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On the distribution of order statistics from generalized logistic samples

Summary - Some distributional properties of a generalization of the logistic distribution are presented in this paper. For instance, such a distribution can be used for modelling binary response data and for discriminating among certain survival models as well. The cumulants of this general distribution are expressed in terms of the generalized Reimann zeta function and its moments are determined via a recursive relationship. Connections to the extreme value model and the type-2 beta distribution are pointed out. Finally, closed form representations of the density function and the moments of order statistics from such a generalized logistic distribution are also provided.

Key Words - Generalized logistic distribution; Order statistics; Moments; Cumulants.

1. A GENERALIZED LOGISTIC DISTRIBUTION

Consider a density function of the form

$$f(x) = c \cdot \frac{e^{\alpha x}}{(1+e^x)^{\alpha+\beta}}, \quad -\infty < x < \infty, \quad \Re(\alpha) > 0, \quad \Re(\beta) > 0,$$

where $\Re(\cdot)$ denotes the real part of $(\cdot)$.

Its normalizing constant, c, is such that

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty} f(x)\mathrm{d}x = c\int_{-\infty}^{\infty} \frac{e^{(\alpha-1)x}e^x}{(1+e^x)^{\alpha+\beta}}\mathrm{d}x \\ &= c\int_0^{\infty} y^{\alpha-1}(1+y)^{-(\alpha+\beta)}\mathrm{d}y \quad (\text{with} \quad y = e^x) \\ &= c\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \quad \text{for} \quad \Re(\alpha) > 0 \quad \text{and} \quad \Re(\beta) > 0. \end{aligned}$$

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The resulting density

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{e^{\alpha x}}{(1+e^{x})^{\alpha+\beta}}, \quad -\infty < x < \infty,\ \Re(\alpha) > 0,\ \Re(\beta) > 0, \quad (1)$$

will be referred to as the generalized logistic density. Clearly, the above density function also has the following representation:

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{e^{-\beta x}}{(1+e^{-x})^{\alpha+\beta}}, \quad -\infty < x < \infty,\ \Re(\alpha) > 0,\ \Re(\beta) > 0. \quad (1')$$

Prentice (1976, equation 2) proposed using the density function given in (1) to model the relationship between response probability and dosage in quantal response bioassay. It is also seen from Kalbfleisch and Prentice (1980, pp. 28, 63 & 197) that this model is connected to the generalized F failure time distribution, which can be used for instance for discriminating among certain parametric survival models.

As particular cases, one has the logistic distribution for which $\alpha = 1$ and $\beta = 1$ and the type I, type II and type III generalized logistic distributions for which ($\alpha = b$, $\beta = 1$), ($\alpha = 1$, $\beta = b$), and ($\alpha = b$, $\beta = b$), respectively, see Balakrishnan (1988a).

For various distributional results on the logistic distribution and applications, the reader is referred to Johnson and Kotz (1970, Chapter 22), Balakrishnan (1992), Balakrishnan and Rao (1998a,b) and Govindarajulu and Nanthakumar (2000).

Perks (1932) used a type III generalized logistic distribution for graduating life data. Gumbel (1944) showed that this distribution turns out to be the limiting distribution of the reduced mid-range in random samples from a large class of symmetrical continuous distributions. In addition, Cutler (1992) proved that, asymptotically, the type III distribution is connected to statistics based on the kth nearest neighbor distance. The type III distribution can also be used as an approximation to the Student's t-distribution as shown by George, El-Saidi and Singh (1986).

The density of the smallest order statistics in a random sample from the standard logistic distribution is given by a type II generalized logistic distribution. This distribution has been discussed by Dubey (1969).

Zelterman (1989) used a type I generalized logistic distribution for modelling the log odds of moderately rare events. Gerstenkorn (1992) proposed a methodology for estimating the parameter in a type I distribution. Representations of the moments of order statistics from the type I generalized logistic distribution are given in Balakrishnan (1988a,b) and Zelterman (1988).

The cumulants as well as certain distributional properties of the generalized logistic distribution as specified by equation (1) are given in Section 2, while several results in connection with the moments of order statistics from that distribution are presented in Section 3.

2. Cumulants and distributional properties

The moment-generating function of the generalized logistic distribution is seen to be

$$\begin{aligned} M_X(t) =& \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_{-\infty}^{\infty} \frac{e^{(\alpha+t)x}}{(1+e^x)^{\alpha+\beta}} \mathrm{d}x \\ =& \frac{\Gamma(\alpha+t)\Gamma(\beta-t)}{\Gamma(\alpha)\Gamma(\beta)}, \quad -\Re(\alpha) < \Re(t) < \Re(\beta), \end{aligned} \tag{2}$$

on writing $(1+e^x)^{\alpha+\beta}$ as $(1+e^x)^{(\alpha+t)+(\beta-t)}$. A special case of this result agrees with the moment-generating function given in Johnson and Kotz (1970), (equation (9), Chapter 22).

Denoting the logarithmic derivative of the gamma function, that is $\Gamma'(\theta)/\Gamma(\theta)$, by $\psi(\theta)$, the mth cumulant K_m, $m = 1, 2, \ldots$, can be determined as follows:

$$\begin{aligned} K_m &= \frac{\partial^{m-1}}{\partial t^{m-1}} [\psi(\alpha+t) - \psi(\beta-t)]|_{t=0}] \\ &= [\psi^{(m-1)}(\alpha+t) + (-1)^m \psi^{(m-1)}(\beta-t)]|_{t=0} \\ &= (m-1)! \left\{ (-1)^m \sum_{r=0}^{\infty} \frac{1}{(\alpha+r)^m} + \sum_{r=0}^{\infty} \frac{1}{(\beta+r)^m} \right\} \\ &= (m-1)! [(-1)^m \zeta(m,\alpha) + \zeta(m,\beta)] \end{aligned} \tag{3}$$

where $\zeta(s,a) = \sum_{r=0}^{\infty} 1/(r+a)^s$ (any term with $r + a = 0$ being excluded), denotes the generalized Reimann zeta function. The mean and the variance of the distribution are respectively given by K_1 and K_2, while the moments $\mu_m = \mathrm{E}(X^m)$, are readily obtained from the cumulants by making use of the following relationship which is available from the well-known recurrence relation between derivatives and logarithmic derivatives as given for instance by Mathai (1993) in equation (1.7.11) with $B(z)$ as the moment-generating function:

$$\mu_n = \sum_{i=0}^{n-1} \binom{n-1}{i} K_{n-i} \mu_i, \tag{4}$$

with initial value $\mu_0 = 1$, see also Smith (1995). For example, $\mu_1 = K_1$; $\mu_2 = K_2 + K_1^2$; $\mu_3 = K_3 + 3K_2K_1 + K_1^3$; and $\mu_4 = K_4 + 4K_3K_1 + 3K_2^2 + 6K_2K_1^2 + K_1^4$.

When $\alpha = \beta$, one has

$$\frac{e^{x\alpha}}{(1+e^x)^{2\alpha}} = \frac{e^{-x\alpha}}{(1+e^{-x})^{2\alpha}},$$

which implies that the distribution is symmetric about zero. Thus in that case, the odd moments (as well as the odd cumulants as can be seen from equation (3)) will be equal to zero, the even cumulants being given by

$$K_{2m} = (2m-1)![2\xi(2m,\alpha)], \; m = 1, 2, \ldots$$

Further, if $\alpha = 1$ then

$$\begin{aligned}\xi(2m,1) &= \xi(2m) \quad \text{(Riemann zeta function)}\\ &= \sum_{n=1}^{\infty} 1/n^{2m}\\ &= \pi^2/6 \quad \text{for } m = 1\\ &= \pi^4/90 \quad \text{for } m = 2\\ &= \frac{(-1)^{m+1}(2\pi)^{2m}}{2(2m)!} B_{2m},\end{aligned}$$

where B_{2m} is a Bernoulli number (see pp. 8-13 of Mathai (1993)), $B_2 = 1/6$, $B_4 = -1/30$, $B_6 = 1/42$, $B_8 = -1/30, \ldots$ Thus, when $\alpha = \beta = 1$, we have $K_2 = \pi^2/3$; $K_4 = 2\pi^4/15$; $K_6 = 16\pi^6/63$; and $K_8 = 16\pi^8/15$.

We now show that the generalized logistic distribution is in fact connected to the extreme value model. Consider the extreme value density for fixed parameter δ

$$f_1(x|\delta) = \frac{e^{-\beta x} e^{-\delta e^{-x}}}{\Gamma(\beta)\delta^{-\beta}}, \quad -\infty < x < \infty, \quad \delta > 0, \quad \Re(\beta) > 0, \tag{5}$$

and let the parameter δ have a gamma-type prior distribution with density function

$$g(\delta) = \frac{\delta^{\alpha-1} e^{-\delta}}{\Gamma(\alpha)}, \quad \delta > 0, \quad \Re(\alpha) > 0. \tag{6}$$

The posterior density of X is then

$$\begin{aligned}f(x) &= \int_0^{\infty} f_1(x|\delta) g(\delta) \mathrm{d}\delta\\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{e^{-\beta x}}{(1+e^{-x})^{\alpha+\beta}}\\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{e^{\alpha x}}{(1+e^{x})^{\alpha+\beta}}, \quad -\infty < x < \infty, \quad \Re(\alpha) > 0, \quad \Re(\beta) > 0,\end{aligned}$$

which is the density of the generalized logistic distribution given in equation (1).

This distribution is also related to type-2 beta (or F) distributions: from the density in (1), observe that $Y_1 = e^X$ is distributed as a real type-2 beta random variable with parameters (α, β) or equivalently as the ratio Z_1/Z_2 where Z_1 and Z_2 are independently distributed real gamma random variables with parameters $(\alpha, 1)$ and $(\beta, 1)$, respectively. Then, one has

$$X = \ln Y_1 = \ln Z_1 - \ln Z_2.$$

But the moment generating function of $\ln Z_1$ is

$$\begin{aligned} M_{\ln Z_1}(t) &= \int_0^\infty z_1^t \frac{z_1^{\alpha-1} e^{-z_1}}{\Gamma(\alpha)} \mathrm{d}z_1 \\ &= \frac{\Gamma(\alpha+t)}{\Gamma(\alpha)} \end{aligned}$$

while that of $-\ln Z_2$ is $M_{-\ln Z_2}(t) = M_{\ln Z_2}(-t) = \Gamma(\beta - t)/\Gamma(\beta)$ $\Re(t) < \Re(\beta)$. The resulting moment-generating function for X is clearly that given in equation (2) for the generalized logistic distribution. Similarly, $Y_2 = e^{-X}$ is distributed as a real type-2 beta random variable with parameters (β, α) or equivalently as the ratio Z_2/Z_1 where Z_1 and Z_2 are as defined above.

The cumulative distribution function of the generalized logistic distribution will involve incomplete type-2 beta functions which can be evaluated in terms of Whittaker functions or confluent hypergeometric functions for general parameters α and β. When α or β is a positive integer, the following technique can be employed for determining the distribution function. First, once more, we observe that

$$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{e^{\alpha x}}{(1+e^x)^{\alpha+\beta}} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{e^{-\beta x}}{(1+e^{-x})^{\alpha+\beta}}. \tag{7}$$

If α or β is a positive integer, we make use of the representation of the density of X given respectively on the left- or right-hand side of equation (7). For example, if α is a positive integer,

$$\begin{aligned} F(u) &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_{-\infty}^{u} \frac{e^{\alpha x}}{(1+e^x)^{\alpha+\beta}} \mathrm{dx} \\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_1^{1+e^u} (y-1)^{(\alpha-1)} y^{-(\alpha+\beta)} \mathrm{d}y \quad (\text{with} \quad y = 1+e^x) \\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \sum_{s=0}^{\alpha-1} \binom{\alpha-1}{s} \frac{(-1)^s}{\beta+s} \left(1 - \frac{1}{(1+e^u)^{\beta+s}}\right), \end{aligned} \tag{8}$$

and when $\alpha = 1$,

$$F(u) = \frac{\Gamma(\alpha+\beta)}{\beta\Gamma(\alpha)\Gamma(\beta)}\left[1 - \frac{1}{(1+e^u)^\beta}\right].$$

For general α and β, $\Re(\alpha) > 0, \Re(\beta) > 0$,

$$F(u) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\sum_{s=0}^{\infty}\frac{(\alpha-1)_s(-1)^s}{s!(\beta+s)}\left[1 - \frac{1}{(1+e^u)^{\beta+s}}\right]$$

where, for example, $(a)_n = \Gamma(a+n)/\Gamma(a)$ is the Pochammer symbol.

3. Distribution of order statistics

First we determine the distribution of U_1, the r_1-th order statistic of a sample of size n generated from a generalized logistic population whose parameter α is assumed to be a positive integer. The density of U_1 is given by

$$f_{U_1}(u_1) = \frac{\Gamma(n+1)}{\Gamma(r_1)\Gamma(r_2)}[F(u_1)]^{r_1-1}[1-F(u_1)]^{r_2-1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{e^{\alpha u_1}}{(1+e^{u_1})^{\alpha+\beta}} \quad (9)$$

where $r_2 = n+1-r_1$ and $F(u_1)$ can be obtained from equation (8). Note that we can express $1 - F(u_1)$ as

$$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)(1+e^{u_1})^\beta}\left[a_0 + \frac{a_1}{1+e^{u_1}} + \ldots + \frac{a_{\alpha-1}}{(1+e^{u_1})^{\alpha-1}}\right] \quad (10)$$

where

$$a_s = \binom{\alpha-1}{s}\frac{(-1)^s}{\beta+s}.$$

Then for a positive integer δ,

$$[1-F(u_1)]^\delta = c_\delta \sum_{m_0+\ldots+m_{\alpha-1}=\delta}\frac{\delta!}{m_0!\ldots m_{\alpha-1}!}a_0^{m_0}\ldots a_{\alpha-1}^{m_{\alpha-1}} \times (1+e^{u_1})^{-(m_1+2m_2+\ldots+(\alpha-1)m_{\alpha-1})}$$

where

$$c_\delta = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^\delta (1+e^{u_1})^{-\beta\delta}.$$

The moment-generating functions of U_1 can be obtained as follows:

$$\begin{aligned}
M_{U_1}(t) &= \frac{\Gamma(n+1)}{\Gamma(r_1)\Gamma(r_2)} \int_{u_1} [F(u_1)]^{r_1-1}[1-F(u_1)]^{r_2-1} e^{tu_1} \\
&\quad \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{e^{\alpha u_1}}{(1+e^{u_1})^{\alpha+\beta}} du_1 \\
&= \frac{\Gamma(n+1)}{\Gamma(r_1)\Gamma(r_2)} \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^{r_2} \sum_{m=0}^{r_1-1} \binom{r_1-1}{m} (-1)^m \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^m \\
&\quad \times \sum_{m_0+\ldots+m_{\alpha-1}=r_2+m-1} (r_2+m-1)! \frac{a_0^{m_0}}{m_0!} \cdots \frac{a_{\alpha-1}^{m_{\alpha-1}}}{m_{\alpha-1}!} \\
&\quad \times \frac{\Gamma(\alpha+t)\Gamma(\beta-t+m_1+2m_2+\ldots+(\alpha-1)m_{\alpha-1}+\beta(r_2+m-1))}{\Gamma(\alpha+\beta+m_1+2m_2+\ldots+(\alpha-1)m_{\alpha-1}+\beta(r_2+m-1))}
\end{aligned} \tag{11}$$

for $r_2 = n+1-r_1$ and $-\Re(\alpha) < \Re(t) < \Re(\beta)$. When $r_1 = 1$, in which case r_1 is the smallest order statistic, $m = 0$ and

$$\begin{aligned}
M_{U_1}(t) &= n \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^n \sum_{m_0+\ldots+m_{\alpha-1}=n-1} (n-1)! \frac{a_0^{m_0}}{m_0!} \cdots \frac{a_{\alpha-1}^{m_{\alpha-1}}}{m_{\alpha-1}!} \\
&\quad \times \frac{\Gamma(\alpha+t)\Gamma(\beta-t+m_1+2m_2+\ldots+(\alpha-1)m_{\alpha-1}+\beta(n-1))}{\Gamma(\alpha+\beta+m_1+2m_2+\ldots+(\alpha-1)m_{\alpha-1}+\beta(n-1))}.
\end{aligned} \tag{12}$$

Further, if $\alpha = 1$, $m_0 = n-1$, and since $a_0 = 1/\beta$, $(n-1)!a_0^{m_0}/m_0! = 1/\beta^{n-1}$, and the ratio of gamma functions becomes

$$\frac{\Gamma(1+t)\Gamma(n\beta-t)}{\Gamma(1+n\beta)}.$$

Thus for $r_1 = 1$ and $\alpha = 1$,

$$M_{U_1}(t) = n\beta \frac{\Gamma(1+t)\Gamma(n\beta-t)}{\Gamma(1+n\beta)}. \tag{13}$$

The h-th integer moment of U_1 can be determined by differentiating (11) and evaluating the result at $t = 0$. We observe that only the product of gamma functions containing t will be affected. Let $B = \Gamma(\alpha+t)\Gamma(\beta-t+m_1+2m_2+\ldots+(\alpha-1)m_{\alpha-1}+\beta(r_2+m-1))$, then

$$\begin{aligned}
\mu^{(h)} &= \frac{\partial^h B}{\partial t^h}\Big|_{t=0} \\
&= \frac{\partial^{h-1} AB}{\partial t^{h-1}}\Big|_{t=0}
\end{aligned}$$

where $A = \partial \ln B/\partial t = \psi(\alpha + t) - \psi(\beta - t + m_1 + 2m_2 + \ldots + (\alpha - 1)m_{\alpha-1} + \beta(r_2 + m - 1))$. Then

$$\mu^{(h)} = \sum_{s_1=0}^{h-1} \binom{h-1}{s_1} \mu^{(h-1-s_1)} \mu_{(s_1)} \tag{14}$$

where

$$\begin{aligned}\mu_{(s_1)} &= \frac{\partial^{s_1} A}{\partial t^{s_1}}\Big|_{t=0} \\ &= (-1)^{s_1+1} s_1! [\zeta(s_1 + 1, \alpha) \\ &\quad + (-1)^{s_1+1} \zeta(s_1+1, \beta + m_1 + 2m_2 + \ldots + (\alpha - 1)m_{\alpha-1} + \beta(r_2 + n - 1))]\end{aligned}$$

with $\mu_{(0)} = \psi(\alpha) - \psi(\beta + m_1 + 2m_2 + \ldots + (\alpha - 1)m_{\alpha-1} + \beta(r_2 + n - 1))$ and $\mu^{(0)} = B|_{t=0}$. Thus for $h = 0, 1, 2, \ldots$, the h-th moment of U_1 is

$$\begin{aligned}E(U_1^h) &= \frac{\Gamma(n+1)}{\Gamma(r_1)\Gamma(r_2)} \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^{r_2} \sum_{m=0}^{r_1-1} \binom{r_1 - 1}{m} (-1)^m \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^m \\ &\quad \times \sum_{m_0+\ldots+m_{\alpha-1}=r_2+m-1} (r_2 + m - 1)! \frac{a_0^{m_1}}{m_0!} \cdots \frac{a_{\alpha-1}^{m_{\alpha-1}}}{m_{\alpha-1}!} \\ &\quad \times \frac{1}{\Gamma(\alpha + \beta + m_1 + 2m_2 + \ldots + (\alpha - 1)m_{\alpha-1} + \beta(r_2 + n - 1))} \mu^{(h)}\end{aligned}$$

where $\mu^{(h)}$ is defined in equation (14).

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