

**KING SAUD UNIVERSITY  
DEPARTMENT OF MATHEMATICS**

**TIME: 1H30**

**FULL MARKS: 40**

**Question 1.** a) Determine whether the initial value problem has a unique solution

$$y' = y + (\sin x)^2, \quad y(\pi) = 1. \quad [6]$$

b) Find  $m$  such that  $y^m$  will be an integrating factor for the DE:  $(y^2 + 2xy)dx = x^2 dy$  and solve the DE. [6]

**Question 2.** a) Solve the initial value problem

$$\frac{dy}{dx} = (x + y + 2)^2, \quad y(-2) = 1. \quad [6]$$

b) Write the differential equation

$$x \frac{dy}{dx} = y^2(1 + 2y^2) \quad [7]$$

into Bernoulli's equation form, and then solve it.

**Question 3** Determine the region of the  $xy$ -plane for which the differential equation

$$\sqrt{y+1} \frac{dy}{dx} = \ln(x-5)$$

Would have a unique solution through a point  $(x_0, y_0)$  in the region. [7]

**Question 4.** A hot bar is moved to a storage room whose temperature is  $20^\circ \text{F}$ . 5 minutes later, the temperature of the bar is  $119^\circ \text{C}$ , and after 15 minutes, it is  $31^\circ \text{C}$ . What was the temperature of the bar when it was placed in the storage room? [8]