

MATH 204

**Question 1. a)** Determine a region of the  $xy$ -plane, for which the differential equation:

$(1 - y) \frac{dy}{dx} = xy$ , has a unique solution through every point  $(x_0, y_0)$  in the region. [6]

**b)** Solve the differential equation  $x^2 \frac{dy}{dx} + xy = 1$ ,  $x > 0$ . [6]

**Question 2. a)** Find the general solution of the differential equation  $\frac{dy}{dx} = \frac{-2xy^3 - 2}{3x^2y^2 + 3}$ . [6]

**b)** Solve the initial value problem  $\begin{cases} \frac{dy}{dx} = \frac{1 - 3x - 3y}{x + y} \\ y(0) = 1 \end{cases}$  [7]

**Question 3.** Solve the initial value problem  $\begin{cases} y^{1/2} \frac{dy}{dx} + y^{3/2} = 1 \\ y(0) = 4 \end{cases}$  [7]

**Question 4.** A culture initially has  $p_0$  number of bacteria. At  $t = 1$  hour, the number is measured to be  $\frac{3}{2}p_0$ . If the rate of growth is proportional to the number of bacteria  $p(t)$  present at time  $t$ , determine the time necessary for the number of bacteria to be  $3p_0$ . [8]