KING SAUD UNIVERSITY
First Midterm Exam Of Math **201**, Spring of 1419-1420
Duration one and half hour

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1. **A-** Describe the set of points in the xy-plane at which f is continuous

$$f(x,y) = \frac{x\,y}{x-5y}$$

B- If $f(x,y) = x^2 + 9y^2$, sketch the **level curves** of f for $k = 0, 4$.

C- Find the following limits if it exists:

$$(\mathbf{i}) lim_{(x,y)\to(-1,0)} \frac{(x^2-1)(y+2)}{(x+1)(y-5)} \quad \text{and} \quad (\mathbf{ii}) lim_{(x,y)\to(1,2)} \frac{xy-2x-y+2}{x^2+y^2-2x-4y+5}$$

2. **A-** Let

$$f(x,y) = \begin{cases} \frac{9x^3\,y}{3x^4+4y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

   (a) Prove that $f_x(0,0)$ and $f_y(0,0)$ exists. (Use the definition of f_x and f_y.)

   (b) Is f differentiable at $(0,0)$. Justify your answer.

B- Let $f(x,y) = sin(x+y)e^{x-y}$, show that

$$(f_x)^2 + (f_y)^2 = 2e^{(2x-2y)}.$$

C- Let $w = f(u,v)$, where $u = t\,sin(x)$ and $v = t\,cos(x)$. Compute $\partial^2 w/\partial x^2$.

3. **A-** Let $V(r,h) = \pi\,r^2 h$. If r changes from 2 to 2.5 and h changes from 4 to 3.7. Use the differential to approximate the change in V.

B- Find the **maximum** and **minimum** values of the function

$$f(x,y) = 3x^2 - 4y + 2y^2 + 1$$

on the region $R = \{(x,y)|x^2+y^2 \leq 16\}$.

KING SAUD UNIVERSITY
First Midterm Exam Of Math **201**, Fall of 1420-1421
Duration one and half hour

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1. **A-** Describe the set of points where f is continuous

$$f(r,s) = \sqrt{3-2r} - e^{r^2/6s}$$

B- If $z = f(x,y) = 4x^2 + y^2$, sketch the **level curves** of f, for $k = 0, 4, 16$.

C- Find the following limits if it exists:

$$(\mathbf{i})\ lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} \quad \text{and} \quad (\mathbf{ii})\ lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz}{x^2+y^2+z^2}$$

$$(\mathbf{iii})\ lim_{(x,y)\to(-1,2)} \frac{xy-2x}{xy-6-2x+3y}.$$

2. **A-** Let

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

(a) Prove that $f_x(0,0)$ and $f_y(0,0)$ exists. (Use the definition of f_x and f_y.)

(b) show that f is not continuous at $(0,0)$

(c) show that f is not differentiable at $(0,0)$.

B- If $w(x,y) = cos(x-y) + ln(x+y)$, show that

$$w_{xx} - w_{yy} = 0$$

3. **A-** Use the differential to approximate the change in $f(x,y) = x^2 - 2xy + 3y^2$, if (x,y) changes from $(1,2)$ to $(1.03, 1.99)$.

B- If $w = f(x,y)$, $x = r\,cos\,\theta$ and $y = r\,sin\,\theta$, show that

$$\frac{\partial w}{\partial x} = (cos\,\theta)\,\frac{\partial w}{\partial r} - (\frac{sin\,\theta}{r})\,\frac{\partial w}{\partial \theta}.$$

Good Luck

KING SAUD UNIVERSITY
Second Midterm Exam Of Math **201**, Fall of 1420-1421
Duration Two hours

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1. Find the **absolute extrema** (absolute maximum and absolute minimum)of

$$f(x,y) = x^2 - 6x + y^2 - 4y$$

on the closed region bounded by the x- axis, y- axis and the line $x + y = 7$.

2. Evaluate the double integral

$$\int_0^1 \int_{\sqrt{X}}^1 sin(y^3 + 1)dy\,dx.$$

3. **A-** Find the area of the region bounded by the graphs of $5x = y^2 - 25$ and $x + y = 5$, using the double integral and sketch the region.

   B- Sketch the region bounded by the graphs of the equations $z = x^2$, $z = 4$, $y = 0$ and $y + z = 4$. Use the **triple integral** to find its volume.

4. **A-** Determine whether the following sequences **converge or diverge**, if it converges then find the **limit**

$$(\mathbf{1})\left\{\frac{3}{n} + (-3)^n\right\}, \qquad (\mathbf{2})\left\{(-1)^n\,\frac{3+n}{5\,n^2}\right\}, \qquad (\mathbf{3})\left\{(-1)^n\,\frac{ln(n)}{n}\right\}.$$

$$(\mathbf{4})\left\{n^{1/n}\right\}, \qquad (\mathbf{5})\left\{\frac{cos(n\pi)}{\sqrt{n}}\right\}.$$

   B- Answer **True** or **False**. Justify your answer :

   (a) Every monotonic sequence converges.
   (b) Every convergent sequence is monotonic.
   (c) If the series $\sum a_n$ converges, then $lim_{n\to\infty} a_n = 0$.
   (d) If $R > 4$, then $lim_{n\to\infty}\frac{1}{R^n} = 0$
   (e) Sum of two divergent sequences is a divergent sequence.

KING SAUD UNIVERSITY
Final Exam Of Math **201**, Spring of 1423-1424
Duration : Three Hours

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1. **A-** Find the following limits if it exists;

$$1)\, lim_{(x,y)\to(2,1)} \frac{x^2-4x+4}{(x-2)(y-1)}, \qquad 2) lim_{(x,y)\to(0,0)}[\, \frac{xy}{\sqrt{x^2+y^2}} + 3\, e^{x+y}\,].$$

   B- Let $f(x,y) = \begin{cases} \frac{xy^2}{y^3+x^3}, & y \neq -x \\ 0, & y = -x \end{cases}$

   1- Show that $f_x(0,0)$ and $f_y(0,0)$ exist.

   2- Is f differentiable at $(0,0)$, $(1,1)$, justify.

   C- Let $z = e^{-t}\, cos(x/\pi)$. Show that

$$\frac{\partial z}{\partial t} = \pi^2 \frac{\partial^2 z}{\partial x^2}.$$

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2. **A-** Use the differetial to approximate the change in the function $w = f(x,y,z) = x^2\, ln(z^2+y^2)$ as (x,y,z) changes from $(1,2,3)$ to $(0.8, 1.78, 3.1)$.

   B- Find the local extrema and saddle points of the function

$$f(x,y) = x^3 - 3\,x\,y + y^3.$$

   C- Change the integral to cylinderical coordinates then evaluate it;

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{(x^2+y^2)^2}^{1} x^2\, dz\, dy\, dx.$$

   D- Evaluate the integral

$$\int_0^8 \int_{x^{1/3}}^{2} \frac{dy\, dx}{y^4+1}.$$

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3. **A-** Determine whether the following series are absolutely convergent, conditionally convergent or divergent:

$$\mathbf{1)}\sum_{n=1}^{\infty}\frac{1+sin(n)}{n^2} \qquad \mathbf{2)}\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\sqrt{ln\, n}}$$

$$\mathbf{3)}\sum_{n=1}^{\infty}\frac{n!}{ln(n+1)} \qquad \mathbf{4)}\sum_{n=0}^{\infty}\frac{(2n+3)^2}{(n+1)^3}$$

B- Find the interval and radius of convergence of the series

$$\sum_{n=1}^{\infty}\frac{ln(n)}{e^n}\,(x-e)^n.$$

C- Find the Maclaurin series for the function $f(x) = e^x$, for all $x \in \mathbf{R}$, then show that the series represents the function $f(x) = e^x$ for all x, i.e show that $lim_n R_n(x) = 0$.

D- Approximate the integral

$$\int_0^1 \sqrt{x}\, e^{-x^2}\, dx$$

to three decimal places.

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4. **A-** Prove the following:

- If $\sum a_n x^n$ has a radius of convergence r, then the series $\sum a_n x^{2n}$ has a radius of convergence $\sqrt{r}$.
- If $\sum a_n$ is absolutely convergent series, then $\sum a_n x^n$ is absolutely convergent for all $x \in [-1, 1]$.

B- The following questions are extra credits:

1- Let $\{a_n\}$, $a_n \neq 0$ for all n, be a divergent sequence, give an example to show that $\{1/a_n\}$ is not necessarily a convergent seqeunce.

2- Let $\sum a_n$ and $\sum b_n$ be two convergent series. Give an example to show that $\sum a_n\, b_n$ is not necessarily a convergent series.

Good Luck.