

APPROXIMATE SOLUTION FOR THE GENERALIZED TIME-DELAYED BURGERS-HUXLEY EQUATION

E. S. FAHMY and I. BAJUNAID

Mathematics Department

Women Students Medical Studies and Sciences Sections

King Saud University

P. O. Box 22452, Riyadh 11495, Kingdom of Saudi Arabia

e-mail: esfahmy@operamail.com

ibajunaid@hotmail.com

Abstract

In this paper, the approximate solution for the generalized time-delayed Burgers-Huxley equation is obtained by using the Adomian decomposition method for any value of $\delta \geq 0$ and $\tau \geq 0$, also the convergence of the decomposition series of solutions was proved.

1. Introduction

It is known now that many reaction diffusion models have been used to describe several branches of sciences such as Biology, Economy, etc. [1, 2, 4, 16, 19]. The equations describing such models are derived from the classical diffusion equations. The most commonly one is the generalized time-delayed Burgers-Huxley equation. As one knows, the Burgers-Huxley equation [5-7, 17], was first introduced to describe turbulence in one space dimension, and has been used in several other physical contexts, including for instance

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sound waves in viscous media. The generalized time-delayed Burgers-Huxley equation is more complicated and makes the system qualitatively different from former Burgers-Huxley equation [5-7, 17].

Recently, several new methods are used to find exact solutions of such nonlinear reaction diffusion equations such as, tanh-function method [9, 13], simplest equation method [12] and Lie symmetries [18].

Here we will find an approximate solution for the modified model using the Adomian Decomposition Method (ADM) [3, 4, 10, 11], this equation is given as

$$\tau u_{tt} + \left(1 - \tau \frac{df}{du}\right) u_t = \alpha u^\delta u_x + u_{xx} + f(u), \quad f(u) = \beta u(1 - u^\delta)(u^\delta - \gamma), \quad (1)$$

where α , β and γ are real constants and $\delta \geq 0$, u is the population density and τ is a time-delayed constant. It is clear that when $\tau = 0$ equation (1) reduced to generalized Burgers-Huxley equation [1].

The exact travelling wave solution of equation (1) is derived in [8] as:

$$u_E(x, t) = \left(\frac{1}{2} - \frac{1}{2} \tanh(b(x - ct)) \right)^{\frac{1}{\delta}}, \quad (2)$$

where $b = \frac{a\delta\sqrt{\beta}}{2(\delta+1)\sqrt{1-\tau c^2}}$, c is the wave speed and

$$a = \frac{(-\alpha - c\tau\beta(1+\gamma)(\delta+1)) + \sqrt{(-\alpha - c\tau\beta(1+\gamma)(\delta+1))^2 - 4\beta(\delta+1)(1-\tau c^2)}}{2(\delta+1)\sqrt{\beta(1-\tau c^2)}}.$$

2. Method of Solution

Equation (1) can be formulated as:

$$\begin{aligned} (1 + \tau\beta\gamma)u_t &= (u_{xx} - \tau u_{tt} - \beta\gamma u) + (\alpha u^\delta u_x + \beta(1 + \gamma)u^{\delta+1} \\ &\quad - \beta u^{2\delta+1} + \tau\beta(1 + \gamma)(\delta + 1)u^\delta u_t \\ &\quad + \tau\beta(2\delta + 1)u^{2\delta} u_t), \end{aligned} \quad (3)$$

with the initial condition

$$u(x, 0) = \left(\frac{1}{2} - \frac{1}{2} \tanh(bx) \right)^{\frac{1}{\delta}}. \tag{4}$$

In an operator form, equation (1) can be written as:

$$L_t u = Ru + F(u), \tag{5}$$

where

$$L_t = q \frac{\partial}{\partial t}, \quad R = \frac{\partial^2}{\partial x^2} - \tau \frac{\partial^2}{\partial t^2} - \gamma \beta \tag{6}$$

are linear operators, $q = (1 + \tau\beta\gamma) \neq 0$, and $F(u)$ is the nonlinear term given as:

$$F(u) = \alpha u^\delta u_x + \beta(1 + \gamma)u^{\delta+1} - \beta u^{2\delta+1} + \tau\beta(1 + \gamma)(\delta + 1)u^\delta u_t + \tau\beta(2\delta + 1)u^{2\delta} u_t. \tag{7}$$

Following the ADM, we define the integration inverse of L_t , by

$$L_t^{-1} = \frac{1}{q} \int_0^t (\cdot) dt. \tag{8}$$

Applying this inverse operator to both sides of (5), yields

$$u(x, t) = g(x) + L_t^{-1}[(Ru + F(u))], \quad u(x, 0) = g(x), \tag{9}$$

where $g(x)$ is the solution of the homogeneous equation $L_t u = 0$.

The ADM decomposes the solution $u(x, t)$ into an infinite series

$$u(x, t) = \frac{1}{q} \sum_{n=0}^{\infty} u_n(x, t), \tag{10}$$

and the nonlinear term $F(u)$ is decomposed into an infinite series of polynomials as

$$F(u) = \sum_{n=0}^{\infty} A_n. \tag{11}$$

The A_n, s are called the *Adomian polynomials*, and the components $u_n(x, t)$ will be determined recursively as

$$\begin{aligned} u_0(x, t) &= u(x, 0) = g(x), \\ u_1(x, t) &= L_t^{-1}[(Ru_0 + A_0)], \\ u_2(x, t) &= L_t^{-1}[(Ru_1 + A_1)], \\ &\vdots \\ u_{n+1}(x, t) &= L_t^{-1}[(Ru_n + A_n)], \end{aligned} \quad (12)$$

where

$$A_k = \frac{1}{k!} \frac{d^k}{d\lambda^k} \left[F \left(\sum_{m=0}^{\infty} \lambda^m u_m \right) \right]_{\lambda=0}, \quad k = 0, 1, \dots \quad (13)$$

That is

$$\begin{aligned} A_0 &= F(u_0), \\ A_1 &= u_1 F'(u_0), \\ A_2 &= u_2 F'(u_0) + \frac{u_1^2}{2} F''(u_0), \\ A_3 &= u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{6} u_1^3 F'''(u_0), \\ &\vdots \end{aligned} \quad (14)$$

and then

$$\begin{aligned} A_0 &= \alpha u_0^\delta (u_0)_x + \beta(1 + \gamma) u_0^{\delta+1} - \beta u_0^{2\delta+1} + \tau\beta(1 + \gamma)(\delta + 1) u_0^\delta (u_0)_t \\ &\quad + \tau\beta(2\delta + 1) u_0^{2\delta} (u_0), \\ A_1 &= \alpha\delta u_0^{\delta-1} u_1 (u_0)_x + \beta_1(\delta + 1) u_0^\delta u_1 + \beta_2(2\delta + 1) u_0^{2\delta} u_1 + \beta_3\delta u_0^{\delta-1} u_1 (u_0)_t \\ &\quad + \beta_3 u_0^\delta (u_1)_t + 2\beta_4\delta u_0^{2\delta-1} u_1 (u_0)_t + \beta_4 u_0^{2\delta} (u_1)_t, \end{aligned}$$

$$\begin{aligned}
 A_2 = & \frac{1}{2} ((\delta(\delta - 1)u_0^{\delta-2}u_1^2 + 2\delta u_0^{\delta-1}u_2)\alpha(u_0)_x + 2\alpha\delta u_0^{\delta-1}u_1(u_1)_x + 2\alpha u_0^\delta(u_2)_x \\
 & + (\delta(\delta + 1)u_0^{\delta-1}u_1^2 + 2(\delta + 1)u_0^\delta u_2)\beta_1 \\
 & + 2\delta(2\delta + 1)u_0^{2\delta-2}u_1^2 + 2(2\delta + 1)u_0^{2\delta}u_2)\beta_2 \\
 & + ((\delta(\delta - 1)u_0^{\delta-2}u_1^2 + 2\delta u_0^{\delta-1}u_2)(u_0)_t \\
 & + 2\delta u_0^{\delta-1}u_1(u_1)_t + 2u_0^\delta(u_2)_t)\beta_3 \\
 & + ((2\delta(2\delta - 1)u_0^{2\delta-2}u_1^2 + 4\delta u_0^{2\delta-1}u_2)(u_0)_t \\
 & + 4\delta u_0^{2\delta-1}u_1(u_1)_t + 2u_0^{2\delta}u_2)(u_2)_t)\beta_4), \\
 & \vdots
 \end{aligned} \tag{15}$$

where

$$\beta_1 = \beta(1 + \gamma), \quad \beta_2 = -\beta, \quad \beta_3 = \varpi\beta(1 + \gamma)(\delta + 1), \quad \beta_4 = \varpi\beta(2\delta + 1).$$

The first few terms $u_n(x, t)$ (for $n \geq 1$) are given as follows:

$$\begin{aligned}
 u_0(x, t) &= \left(\frac{1}{2} - \frac{1}{2} \tanh(kx)\right)^{\frac{1}{\delta}}, \\
 u_1(x, t) &= 2^{-\frac{1}{\delta}} \left(\frac{1}{2} - \frac{1}{2} \tanh(kx)\right)^{\frac{1}{\delta}} [-B + \beta_1 \left(\frac{1}{2} - \frac{1}{2} \tanh(kx)\right)^{\frac{1}{\delta}}] \\
 &+ \beta_2 \left(\frac{1}{2} - \frac{1}{2} \tanh(kx)\right)^{\frac{1}{\delta}}^{2\delta} + \frac{k^2 \operatorname{sech}^4(kx)(1 - \delta)}{\delta^2(-1 + \tanh(kx))^2} \\
 &+ \frac{k \operatorname{sech}^2(kx) \left(\frac{1}{2} - \frac{1}{2} \tanh(kx)\right)^{\frac{1}{\delta}}}{\delta(-1 + \tanh(kx))} + \frac{2k^2 \operatorname{sech}^2(kx) \tanh(kx)}{\delta(-1 + \tanh(kx))},
 \end{aligned}$$

$$\begin{aligned}
u_2(x, t) = & \frac{1}{4} t \frac{1}{\delta^3} \left(2^{2-\frac{1}{\delta}} k^2 \operatorname{sech}^8(2kx) (\cosh(2kx) - \sinh(2kx)) (k - k(2 + \delta)) \right. \\
& + k\delta \cosh(2kx) - \delta \sinh(2kx) \left. \left(k + \alpha \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) \right) \right) \\
& + \delta^2 \cosh^2(2kx) \beta_1 \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) \\
& + 2\delta^2 \cosh^2(2kx) \beta_2 \left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \left. \right) (1 - \tanh(kx))^{-4+\frac{1}{\delta}} \\
& + \frac{1}{\delta^2} \left(2^{\frac{-1+\delta}{\delta}} k^2 t \operatorname{sech}^8(kx) (\cosh(3kx) - \sinh(3kx)) \right. \\
& \left. (k(2k(-3 - 2\delta + \delta \cosh(2kx)) \sinh(kx) - \delta \cosh(3kx))) \right. \\
& \left. \left(k + \alpha \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) \right) + \cosh(kx) \right. \\
& \left. \left(k(2 + 3\delta) + 3\delta\alpha \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) + 2\delta^2 \beta_1 \cosh^2(kx) \sinh(kx) \right) \right. \\
& \left. \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) + 2\delta^2 \cosh^2(kx) \right. \\
& \left. (\cosh(kx) + 3 \sinh(kx)) \beta_2 \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) \right) (1 - \tanh(kx))^{-4+\frac{1}{\delta}} \\
& - \frac{1}{\delta^3} \left(2^{\frac{-1}{\delta}} k t \alpha \operatorname{sech}^6(kx) \right) (\cosh(2kx) - \sinh(2kx)) (-6k^2\delta - B\delta^2 \\
& - 2k^2\delta^2 + 2k^2 \cosh^2(2kx) - B\delta^2 \cosh(2kx) + 2k^2\delta^2 \cosh(2kx) \\
& + 2k^2 \sinh(2kx) - 2k^2\delta^2 \sinh(2kx) - k\alpha\delta \left. \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & -2k\delta \sinh(2kx) \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) \\
 & + 2\alpha k \delta^2 \sinh(2kx) \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) \\
 & + \frac{4k\alpha \cosh^2(2kx) \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right)}{-1 + \tanh(kx)} \\
 & + \frac{1}{\delta} \left(2^{1-\frac{1}{\delta}} k^2 \left(-1 + \frac{1}{\delta} \right) t \operatorname{sech}^4(kx) (1 - \tanh(kx))^{-3+\frac{1}{\delta}} \right) \\
 & + \frac{1}{\delta} \left(2^{2-\frac{1}{\delta}} k^2 t \operatorname{sech}^2(kx) (1 - \tanh(kx))^{-1+\frac{1}{\delta}} \right) \tanh(kx) \\
 & (-B + \beta_4 \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^\delta \right) \\
 & + \beta_2 \left(\left(\frac{1}{2} - \frac{1}{2} \tanh(kx) \frac{1}{\delta} \right)^{2\delta} \right) + \frac{k^2 \operatorname{sech}^4(kx) (1 - \delta)}{\delta^2 (-1 + \tanh(kx))^2} \\
 & + \frac{2k^2 \operatorname{sech}^4(kx) \tanh(kx)}{\delta (-1 + \tanh(kx))}), \quad \text{☰} \\
 & \vdots \tag{16}
 \end{aligned}$$

Using (12), we easily determine the components $u_n(x, t)$, $n \geq 0$. It is, in principle, possible to calculate more components in the decomposition series to enhance the approximation. Consequently, we can recursively determine every term of the series (12), and hence the solution $u(x, t)$ is readily obtained in a series form. It is interesting to note that we obtain the solution by using the initial conditions only.

In order to prove numerically whether the Adomian decomposition method for generalized time-delayed Burgers-Huxley equation leads to higher accuracy, we evaluate the approximate solution, exact solution and

the absolute error in the following table, where $t = 0.3$, $\tau = 0.5$, $\delta = 1$. Also, we illustrate the accuracy by using a set of figures for different values of δ . Namely, Figure 1. Normal graph (Exact solution u_E) and dashed graph (Approximate solution u), where $t = 0.3$, $\tau = 0.5$, $\alpha = \beta = \gamma = 1$ and $\delta = 1$, respectively. Figure 2. Normal graph (Exact solution u_E) and dashed graph (Approximate solution u), where $t = 0.3$, $\tau = 0.5$, $\alpha = \beta = \gamma = 1$ and $\delta = 2$, respectively. Figure 3. Normal graph (Exact solution u_E) and dashed graph (Approximate solution u), where $t = 0.3$, $\tau = 0.5$, $\alpha = \beta = \gamma = 1$ and $\delta = 3$, respectively.

x_i	u_{Exact}	$u_{Apprxmit}$	$ u_{Apprxmit} - u_{Exact} $
-10	1	1.01441	1.44125×10^{-2}
-8	1	1.044124	1.44124×10^{-2}
-6	0.999940	1.01440	1.44076×10^{-2}
-4	0.999632	1.01384	1.41469×10^{-2}
-2	0.983374	0.984511	1.1376×10^{-3}
0	0.519890	0.507792	1.2972×10^{-3}
2	0.019455	0.233822	3.9271×10^{-3}
4	0.00036327	0.0044205	7.8724×10^{-5}
6	6.6559×10^{-6}	8.10128×10^{-6}	1.44538×10^{-6}
8	1.2190×10^{-6}	1.48382×10^{-7}	2.64739×10^{-8}
10	2.85512×10^{-7}	1.57648×10^{-7}	4.84887×10^{-10}

3. Convergence of the Method

One aim of this paper is to provide a sufficient condition for the convergence of the series decomposition. Let us consider the Hilbert space \mathcal{H} which may be defined by $\mathcal{H} = \mathcal{L}^2((a, b) \times [0, T])$ and define $u : (a, b) \times [0, T] \rightarrow \mathbb{R}$ with

$$\int_{(a, b) \times [0, T]} u^2(x, s) ds dx < \infty, \quad (17)$$

the scalar product

$$(u, v)_{\mathcal{H}} = \int_{(a, b) \times [0, T]} u(x, s)v(x, s) ds dx, \quad (18)$$

and the associated norm

$$\|u\|_{\mathcal{H}}^2 = \int_{(a, b) \times [0, T]} u^2(x, s) ds dx. \quad (19)$$

According to [15], the ADM is convergent if the following two hypotheses are satisfied:

$$H_1: ((L_t(u) - L_t(v)), u - v) \geq k \|u - v\|^2; k > 0, \forall u, v \in \mathcal{H},$$

H_2 : Whatever may be $M > 0$, there exists a constant $c(M)$ such that for $u, v \in \mathcal{H}$ with $\|u\| \leq M, \|v\| \leq M$, we have

$$((L_t(u) - L_t(v)), w) \leq c(M) \|u - v\| \|w\| \text{ for every } w \in \mathcal{H}.$$

Lemma 1. *The function $F(u)$, represents the nonlinear term in equation (5) is a Lipschitzian function.*

Proof. Our claim is to prove that there exists a positive constant k such that

$$\|F(u) - F(v)\| \leq k \|u - v\|. \quad (20)$$

Notice that $F(u)$ can be written in the following form

$$\begin{aligned} F(u) &= \frac{\alpha}{\delta + 1} (u^{\delta+1})_x + \beta(1 + \gamma)u^{\delta+1} - \beta u^{2\delta+1} \\ &\quad + \mathfrak{p}(1 + \gamma)(u^{\delta+1})_t + \mathfrak{p}(u^{2\delta+1})_t. \end{aligned} \quad (21)$$

So

$$\begin{aligned} F(u) - F(v) &= \frac{\alpha}{\delta + 1} (u^{\delta+1} - v^{\delta+1})_x + \beta(1 + \gamma)(u^{\delta+1} - v^{\delta+1}) \\ &\quad - \beta(u^{2\delta+1} - v^{2\delta+1}) + \mathfrak{p}(1 + \gamma)(u^{\delta+1} - v^{\delta+1})_t \\ &\quad + \mathfrak{p}(u^{2\delta+1} - v^{2\delta+1})_t, \end{aligned} \quad (22)$$

then

$$\begin{aligned} \|F(u) - F(v)\| &\leq \left| \frac{\alpha}{\delta+1} \right| \|u^{\delta+1} - v^{\delta+1}\|_x + |\beta(1+\gamma)| \|u^{\delta+1} - v^{\delta+1}\| \\ &\quad + |\beta| \|u^{2\delta+1} - v^{2\delta+1}\| + |\wp\beta(1+\gamma)| \|u^{\delta+1} - v^{\delta+1}\|_t \\ &\quad + |\wp\beta| \|u^{2\delta+1} - v^{2\delta+1}\|_t. \end{aligned} \quad (23)$$

Since ∂_x, ∂_t are linear differential operators in \mathcal{H} , there exist positive constants $\delta_1, \delta_2, \delta_3$ such that

$$\begin{aligned} \|F(u) - F(v)\| &\leq \delta_1 \left| \frac{\alpha}{\delta+1} \right| \|u^{\delta+1} - v^{\delta+1}\| + |\beta(1+\gamma)| \|u^{\delta+1} - v^{\delta+1}\| \\ &\quad + |\beta| \|u^{2\delta+1} - v^{2\delta+1}\| + \delta_2 |\wp\beta(1+\gamma)| \|u^{\delta+1} - v^{\delta+1}\| \\ &\quad + \delta_3 |\wp\beta| \|u^{2\delta+1} - v^{2\delta+1}\|. \end{aligned} \quad (24)$$

Now applying the mean-value theorem, there exists η such that $u \leq \eta \leq v$, such that

$$\|u^{\delta+1} - v^{\delta+1}\| \leq (1+\delta) \|\eta^\delta\| \|u - v\|.$$

Hence

$$\begin{aligned} \|F(u) - F(v)\| &\leq [\delta_1 |\alpha| M^\delta + (1+\delta) |\beta(1+\gamma)| M^\delta + (1+2\delta) |\beta| M^{2\delta} \\ &\quad + (1+\delta) \delta_2 |\wp\beta(1+\gamma)| M^\delta \\ &\quad + (1+2\delta) \delta_3 |\wp\beta| M^{2\delta}] \|u - v\|, \end{aligned} \quad (25)$$

where the constant M satisfies $\|u\| \leq M$ and $\|v\| \leq M$. Then

$$\|F(u) - F(v)\| \leq k \|u - v\|,$$

where

$$\begin{aligned} k &= \delta_1 |\alpha| M^\delta + (1+\delta) |\beta(1+\gamma)| M^\delta + (1+2\delta) |\beta| M^{2\delta} \\ &\quad + (1+\delta) \delta_2 |\wp\beta(1+\gamma)| M^\delta + (1+2\delta) \delta_3 |\wp\beta| M^{2\delta}. \end{aligned} \quad (26)$$

Theorem 2 (Sufficient condition for convergence). *For equation (5) without initial and boundary conditions, the ADM converges towards a particular solution.*

Proof. First, we will verify the hypothesis of convergence (H_1) . From equation (5), we have

$$L_t(u) - L_t(v) = R(u - v) + (F(u) - F(v)) \quad \forall u, v \in \mathcal{H},$$

where

$$R = \frac{\partial^2}{\partial x^2} - \tau \frac{\partial^2}{\partial t^2} - \gamma\beta.$$

Now,

$$\begin{aligned} (L_t(u) - L_t(v), u - v) &= (R(u - v), u - v) + (F(u) - F(v), u - v) \\ &= ((u - v)_{xx}, u - v) + (-\tau(u - v)_{tt}, u - v) \\ &\quad + (-\gamma\beta(u - v), u - v) + (F(u) - F(v), u - v), \end{aligned}$$

since $\partial_{xx}, \partial_{tt}$ are linear differential operators, there exist β_1, β_2 such that

$$(R(u - v), u - v) \geq -\beta_1 \|u - v\|^2 + \tau\beta_2 \|u - v\|^2 + \gamma\beta \|u - v\|^2. \quad (27)$$

According to Schwartz inequality, and the above Lemma, we get

$$(F(u) - F(v), u - v) \geq -k \|u - v\|^2, \quad (28)$$

then from equations (28), (29), we get

$$((L_t(u) - L_t(v)), u - v) \geq K \|u - v\|^2, \quad (29)$$

where

$$K = -\beta_1 + \tau\beta_2 + \alpha\beta - k, \quad (30)$$

with $\tau\beta_2 + \alpha\beta > \beta_1 + k$.

Now, for H_2 ,

$$\begin{aligned} (L_t(u) - L_t(v), w) &= (R(u - v), w) + (F(u) - F(v), w) \\ &\leq |1 - \tau - \gamma\beta| \|u - v\| \|w\| + k \|u - v\| \|w\| \\ &\leq (|1 - \tau - \gamma\beta| + k) \|u - v\| \|w\| \\ &= c(M) \|u - v\| \|w\|, \end{aligned}$$

where

$$c(M) = |1 - \tau - \gamma\beta| + k,$$

therefore, H_2 holds, and the proof is complete.

4. Conclusions

In this work, we presented approximate solution for the generalized time-delayed Burgers-Huxley equation in a series form by using the Adomian decomposition method. The absolute error and the approximate solution are presented and compared with the exact solution for some values of x and $\delta = 1, \delta = 2, \delta = 3$. Also, we presented a convergence proof of the ADM applied to this equation for $\delta \geq 0$.

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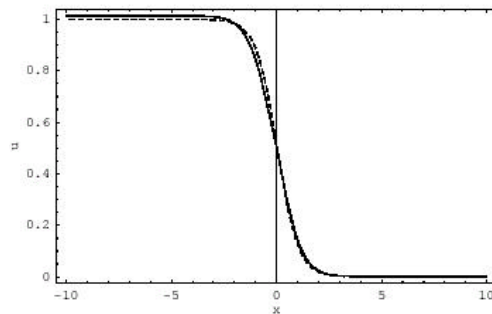


Figure 1. Normal graph (Exact solution u_E) and dashed graph (Approximate solution u), where $t = 0.3$, $\tau = 0.5$, $\alpha = \beta = \gamma = 1$ and $\delta = 1$, respectively.

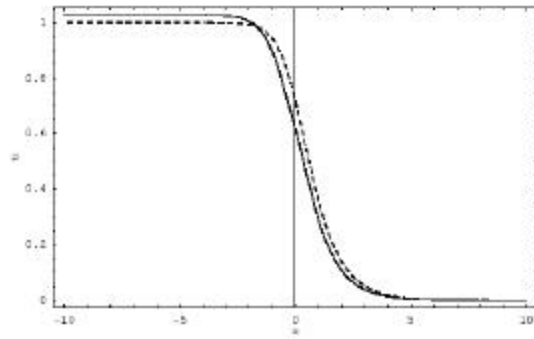


Figure 2. Normal graph (Exact solution u_E) and dashed graph (Approximate solution u), where $t = 0.3$, $\tau = 0.5$, $\alpha = \beta = \gamma = 1$ and $\delta = 2$, respectively.

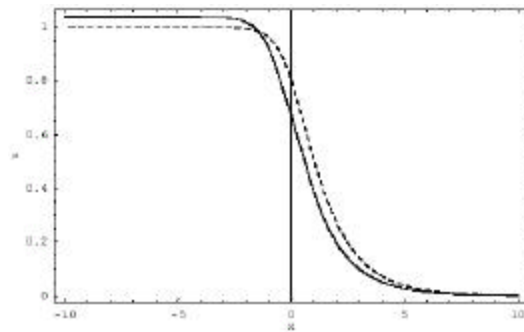


Figure 3. Normal graph (Exact solution u_E) and dashed graph (Approximate solution u), where $t = 0.3$, $\tau = 0.5$, $\alpha = \beta = \gamma = 1$ and $\delta = 3$, respectively.

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