اجابات الاختبار الفصلي
الأول - الفصل الأول
١٤٣١ - ١٤٣٠ هـ

$$f(x;\theta) = \frac{x^{3-1}}{\theta^3 \Gamma(3)} e^{-x/\theta} \quad , \; x \geq 0 \; , \theta > 0 \qquad \boxed{1}$$

$$H_0 : \theta = 2 \quad \text{vs} \quad H_1 : \theta = 1$$

$$\{ x_1 < 2 \, , \, x_2 < 2 \}$$

$$\alpha = P(x_1 < 2 \, , \, x_2 < 2 \mid \theta = 2)$$

$$= \left(\int_0^2 \frac{x^{3-1}}{2^3 \Gamma(3)} e^{-x/2} dx \right)^2$$

$$= \left[1 - \int_0^2 \frac{x^{\frac{6}{2}-1}}{2^{6/2} \Gamma(6/2)} e^{-x/2} dx \right]^2$$

$$= \left[1 - P(\chi^2_6 > 2) \right]^2$$

$$= \left[1 - 0.9 \right]^2 = \underline{\underline{0.01}}$$

$$\beta = 1 - \left(P(x_1 < 2 \mid \theta = 1) \right)^2$$

$$= 1 - \left[\int_0^2 \frac{x^2}{2} e^{-x} dx \right]^2$$

$$= 1 - (0.323)^2 = 1 - 0.104 = \underline{\underline{0.896}}$$

(2)

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}, \quad x = 0, 1, 2, \ldots, \mu > 0$$ $\boxed{2}$

$$L(\mu) = \frac{\mu^{\sum x_i} e^{-n\mu}}{\prod_i x_i!}$$

(أ)

وباستخدام نيمان بيرسون نحصل على $H_0: \mu = \mu_0$ vs $H_1^*: \mu = \mu_1 > \mu_0$

$$\frac{L(\mu_0)}{L(\mu_1)} \leq k$$

$$\Rightarrow \left(\frac{\mu_0}{\mu_1}\right)^{\sum x_i} e^{-n(\mu_0 - \mu_1)} \leq k$$

$$\sum_{i=1}^{n} x_i \ln\left(\frac{\mu_0}{\mu_1}\right) - n(\mu_0 - \mu_1) \leq \ln k$$

$$\sum_{i=1}^{n} x_i \geq \frac{\ln k + n(\mu_0 - \mu_1)}{\ln\left(\frac{\mu_0}{\mu_1}\right)} = A$$

المنطقة الحرجة هي $\{(x_1, \ldots, x_n) : \sum_{i=1}^{n} x_i \geq A\}$

حيث $0.05 = P(Y \geq A)$, $Y \sim Poisson(n\mu_0)$

(ب) عندما $\mu_0 = 3$, $n = 3$ فإن

$$0.05 = P(Y \geq A), \quad Y \sim Poisson(9)$$

$$\Rightarrow P(Y \leq A-1) = 0.95 \Rightarrow A - 1 = 14$$

$$\Rightarrow A = 15$$

وحيث ان $\sum x_i = 18$

لذلك $\sum x_i > A \Leftarrow$ نرفض H_0

③

$$f(x) = \binom{M}{x} p^{x}(1-p)^{M-x}$$ ٣

$$H_0: p = \frac{1}{2} \quad vs \quad H_1: p > \frac{1}{2}$$

المنطقة الحرجة هي

$$\{(x_1, x_2, \dots, x_n) : x_i \geq C\}$$

قوة الاختبار

$$\pi(p) = P(x_i \geq C \mid p)$$

$$= P\left(\frac{x_i - Mp}{\sqrt{Mp(1-p)}} \geq \frac{C - Mp}{\sqrt{Mp(1-p)}}\right)$$

$$\pi(p) = P\left(Z \geq \frac{C - Mp}{\sqrt{Mp(1-p)}}\right)$$

$$\pi\left(\frac{1}{2}\right) = 0.1 \Rightarrow P\left(Z \geq \frac{C - M/2}{\sqrt{M/4}}\right) = 0.1$$

$$\Rightarrow \frac{C - M/2}{\sqrt{M/4}} = \frac{2C - M}{\sqrt{M}} = \boxed{1.28} \rightarrow ①$$

$$\pi\left(\frac{2}{3}\right) = P\left(Z \geq \frac{C - 2M/3}{\sqrt{\frac{2M}{9}}}\right) = 0.95$$

$$\Rightarrow \frac{C - 2M/3}{\sqrt{2M/9}} = \frac{3C - 2M}{\sqrt{2M}} = -1.645 \rightarrow ②$$

من ① نجد ان

$$2C = M + 1.28\sqrt{M} \Rightarrow C = \frac{M}{2} + 0.64\sqrt{M}$$

$$3C - 2M = -1.645\sqrt{2M}$$ من ②

$$\Rightarrow \frac{3M}{2} + 1.92\sqrt{M} - 2M = -1.645\sqrt{2M}$$

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$$\Rightarrow \frac{M}{2} + \sqrt{M}\,(-1.645\sqrt{2} - 1.92) = 0$$

$$M + 2\sqrt{M}\,(-1.645\sqrt{2} - 1.92) = 0$$

$$\sqrt{M}\left[\sqrt{M} + 2(-1.645\sqrt{2} - 1.92)\right] = 0$$

$$\Rightarrow \sqrt{M} - 2(1.645\sqrt{2} + 1.92) = 0$$

$$\sqrt{M} = 2(1.645\sqrt{2} + 1.92) = 8.5$$

$$\Rightarrow M = 72.25 \simeq 73$$

$$C = \frac{M}{2} + 0.64\sqrt{n} \simeq 42$$ من ①