

MATH 204

**Question 1. a)** Find an interval centered about  $x = 0$ , for which the following initial value problem has a unique solution.

$$\begin{cases} (\cos x)y'' + (\sin x)y' = e^x \cos x \\ y(0) = 1, \quad y'(0) = 0 \end{cases} \quad [6]$$

**b)** Determine whether the following set of functions is linearly independent on  $\mathbb{R}$

$$f(x) = x + 2x^2, \quad g(x) = -x^2, \quad h(x) = 3x^3. \quad [6]$$

**Question 2. a)** If  $y_1 = \frac{\sin x}{\sqrt{x}}$  is a solution of  $x^2 y'' + xy' + (x^2 - 1/4)y = 0$ ,  $0 < x < \pi$ .

Find the second solution  $y_2$ . [6]

**b)** Find only the form of the particular solution of the differential equation

$$y''' + y' = 2 + \sin x + xe^x. \quad [6]$$

**Question 3.** Find the general solution of the differential equation

$$xy'' - 2xy' + xy = e^x, \quad x > 0. \quad [8]$$

**Question 4.** Use the method of power series to find the general solution about the point  $x = 0$  of the differential equation  $2y'' - xy' - 3y = 0$ . [8]