increasing and decreasing.
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Question 2:

(4 + 4 + 9 + 4 + 3 + 6)

- (a) A rectangular window of constant perimeter equals 120 cms. Find the dimensions of the window which allows maximum area.

- (b) Let $f(x, y) = e^{-2y} \cos 2x$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

- (c) Evaluate the following integrals:

$$(i) \int_1^4 \frac{dx}{2\sqrt{x}(1 + \sqrt{x})^2} \quad (ii) \int \frac{x+4}{x^2 + 5x - 6} dx, \quad (iii) \int e^{2x} \cos 3x dx$$

- (d) Evaluate the integral $\int_0^1 x\sqrt{1-x^2} dx$ by using Trapezoidal rule for $n = 5$.

- (e) Find the area of the region bounded by the graphs of the equations $y = x^2$ and $y = -x^2 + 4x$.

- (f) Solve the following differential equations

$$(i) \frac{dy}{dx} = \frac{4x}{(x^2 + 8)^{1/3}}, \quad \text{subject to the initial condition } y(0) = 0.$$

$$(ii) (x+1) \frac{dy}{dx} - 2(x^2 + x)y = \frac{e^{x^2}}{(x+1)^2}.$$

Question 1:

(3 + 3 + 6 + 3 + 4 + 6)

(a) Find the domain of the function $f(x) = \frac{\sqrt{3x-4}}{x^2-5x+4}$.

(b) For what values of k the following function is continuous at $x = 1$?

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1}, & \text{if } x \neq 1 \\ 1 - k, & \text{if } x = 1. \end{cases}$$

(c) Find the horizontal asymptote and the vertical asymptote for the graph of the function $f(x) = \frac{2x^2 + x - 1}{x^2 - 1}$.

(d) Find the limit, if it exists $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x}$.

(e) Find an equation of the tangent line to the graph of the equation $e^{2x} \cos \pi x + 4 \cos x \sin y = 1$ at the point $(0, 0)$.

(f) Let $f(x) = 2x^5 - 5x^2 + 1$ be a function

(i) Find critical number of the function.

(ii) Find intervals over which the given function is increasing and decreasing.

(iii) Locate local extrema of the given function.

(iv) Sketch the graph of the given function.

Question 2:

(3 + 3 + 6 + 4 + 3 + 6)

(a) Use logarithmic differentiation to find y' if $y = \frac{(x^3 + 3)(\sin^2 x \cos x)}{\sqrt[3]{(4x + 1)(x^3 + 2)}}$.

(b) Let $f(x, y, z) = xz^2 e^{xy} \cos(yz)$. Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$.

(c) Evaluate the following integrals:

$$(i) \int 7^{\tan 2x} \sec^2 2x \, dx, \quad (ii) \int \frac{x + 10}{(x + 1)(x^2 - 100)} \, dx.$$

(d) Evaluate the integral $\int_0^3 \sqrt{9 - x^2} \, dx$ by using Trapezoidal's rule for $n = 6$.

(e) Find the area of the region bounded by the graphs of the equations $y = -x$ and $y = 2x - 3x^2$.

(f) Solve the following differential equations

$$(i) \quad e^{-y} \sin x - y \frac{dy}{dx} \cos^2 x = 0,$$

$$(ii) \quad \cos x \frac{dy}{dx} + \sin xy = \cos^3 x, \quad \text{subject to initial condition } y(0) = -1.$$

Question 1:

(3 + 4 + 4)

- (a) Solve the inequality and express your answer in terms of intervals: $x^2 - 6x + 8 \geq 0$.
- (b) Let $f(x) = \frac{1}{x+1}$ ($x \neq -1$) and $g(x) = 2x^2$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.
- (c) Find the horizontal asymptote and the vertical asymptote to the graph of the function $f(x) = \frac{x+2}{x^2 - 2x + 5}$.
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Question 2:

(4 + 3 + 5)

- (a) Use logarithmic differentiation to find y' if $y = \frac{e^{2x}(9x-2)^3}{\sqrt[4]{(x^2+1)(3x^3+1)}}$.
- (b) Find an equation of the tangent line to the graph of the equation $x \ln y = y \ln x$ at $x = 1$.
- (c) Let $f(x) = \frac{2}{3}x^3 - 2x + 1$ be a function. Sketch the graph of the function $f(x)$ showing the local extrema, concavity and the point of inflection.
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Question 3:

(4 + 3 + 6)

- (a) A field biologist wants to enclose a rectangular plot. He has 1600 meters of fencing (take this as perimeter). Using that material, find the dimensions of the study plot that will have the largest area.
- (b) If $f(x, y) = e^{-2y} \cos 2x$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
- (c) Evaluate the following integrals:

$$(i) \int \sin(\ln x) dx; \quad (ii) \int \frac{dx}{x^2(x-1)} dx.$$

Question 4:

(4 + 4 + 6)

- (a) Evaluate the integral $\int_0^2 \sqrt{x^2+1} dx$ by using Simpson's rule for $n = 4$.
- (b) Find the area of the region bounded by the graphs of $y = x^2$ and $y = 2 - x^2$ on the interval $[0, 2]$.
- (c) (i) Find the solution of the differential equation subject to the given initial condition:

$$\frac{dy}{dx} = \frac{y \cos x}{1 + y^2}; \quad y(0) = 1.$$

- (ii) Solve the differential equation:

$$x^2 \frac{dy}{dx} + 2xy = \sec^2 x.$$

Question 1: (3 + 3 + 6 + 3 + 4 + 6)

(b) Find $(f \circ g)(x)$ and the domain of $\frac{f(x)}{g(x)}$, where

$$f(x) = \sqrt{3-x} \quad \text{and} \quad g(x) = \sqrt{x^2-16}$$

(b) For what values of a and b the following function is continuous at $x = -4$ and $x = 4$?

$$f(x) = \begin{cases} a, & \text{if } x = -4 \\ \frac{16-x^2}{5-\sqrt{x^2+9}}, & \text{if } |x| < 4 \\ b, & \text{if } x = 4. \end{cases}$$

(c) Find the horizontal asymptote and the vertical asymptote for the graph of the function $f(x) = \frac{2x}{\sqrt{4x^2-12x}}$.

(d) Use sandwich theorem to prove that $\lim_{x \rightarrow 0} x^3 \sin(1/x^3) = 0$.

(e) Find an equation of the tangent line to the graph of the equation $3y^4 + 4x - x^2 \sin y = 4$ at the point $(1, 0)$.

(f) Let $f(x) = x^4 - 2x^2$ be a function, then

(i) Find local extrema of $f(x)$ using second derivative test.

(ii) Find the intervals where the graph of $f(x)$ is concave upward and concave downward.

(iii) Find the point of inflection in the graph of $f(x)$.

(iv) Sketch the graph of the given function.

Question 2: (3 + 3 + 6 + 4 + 3 + 6)

(a) Use logarithmic differentiation to find y' if $y = \frac{e^{x-2} \cos^2 x}{(x^2+1)^{3x}}$.

(b) Let $f(x, y) = \sin x + ye^x$. Find the partial derivatives $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$.

(c) Evaluate the following integral:

$$\int \frac{3x^2 - 7x - 2}{x^3 + x} dx.$$

(d) Evaluate the integral $\int_0^1 \sqrt{x^2+4} dx$ by using Simpson's rule for $n = 4$.

(e) Find the area of the region bounded by the graphs of the equations $y = x^2 - 1$ and $y = 7 - x^2$.

(f) Solve the following differential equations

$$(i) \quad \cos x = \left(\frac{2y + e^{3y}}{x} \right) \frac{dy}{dx}; \quad y(0) = 0,$$

$$(ii) \quad x \frac{dy}{dx} = y + x^2 \sin x.$$

Question 1: (3 + 3 + 3 + 3)

(a) Solve the inequality and express your answer in terms of intervals $|2x - 3| > 3$.

(b) Find the domain of the function $f(x) = \frac{1}{\sqrt{6x - 8 - x^2}}$.

(c) Find the vertical and horizontal asymptotes to the graph of the function $f(x) = \frac{x^2 + 1}{x^2 - 4}$.

(d) For what value of k , the following function is continuous for \mathbf{R} .

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x \leq 1 \\ k(x + 1), & \text{if } x > 1. \end{cases}$$

Question 2: (4 + 4 + 4 + 4)

(a) Use logarithmic differentiation to find y' if

$$y = \frac{\sin^2 x \cos x \tan^2 x}{\sqrt{x}}.$$

(b) Let $f(x) = x^3 + 6x^2 - 14$ be a function

(i) Find local extrema of $f(x)$ using second derivative test.

(ii) Find the intervals where the graph of $f(x)$ is concave upward and concave downward.

(iii) Find the point of inflection in the graph of $f(x)$.

(iv) Sketch the graph of the given function.

(c) A rectangular window of constant perimeter equals 200 cms. Find the dimensions of the window which allows maximum length.

(d) Let $f(x, y) = x^2y^3 - 3xy^2 + 2y$. Calculate the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(2, -3)$.

Question 3: (9 + 4 + 3 + 6)

(a) Evaluate the following integrals:

$$(i) \int 5^{\sin 2x} \cos 2x \, dx, \quad (ii) \int \frac{3x - 2}{x^3 - x^2} \, dx, \quad (iii) \int e^{2x} \sin x \, dx.$$

(b) Evaluate the integral $\int_0^1 \frac{1}{1+x} \, dx$ by using Simpson's rule for $n = 4$.

(c) Find the area of the region bounded by the graphs of the equations $y = x^2 - 1$ and $y = 7 - x^2$.

(d) Solve the following differential equations

$$(i) \frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8}.$$

$$(ii) x \frac{dy}{dx} - 4y = x^6 e^x.$$

Question 1: (4 + 6 + 3 + 4 + 4 + 4)

(a) Find the domain of $(f \circ g)(x)$ and $(f \circ g)'(5)$, where

$$f(x) = x + x^{-1}, \quad g(x) = \sqrt{x-1}$$

(b) Find the following limits, if exist

$$(i) \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta}, \quad (ii) \lim_{x \rightarrow \infty} \sqrt{9x^2 + x} - 3x$$

(c) Find the vertical asymptotes for the graph of the function $f(x) = \frac{4x^2}{\sqrt{4x + 5x^2 + x^3}}$.

(d) Use implicit differentiation to find $\frac{dy}{dx}$ if

$$x2^y + y^2 = \log_3(x).$$

(e) Use logarithmic differentiation to find y' if

$$y = \frac{\sin x \sqrt[3]{1 + \cos x}}{\sqrt{\cos x}}.$$

(f) Find the extrema of the function $f(x) = -2 \sin x$ on $[0, \frac{3\pi}{4}]$.

Question 2: (8 + 5 + 4 + 8)

(a) Evaluate the following integrals:

$$(i) \int \sin(\ln x) dx, \quad (ii) \int \frac{2x^2 + 4x - 8}{x^3 - 4x} dx$$

(b) Evaluate the integral $\int_0^2 e^{-x^2} dx$ by using Simpson's rule for $n = 4$.

(c) Find the area of the region bounded by the graphs of the equations $y = x^2$ and $y = 4x$.

(d) Solve the differential equations

$$(i) \left(\frac{y}{y-1} \right) \frac{dy}{dx} - \frac{x+1}{x} = 0.$$

$$(ii) e^{x-y} \frac{dy}{dx} + 1 = 0.$$