Department of Physics and Astronomy
College of Science, King Saud University Riyadh

Final Examination
2nd Semester 1428-1429 H

Solid State Physics, **Phys 570**

Time: Three hours **Maximum marks- 50**

Mass of the electron m	$= 9.11 \times 10^{-31}$ kg
Planck's constant h	$= 2\pi \times 1.05 \times 10^{-34}$ Js
elementary charge e	$= 1.60 \times 10^{-19}$ C
one electron volt	$= 1.60 \times 10^{-19}$ J
Boltzmann's constant k_B	$= 1.38 \times 10^{-23}$ J-K^{-1}
permittivity of vacuum ε_0	$= 8.85 \times 10^{-12}$ Fm^{-1}
Avogadro's number	$= 6.02 \times 10^{23}$

Answer all questions. The marks are written against each question

Q. 1 a What is the nearly free electron model? 2

b According to nearly free electron model what is the ψ (+) and the ψ(-)? Write it down in one dimension. 3

c According to nearly free electron model the potential energy due to one dimensional lattice is given by 5

$$V(x) = -\sum_{n=1}^{\infty} V_n \cos(2n\pi x/a)$$

This potential energy is small in comparison to the kinetic energy in the free electron model. Using first-order perturbation theory show that there exist an energy gap of order $\boldsymbol{V_1}$ at k= $\pm\pi/a$

Q.2 Theory shows that the density of electrons in the conduction band of a semiconductor (whether intrinsic or extrinsic) is given by

Cont..

$$n_e = A(m_e T)^{3/2} \exp\left(\frac{\mu - E_C}{k_B T}\right)$$

where μ is the chemical potential of the electrons in the material, E_C is the energy of the bottom of the conduction band, k_B is Boltzmann's constant, T is temperature and me is the effective mass of the electrons. The constant A is given by

$$\frac{1}{4}(2k_B/(\pi\hbar^2))^{3/2}$$

The density of holes in the valence band is similarly given by

$$n_h = A(m_h T)^{3/2} \exp\left(\frac{E_V - \mu}{k_B T}\right)$$

where m_h is the effective mass of the holes, and E_V is the energy of the top of the valence band.

a. For an intrinsic semiconductor, show that the chemical potential of the electrons is given by 4

$$\mu = \frac{1}{2}(E_C + E_V) + \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$$

b. Calculate the chemical potential of electrons relative to the top of the valence band $(\mu - E_V)$ for intrinsic silicon at 300 K, if E_C-E_V=1.1 eV, and assuming that both m_e and m_h are equal to the mass of a free electron 3

c If the hole density in a sample of p-type silicon is 10^{22} m^{-3}, calculate the 3

. chemical potential, relative to the top of the valence band, of the electrons in this material at 300 K.

Q.3. a. Consider N atoms arranged in a ring with the atoms spaced a distance a apart. Electrons moving in this structure may be modeled by a one dimensional wave function $\psi(x)$, where x lies in the range $0 \leq x \leq Na$. The atoms lie at positions $x = \ell a$ where ℓ is an integer. 5

Show that symmetry implies that

$$\psi(x + a) = \exp(i\phi)\psi(x)$$

where ϕ can take the values

$$\phi = \frac{2\pi p}{N}$$

only, where p is an integer.

Hence show that an expression of the form

$$\psi(x) = \exp(ikx)u(x)$$

with $k = 2\pi p/(Na)$ can be used to represent the electron, as long as the function u(x) satisfies a certain condition. What is this condition?

b. The response of a set of free electrons in a material to an external force F in the x direction can be modelled using the following transport equation: 5

$$\frac{dv_D}{dt} + \frac{v_D}{\tau} = \frac{F}{m}$$

where v_D is the drift velocity in the x direction, t is time, τ is the relaxation time and m is the electron mass.

Solve this equation in the steady state for a constant applied electric field to show that the electrical conductivity of the material, according to the model, is given by

$$\sigma = ne^2\tau/m$$

where n is the free electron density and e is the electronic charge.

Q.4. a Indium antimonide has E_g = 0.023 eV, dielectric constant ε = 18; electron effective mass m_e=0.015 m. where m is the free electron mass Calculate

i. The donor ionization energy E_d 1.5

ii. The radius of the ground state orbit 1.5

iii. At what minimum donor concentration will appreciable overlap effects between the donor orbits of adjacent impurity atoms occur? 2.0

b. Apply the Hund's rule to find the ground state of Eu^{++} ($4f^7 5s^2 p^6$). Write the answer step by step applying the rules. Find the value of effective Bohr magnetons (p). 5

Q.5. a. The wave function of hydrogen atom in its ground state 1s is $\psi = (\pi a_o^3)^{-1/2} \exp(-r/a_o)$, where a_o =0.529X10^{-8} cm. The charge density is $\rho(x,y,z) = -e|\psi|^2$, according to statistical interpretation of wave function. Show that for this state $\langle r^2 \rangle = 3\, a_o^2$ and calculate the molar diamagnetic susceptibility of atomic hydrogen is (-2.36×10^{-6} cm^3 /mole). 7

b i. In what material the first superconductivity was observed? 3

ii. What is the Meissner effect?

iii. What is the temperature of liquid helium and liquid nitrogen?