

Brief solutions to selected problems from the textbook:

6.4.1: $n_1 = 5$, $\bar{x}_1 = 48$, $s_1 = 8.1$; $n_2 = 7$, $\bar{x}_2 = 56.7$, $s_2 = 8.1$

$$t_{10,0.95} = 1.8125, s_p^2 = \frac{4(8.1)^2 + 6(8.1)^2}{10} = 65.61, \sqrt{\frac{s_p^2}{5} + \frac{s_p^2}{7}} = 4.7429$$

$$\mu_1 - \mu_2 = (-8.7) \pm (8.5965) = (-17.2965, -0.1035)$$

6.4.2: $n_1 = 8$, $\bar{x}_1 = 74$, $\sqrt{\frac{\sigma_1^2}{n_1}} = 6$; $n_2 = 12$, $\bar{x}_2 = 101$, $\sqrt{\frac{\sigma_2^2}{n_2}} = 5$

$$Z_{0.95} = 1.645, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{36 + 25} = 7.8103$$

$$\mu_1 - \mu_2 = (-27) \pm (12.8479) = (-39.8479, -14.1521)$$

6.4.3: $n_1 = 120$, $\bar{x}_1 = 21.4444$, $s_1 = 15.392$; $n_2 = 42$, $\bar{x}_2 = 3.3333$, $s_2 = 14.595$

$$Z_{0.95} = 1.645, \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{7.046} = 2.6544$$

$$\mu_1 - \mu_2 = (18.1111) \pm (4.3665) = (13.7446, 22.4776)$$

6.4.4: $n_1 = 5$, $\bar{x}_1 = 6.02$, $s_1 = 1.217$; $n_2 = 5$, $\bar{x}_2 = 3.378$, $s_2 = 1.687$; equal vars.

$$t_{8,0.95} = 1.86, s_p^2 = \frac{4(1.217)^2 + 4(1.687)^2}{8} = 2.1635, \sqrt{\frac{s_p^2}{5} + \frac{s_p^2}{7}} = 0.9303$$

$$\mu_1 - \mu_2 = (2.642) \pm (1.7304) = (0.9116, 4.3724)$$

6.4.5: $n_1 = 123$, $\bar{x}_1 = 0.83$, $\sqrt{\frac{\sigma_1^2}{n_1}} = 0.16$; $n_2 = 290$, $\bar{x}_2 = 1.39$, $\sqrt{\frac{\sigma_2^2}{n_2}} = 0.18$

$$Z_{0.95} = 1.645, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{(0.16)^2 + (0.18)^2} = 0.2408$$

$$\mu_1 - \mu_2 = (-0.56) \pm (0.3961) = (-0.9561, -0.1639)$$

6.4.6: $n_1 = 12$, $\bar{x}_1 = 13.21$, $s_1 = 1.05$; $n_2 = 9$, $\bar{x}_2 = 11$, $s_2 = 1.01$; equal vars.

$$t_{19,0.95} = 1.7291, s_p^2 = \frac{11(1.05)^2 + 8(1.01)^2}{19} = 1.0678, \sqrt{\frac{s_p^2}{12} + \frac{s_p^2}{9}} = 0.4557$$

$$\mu_1 - \mu_2 = (2.21) \pm (0.788) = (1.422, 2.998)$$

6.4.7: $n_1 = 12$, $\bar{x}_1 = 11.1$, $s_1 = 1.5$; $n_2 = 12$, $\bar{x}_2 = 7.8$, $s_2 = 2$; equal vars.

$$t_{22,0.95} = 1.7171, s_p^2 = \frac{11(1.5)^2 + 11(2)^2}{22} = 3.125, \sqrt{\frac{s_p^2}{12} + \frac{s_p^2}{12}} = 0.7217$$

$$\mu_1 - \mu_2 = (3.3) \pm (1.2392) = (2.0608, 4.5392)$$

6.4.8: $n_1 = 11$, $\bar{x}_1 = 26$, $s_1 = 5$; $n_2 = 14$, $\bar{x}_2 = 21$, $s_2 = 6$; equal vars.

$$t_{23,0.95} = 1.7139, s_p^2 = \frac{10(5)^2 + 13(6)^2}{23} = 31.2174, \sqrt{\frac{s_p^2}{11} + \frac{s_p^2}{14}} = 2.2512$$

$$\mu_1 - \mu_2 = (5) \pm (3.8583) = (1.1417, 8.8583)$$

6.4.9: $n_1 = 20$, $\bar{x}_1 = 7$, $s_1 = 2$; $n_2 = 24$, $\bar{x}_2 = 36$, $s_2 = 10$, $w_1 = 0.2$, $w_2 = 4.1667$

$$t_{19,0.95} = 1.7291, t_{23,0.95} = 1.7139, t' = 1.7146, \sqrt{w_1 + w_2} = 2.0897$$

$$\mu_1 - \mu_2 = (-29) \pm (3.583) = (-32.583, -25.417)$$

6.4.10: $n_1 = 14, \bar{x}_1 = 20.41, s_1 = 6.81; n_2 = 18, \bar{x}_2 = 14.72, s_2 = 6.2$; equal vars.

$$t_{30,0.95} = 1.6973, s_p^2 = \frac{13(6.81)^2 + 17(6.2)^2}{30} = 41.879, \text{ s.e.} = \sqrt{\frac{s_p^2}{11} + \frac{s_p^2}{14}} = 2.3061$$

$\mu_1 - \mu_2 = (5.69) \pm (3.9141) = (1.7759, 9.6041)$, yes, by between 1.8, and 9.6, because the interval does not include 0.

6.5.1 $X = 166, n = 947, \text{ p.e.} = \hat{p} = 0.1753, 1 - \hat{p} = 0.8247, \text{ r.c.} = Z_{0.95} = 1.645,$

$$\text{ s.e.} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.01236; 90\% \text{ c.i.} = p = 0.1753 \pm 0.0203 = (0.155, 0.1956)$$

6.5.2 $n = 1229, \text{ p.e.} = \hat{p} = 0.5, 1 - \hat{p} = 0.5, \text{ r.c.} = Z_{0.975} = 1.96,$

$$\text{ s.e.} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.01426; 95\% \text{ c.i.} = p = 0.5 \pm 0.02795 = (0.47205, 0.52795)$$

6.5.3 $n = 86, \text{ p.e.} = \hat{p} = 0.128, 1 - \hat{p} = 0.872, \text{ r.c.} = Z_{0.995} = 2.58,$

$$\text{ s.e.} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.03603; 99\% \text{ c.i.} = p = 0.128 \pm 0.09296 = (0.03504, 0.22096)$$

6.5.4 $X = 88, n = 125, \text{ p.e.} = \hat{p} = 0.704, 1 - \hat{p} = 0.296, \text{ r.c.} = Z_{0.975} = 1.96,$

$$\text{ s.e.} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.04083; 95\% \text{ c.i.} = p = 0.704 \pm 0.08 = (0.624, 0.784)$$

6.6.1 $X_1 = 145, n_1 = 194, \hat{p}_1 = 0.74742, X_2 = 113, n_2 = 189, \hat{p}_2 = 0.59788$

$$\text{ p.e.} = \hat{p}_1 - \hat{p}_2 = 0.14954, \text{ r.c.} = Z_{0.975} = 1.96, \text{ s.e.} = \sqrt{0.0022452} = 0.047384$$

$$p_1 - p_2 = 0.14954 \pm 0.092873 = (0.056667, 0.242413)$$

6.6.2 $n_1 = 404, \hat{p}_1 = 0.592, n_2 = 795, \hat{p}_2 = 0.449, \text{ r.c.} = Z_{0.995} = 2.58,$

$$\text{ p.e.} = \hat{p}_1 - \hat{p}_2 = 0.143, \text{ s.e.} = 0.030105$$

$$p_1 - p_2 = 0.14253 \pm 0.077671 = (0.064857, 0.2202)$$

6.6.3 $X_1 = 52, n_1 = 67, \hat{p}_1 = 0.77612, X_2 = 89, n_2 = 133, \hat{p}_2 = 0.66917$

$$\text{ p.e.} = \hat{p}_1 - \hat{p}_2 = 0.10695, \text{ r.c.} = Z_{0.975} = 1.96, \text{ s.e.} = \sqrt{0.0042578} = 0.0652515$$

$$p_1 - p_2 = 0.10695 \pm 0.12789 = (-0.02094, 0.23484)$$

6.6.4 $X_1 = 58, n_1 = 215, \hat{p}_1 = 0.269767, X_2 = 217, n_2 = 1140, \hat{p}_2 = 0.190351$

$$\text{ p.e.} = \hat{p}_1 - \hat{p}_2 = 0.079416, \text{ r.c.} = Z_{0.95} = 1.645, \text{ s.e.} = \sqrt{0.0010512} = 0.0324225$$

$$p_1 - p_2 = 0.079416 \pm 0.053335 = (0.02608, 0.132751)$$

7.2.1 $n = 211, \bar{x} = 59.1, s = 25.6; H_0 : \mu \geq 60, H_1 : \mu < 60 ; \alpha = 0.1$

$$z_0 = \frac{59.1 - 60}{\frac{25.6}{\sqrt{211}}} = -.51, z_{0.9} = 1.28, p = 0.305, \text{ do not rej. } H_0, \text{ no less than } 60.$$

7.2.2 $n = 3918, \bar{x} = .9, s = .96; H_0 : \mu \geq 1, H_1 : \mu < 1 ; \alpha = 0.01$

$$z_0 = \frac{0.9 - 1}{\frac{0.96}{\sqrt{3918}}} = -0.51, \quad z_{0.99} = 2.33, \quad p = 0.0, \quad \text{rej. } H_0, \text{ the mean is } < 1.$$

$$7.2.3 \quad n = 8, \quad \bar{x} = 25, \quad s = 4; \quad H_0 : \mu \leq 20, \quad H_1 : \mu > 20 \quad ; \alpha = 0.01$$

$$t_0 = \frac{25 - 20}{\frac{4}{\sqrt{8}}} = 3.54, \quad t_{7,0.99} = 3, \quad p = 0.005, \quad \text{rej. } H_0, \text{ the mean is } > 20.$$

$$7.2.4 \quad n = 25, \quad \bar{x} = 4.8, \quad s = 2; \quad H_0 : \mu \leq 4, \quad H_1 : \mu > 4 \quad ; \alpha = 0.05$$

$$t_0 = \frac{4.8 - 4}{\frac{2}{\sqrt{25}}} = 2, \quad t_{24,0.95} = 1.71, \quad p = 0.028, \quad \text{rej. } H_0, \text{ the mean is } > 4.$$

$$7.2.5 \quad n = 49, \quad \bar{x} = 21, \quad s = 11; \quad H_0 : \mu \geq 30, \quad H_1 : \mu < 30 \quad ; \alpha = 0.05$$

$$z_0 = \frac{21 - 30}{\frac{11}{\sqrt{49}}} = -5.73, \quad z_{0.95} = 1.645, \quad p = 0.0, \quad \text{rej. } H_0, \text{ the mean is } < 30.$$

$$7.2.6 \quad n = 9, \quad \bar{x} = 6.5, \quad s = 0.6; \quad H_0 : \mu \leq 6, \quad H_1 : \mu > 6 \quad ; \alpha = 0.05$$

$$t_0 = \frac{6.5 - 6}{\frac{0.6}{\sqrt{9}}} = 2.5, \quad t_{8,0.95} = 1.86, \quad p = 0.018, \quad \text{rej. } H_0, \text{ the mean is } > 6.$$

$$7.2.7 \quad n = 25, \quad \bar{x} = 77, \quad s = 10; \quad H_0 : \mu \geq 80, \quad H_1 : \mu < 80 \quad ; \alpha = 0.05$$

$$t_0 = \frac{77 - 80}{\frac{10}{\sqrt{25}}} = -1.5, \quad t_{24,0.95} = 1.71, \quad p = 0.073, \quad \text{do not rej. } H_0, \text{ the mean is } < 80.$$

$$7.3.1 \quad n_1 = 79, \quad \bar{x}_1 = 2.1, \quad s_1 = 1.1; \quad H_0 : \mu_1 \geq \mu_2, \quad \mu_1 - \mu_2 \geq 0 \quad ; \alpha = 0.01$$

$$n_2 = 74, \quad \bar{x}_2 = 1.6, \quad s_2 = 1.2; \quad H_1 : \mu_1 < \mu_2, \quad \mu_1 - \mu_2 < 0$$

$$z_0 = \frac{2.1 - 1.6}{\sqrt{\frac{1.1^2}{79} + \frac{1.2^2}{74}}} = 2.68121, \quad z_{0.99} = 2.33, \quad p = 0.004, \quad \text{rej. } H_0, \quad \mu_1 < \mu_2$$

$$7.3.2 \quad n_1 = 11, \quad \bar{x}_1 = 7116, \quad \sqrt{\frac{\sigma_1^2}{n_1}} = 173; \quad n_2 = 11, \quad \bar{x}_2 = 7058, \quad \sqrt{\frac{\sigma_2^2}{n_2}} = 205;$$

$$H_0 : \mu_1 \leq \mu_2, \quad \mu_1 - \mu_2 \leq 0 \quad ; \alpha = 0.05; \quad H_1 : \mu_1 > \mu_2, \quad \mu_1 - \mu_2 > 0$$

$$z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{58}{268.2424} = 0.2162, \quad z_{0.95} = 1.645, \quad p = 0.414, \quad \text{do not rej. } H_0,$$

$$7.3.3 \quad n_1 = 16, \quad \bar{x}_1 = 6218, \quad s_1 = 693; \quad H_0 : \mu_1 = \mu_2, \quad \mu_1 - \mu_2 = 0 \quad ; \alpha = 0.05$$

$$n_2 = 16, \quad \bar{x}_2 = 8421, \quad s_2 = 1398; \quad H_1 : \mu_1 \neq \mu_2, \quad \mu_1 - \mu_2 \neq 0$$

$$t_0 = \frac{6218 - 8421}{\sqrt{\frac{693^2}{16} + \frac{1398^2}{16}}} = -5.65, \quad t' = t_{15,0.975} = 2.1315, \quad p = 0.0, \quad \text{rej. } H_0,$$

7.3.4 $n_1 = 16, \bar{x}_1 = 22.5, s_1 = 4.1; H_0 : \mu_1 \geq \mu_2, \mu_1 - \mu_2 \geq 0 ; \alpha = 0.05$
 $n_2 = 21, \bar{x}_2 = 26.9, s_2 = 3.2; H_1 : \mu_1 < \mu_2, \mu_1 - \mu_2 < 0$
 $t_0 = -3.67, t_{35,0.95} = 1.69, p = 0.0, \text{ rej. } H_0,$

 7.4.1 $H_0 : \mu_1 - \mu_2 \geq 0, \mu_d \geq 0 ; \alpha = 0.01, H_1 : \mu_1 - \mu_2 < 0, \mu_d < 0$
 $\mu_1 - \mu_2 = d = -4, -8, -6, -1, -1, \dots n = 17, \bar{d} = -3.35294, s_d^2 = 5.117647$
 $s_d = 2.262221; t_0 = \frac{\bar{d}}{s_d} = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = -6.11104, t_{16,0.99} = 2.58349; p = 0.0, \text{ rej. } H_0.$

7.4.2 $H_0 : \mu_1 - \mu_2 = 0, \mu_d = 0 ; \alpha = 0.01, H_1 : \mu_1 - \mu_2 > 0, \mu_d > 0$
 $n = 13, \bar{d} = 5.46923, s_d^2 = 9.687$
 $s_d = 3.11244; t_0 = 6.335725, t_{12,0.99} = 2.68; p = 0.0, \text{ rej. } H_0.$

7.4.3 $H_0 : \mu_1 - \mu_2 = 0, \mu_d = 0 ; \alpha = 0.05, H_1 : \mu_1 - \mu_2 \neq 0, \mu_d \neq 0$
 $\mu_1 - \mu_2 = d = -9, 0, -1, -7, -7, 2, -1, -3 \quad n = 8, \bar{d} = -3.25, s_d^2 = 15.6429$
 $s_d = 3.95511; t_0 = \frac{\bar{d}}{s_d} = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = -2.32418, t_{7,0.975} = 2.364; p = 0.053, \text{ do not rej. } H_0.$

 7.5.1 $\hat{p} = 0.63, H_0 : p \leq p_0 = 0.6, H_1 : p > 0.6, n = 696, \alpha = 0.05$
 $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = 1.61555, z_{0.95} = 1.645, p = 0.053 ; \text{ do not rej.}$

H_0 ; not more than 60%.

7.5.2 $\hat{p} = 0.216, H_0 : p \leq p_0 = 0.15, H_1 : p > 0.15, n = 119, \alpha = 0.05$
 $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = 2.01633, z_{0.95} = 1.645, p = 0.022 ; \text{ rej. } H_0; \text{ more}$

than 15%.

7.5.3 $\hat{p} = 0.66, H_0 : p \leq p_0 = 0.6, H_1 : p > 0.6, n = 670, \alpha = 0.05$
 $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = 3.17, z_{0.95} = 1.645, p = 0.001 ; \text{ rej. } H_0; \text{ yes.}$

7.5.5 $\hat{p} = 0.1,$
 $H_0 : p \geq p_0 = 0.15, H_1 : p < 0.15, n = 250, \alpha = 0.05$
 $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = -2.214, z_{0.95} = 1.645, p = 0.0134 ; \text{ rej. } H_0; \text{ yes.}$

7.6.1

$n_1 = 67, x_1 = 46; n_2 = 57, x_2 = 10; \hat{p}_1 = 0.686567, \hat{p}_2 = 0.175439; \alpha = 0.01$

$H_0 : p_1 \leq p_2, H_1 : p_1 > p_2; \bar{p} = \frac{56}{124} = 0.451613$

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} = 5.699, z_{0.99} = 2.326, p = 0.0$$

7.6.4 $n_1 = 150, x_1 = 21; n_2 = 200, x_2 = 48; \hat{p}_1 = 0.14, \hat{p}_2 = 0.24; \alpha = 0.05$

$H_0 : p_1 = p_2, H_1 : p_1 \neq p_2; \bar{p} = \frac{69}{350} = 0.197143$

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} = -2.32711, z_{0.975} = 1.96, p = 0.01$$
