

**KING SAUD UNIVERSITY
DEPARTMENT OF MATHEMATICS**

TIME: 3 HOURS

FULL MARKS: 100

Question 1. a) Solve the initial value problem

$$x^2 y' = y - xy, \quad y(-1) = -1. \quad [10]$$

b) Find the differential equation of the family $y = -x - 1 + ce^x$. Find the orthogonal trajectories of this family. [12]

Question 2. a) Solve the Cauchy Euler differential equation

$$x^2 y'' - 4xy' + 6y = \ln x^2 \quad [10]$$

b) Solve the system [12]

$$x' + y' + 2x + 2y = \sin 2t$$

$$y' + 3y + 5x = \cos 2t.$$

Question 3. a) Solve the differential equation

$$3(1+t^2)y' = 2ty(y^3 - 1) \quad [11]$$

b) Solve the initial value problem

$$\frac{dy}{dx} = (x + y + 2)^2, \quad y(-2) = 1. \quad [11]$$

Question 4. Find two linearly independent power series solutions for the differential equation $(x^2 - 1)y'' + xy' - y = 0$. [11]

Question 5. Find the Fourier series of the function

$$f(x) = 2|\sin x|, \quad -\pi < x < \pi, \quad f(x + 2\pi) = f(x) \quad [11]$$

Question 6. A thermometer reading 70° F is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the temperature reads 110° F after $1/2$ minute and 145° F after 1 minute. How hot is the oven. [12]