

King Saud University
Faculty of Science
Department of Statistics & OR

Midterm Exam 1
Time (2 hours)
Stat 321 (Hypotheses Testing)

Semester 1
1431-1432

Answer the following Questions:

1. Let X_1, X_2 and X_3 be a random sample of size 3 from the exponential population $\exp(1/\theta)$. A test of $H_0: \theta = 1$ vs $H_1: \theta = 0.5$ uses the critical region $\{(X_1, X_2, X_3): X_1 > 1, X_2 > 1, X_3 > 1\}$. Find the probabilities of Type-I and Type-II errors.
2. Let X_1, X_2, \dots, X_{10} be a random sample from $\exp(1/\theta)$. A test of $H_0: \theta = 0.5$ vs $H_1: \theta > 0.5$ uses the critical region $\{(X_1, X_2, \dots, X_{10}): \sum_{i=1}^{10} X_i \geq C\}$.
 - a- Find the values of C if $\alpha = 0.01$
 - b- Find the values of α if $C = 31.41$.
3. Let X_1, X_2, \dots, X_{11} be a random sample from *Bernoulli* (θ)
 - a- Find the best critical region of size 0.0302 for testing $H_0: \theta = 0.4$ vs $H_1: \theta < 0.4$.
 - b- Show that the power function of the test obtained in part (a) is $\pi(\theta) = (1-\theta)^{10}(1+10\theta)$.
4. Let X_1, X_2, \dots, X_{15} denote a random sample from $N(0, \sigma^2)$. Find the uniformly most powerful test of size 0.01 for testing $H_0: \sigma = 2$ vs $H_1: \sigma > 2$.
5. Let X_1, X_2, \dots, X_n denote a random sample from $N(\mu, 225)$. For testing $H_0: \mu = 75$ vs $H_1: \mu > 75$, we choose to use $\{(X_1, X_2, \dots, X_n): \bar{X} \geq C\}$ as the critical region.
 - a- Derive the power of the test $\pi(\theta)$.
 - b- If $\pi(75) = 0.025$ and $\pi(80) = 0.90$, then find n and C .