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اجوبة الاختبار الفصلي
صيف ١٤٣٠-١٤٣١هـ

السؤال الاول :

(أ) —

$$f(x) = \frac{\theta^x e^{-\theta}}{x!}, \quad x = 0,1,2,\dots$$

$$L(\theta) = \frac{\theta^{\sum x_i} e^{-n\theta}}{\prod_{i=1}^{n} x_i!}$$

let $H_0: \theta = 0.25$ vs $H_1^*: \theta = \theta_1, \ \theta_1 > 0.25$

باستخدام تمهيدية نيمان بيرسون نحصل على

$$\frac{L(\theta_0)}{L(\theta_1)} = \frac{L(0.25)}{L(\theta_1)} \le k$$

$$\Rightarrow \frac{(0.25)^{\sum x_i} e^{-0.25n}}{(\theta_1)^{\sum x_i} e^{-n\theta_1}} \le k$$

$$\Rightarrow \left(\frac{0.25}{\theta_1}\right)^{\sum x_i} e^{-n(0.25-\theta_1)} \le k$$

$$\Rightarrow \sum x_i \ln\left(\frac{0.25}{\theta_1}\right) \le \ln k + n(0.25-\theta_1)$$

since $\theta_1 > 0.25 \Rightarrow \ln\left(\frac{0.25}{\theta_1}\right) < 0$

$$\Rightarrow \sum_{i=1}^{n} x_i \geq \frac{\ln k + n(0.25-\theta_1)}{\ln\left(\frac{0.25}{\theta_1}\right)}$$

$$\Rightarrow \left\{(x_1,\dots,x_{20}) : \sum_{i=1}^{20} x_i \geq c\right\}, \quad c = \frac{\ln k + n(0.25-\theta_1)}{\ln(0.25/\theta_1)}$$

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$$\alpha = p\left(\sum_{i=1}^{20} X_i \geq 11 \,\middle|\, \theta = 0.25\right)$$ (ب)

$$= p(Y \geq 11), \quad Y \sim \text{Poisson}(5)$$

$$= 1 - p(Y \leq 10) = \underline{0.014}$$

(ت)

$$\pi(\theta) = p\left(\sum_{i=1}^{20} X_i \geq 11 \,\middle|\, \theta = .5\right)$$

$$= 1 - p(Y \leq 10), \quad Y \sim \text{Poisson}(10)$$

$$= 0.42$$

(٣)

السؤال الثاني

(أ)

$$f(x) = \frac{\beta}{x^{\beta+1}} \ , \ 1 \leq x < \infty \ , \ \beta > 0$$

$$= a(\beta)\, b(x)\, e^{c(\beta) d(x)}$$

حيث

$$a(\beta) = \beta \ , \ b(x) = \frac{1}{x}$$

$$c(\beta) = \beta \ , \ d(x) = -\ln x$$

∴ $f(x)$ تنتمي للعائلة الاسية بمعلمة واحدة

(ب):

$c(\beta) = \beta$ دالة تزايدية في β لذلك فإنه

اختبار $H_0: \beta = \beta_0$ vs $H_1: \beta < \beta_0$

أفضل المنطقة الحرجة

$$\left\{ (x_1, x_2, \dots, x_n) : -\sum_{i=1}^{n} \ln(x_i) \leq k_1 \right\}$$

حيث

$$\alpha = P\left(+\sum \ln x_i \geq k_1 \mid \beta = \beta_0\right)$$

$$= P\left(\sum \ln x_i \geq k_1 \mid \beta = \beta_0\right)$$

$$= P\left(W \geq k_1 \mid \beta = \beta_0\right) \ , \ W = \sum_{1}^{n} \ln x_i$$

let $y = \ln x \Rightarrow f_Y(y) = \frac{\beta}{(e^y)^{\beta+1}} e^{y} \ , \ y \geq 0$

$$= \beta e^{-\beta y} \ , \ y \geq 0$$

(3)

all i $Y \sim$ exponential$\left(\frac{1}{\beta}\right)$ aup,

$$W \sim \text{gamma}\left(n, \frac{1}{\beta}\right)$$

$$\alpha = P(W \geq R_1 \mid \beta = \beta_0)$$ all i

$$= \int_{R_1}^{\infty} \frac{w^{n-1}\beta_0^n}{\Gamma(n)} e^{-\beta w}\, dw$$

let $\beta w = \frac{t}{2} \Rightarrow \beta\, dw = \frac{dt}{2}$

$w = R_1 \to t = 2\beta R_1$, $w = \infty \Rightarrow t = \infty$

$$= \int_{2\beta R_1}^{\infty} \left(\frac{t}{2\beta_0}\right)^{n-1} \frac{\beta_0^n}{\Gamma(n)}\, e^{-\frac{t}{2}} \frac{dt}{2\beta_0}$$

$$= \int_{2\beta R_1}^{\infty} \frac{t^{n-1}\beta_0^n e^{-\frac{t}{2}}}{\Gamma(n)(2\beta_0)^{n-1}}\, dt$$

$$= \int_{2\beta_0 R_1}^{\infty} \frac{t^{n-1} e^{-t/2}}{\Gamma(n)\, 2^{n-1}}\, dt$$

$$= \int_{2\beta R_1}^{\infty} \frac{t^{\frac{k}{2}-1} e^{-t/2}}{\Gamma\left(\frac{k}{2}\right) 2^{\frac{k}{2}-1}}\, dt \quad , k = 2n$$

$$= P\left(\chi^2_k \geq 2\beta_0 R_1\right)$$

$\to c$

(5)

$n = 10 \Rightarrow K = 20 \ , \ \beta_0 = \frac{1}{4}$ (ب)

$\alpha = P(\chi^2_{20} \geq C) \Rightarrow 0.05 = P(\chi^2_{20} \geq C)$

$\Rightarrow C = 31.41 \Rightarrow 2\beta_0 R_1 = 31.41$

$\Rightarrow R_1 = \frac{31.41}{2(\frac{1}{4})} = 62.82$

$\Sigma \ln x_i = 32.96$ ومن البيانات نجد أن

لذلك

$\Sigma \ln x_i = 32.96 < R_1 = 63.82$

∴ لا نستطيع رفض Ho.

السؤال الثالث

$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$, $-\infty < x < \infty$

$\sigma > 0$

$-\infty < \mu < \infty$

$\Omega = \{(\mu, \sigma^2), -\infty < \mu < \infty, \sigma > 0\}$

$W = \{(\mu, \sigma^2); \mu = \mu_0 \ , \sigma > 0\}$

$L(\mu, \sigma^2) = \left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left[-\frac{1}{2\sigma^2}\Sigma(x_i - \mu)^2\right]$

$\hat{\mu} = \bar{x} \ , \ \hat{\sigma}^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2$

(7)

$$L(\hat{\omega}) = L(\mu_0, \hat{\sigma}^2) = \left(\frac{n}{2\pi \sum (x_i - \mu_0)^2}\right)^{n/2} e^{-\frac{n}{2}}$$

$$L(\hat{\Omega}) = L(\bar{x}, \hat{\sigma}^2) = \left(\frac{n}{2\pi \sum (x_i - \bar{x})^2}\right)^{n/2} e^{-\frac{n}{2}}$$

$$\frac{L(\hat{\omega})}{L(\hat{\Omega})} \le \lambda_0, \quad \lambda_0 < 1$$

$$\Rightarrow \frac{L(\hat{\Omega})}{L(\hat{\omega})} \geq \lambda_0^{-1}$$

$$\frac{\sum (x_i - \bar{x})^2 + n(\bar{x} - \mu_0)^2}{\sum (x_i - \bar{x})^2} \geq \lambda_0^{-\frac{2}{n}}$$

$$\Rightarrow \frac{(\bar{x} - \mu_0)^2}{\frac{1}{n} \sum (x_i - \bar{x})^2} \geq d, \quad d = \lambda_0^{-\frac{2}{n}} - 1$$

$$\Rightarrow \frac{(\bar{x} - \mu_0)^2}{\frac{n-1}{n} s^2} \geq d, \quad d = \lambda_0^{-\frac{2}{n}} - 1$$

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}} \geq c \quad \text{or} \quad \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \le -c, \quad \sqrt{(n-1)d}$$

وبالتالى فإن المنطقة الحرجة هى

$$\left\{(x_1, \ldots, x_n) : \frac{\bar{x} - \mu_1}{s/\sqrt{n}} \geq c \text{ or } \frac{\bar{x} - \mu_1}{s/\sqrt{n}} \le c\right\}$$

حيث يمكن حساب c كالتالى

$$\alpha = P\left(\frac{\bar{x} - \mu_0}{s/\sqrt{n}} \geq c \,\middle|\, \mu = 0\right) + P\left(\frac{\bar{x} - \mu_0}{s/\sqrt{n}} \le -c\right)$$

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عندما $n=10$ فإن

$$\frac{\bar{x}-\mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

$$\Rightarrow \frac{\alpha}{2} = P(T \geq c) \Rightarrow c = t_{\alpha/2}(n-1) = t_{0.005}(9) = 3.25$$

المنطقة الحرجة هي :

$$\left\{(x_1, \ldots, x_{10}) : \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \geq 3.25 \text{ or } \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \leq -3.25\right\}$$