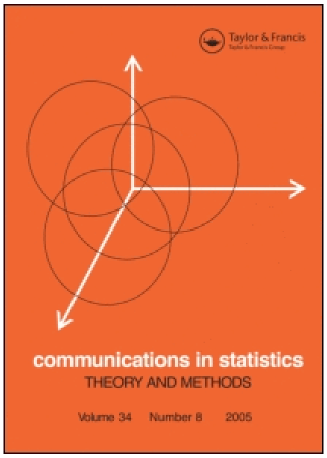

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Order Statistics from the Generalized Exponential Distribution and Applications

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Recently, a new distribution, called generalized exponential distribution (GED), has been introduced and studied quite extensively by authors. The GED can be used as an alternative to gamma and Weibull distributions in many situations. In this article, we use the moments of order statistics from the GED derived by Raqab and Ahsanullah (2001) and Raqab (2004) to develop the correlation goodness -of-fit test for the GED. In addition, we calculate the power of the test based on some other alternative distributions. Further, we construct approximate confidence intervals for the location and scale parameters of the GED. Finally, we apply the procedures developed in the paper to real data set.

Keywords Average width; BLUEs; Confidence intervals; Correlation coefficient; Goodness-of-fit test; Monte Carlo simulation; Moments of order statistics; Power of the test; Pivotal quantity.

Mathematics Subject Classification Primary 60E15; Secondary 62G32.

1. Introduction

Order statistics arise naturally in many real life applications involving data relating to different files such as life testing and economics. Many authors have studied order statistics and the associated inference. Among those are David (1981), David and Nagaraja (2003), Balakrishnan and Cohen (1991), and Arnold et al. (1992). For an extensive survey of moments of order statistics, we refer to Balakrishnan and Sultan (1998).

The three-parameter GED has its probability density function (pdf) as

$$f(x) = \frac{\alpha}{\sigma}(1 - e^{-(x-\mu)/\sigma})^{\alpha-1} e^{-(x-\mu)/\sigma}, \quad x > \mu, \ \mu > 0, \ \alpha > 0, \ \sigma > 0, \tag{1.1}$$

where α, σ, and μ are the shape, scale, and location parameters, respectively.

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The two-parameter pdf of the GED is given by

$$f(x) = \frac{\alpha}{\sigma}(1 - e^{-x/\sigma})^{\alpha-1}e^{-x/\sigma}, \quad x > 0, \ \alpha > 0, \ \sigma > 0, \tag{1.2}$$

while the one-parameter case is given by

$$f(x) = \alpha(1 - e^{-x})^{\alpha-1}e^{-x}, \quad x > 0, \quad \alpha > 0, \quad \sigma > 0. \tag{1.3}$$

The GED was introduced by Gupta and Kundu (1999) as an alternative life model for the gamma and Weibull distributions. They also discuss some theoretical properties of the GED and compared them with respect to the well studied properties of the gamma distribution and Weibull distribution. Finally, they fitted real data set with all three distributions, namely three-parameter gamma, three-parameter Weibull, and three-parameter GED. Gupta and Kundu (2003a) have shown the closeness of gamma and generalized exponential distributions based on different tests. Gupta and Kundu (2003b) have used the ratio of the maximized likelihoods in discriminating between Weibull and generalized exponential distributions while Gupta and Kundu (2004) have used the ratio of the maximized likelihoods in choosing between a generalized exponential and a gamma distributions. Raqab and Ahsanullah (2001) have calculated the coefficient of the best linear estimates (BLUEs) of the location and scale parameters of the GED.

Goodness-of-fit tests are very important techniques for data analysis in the sense of checking whether the given data fits the distributional assumptions of the statistical model. A variety of goodness-of-tests are available in the literature and recently there seems to be significant research on this topic, for more details, see, D'Agostino and Stephens (1986) and Huber-Carol et al. (2002). Correlation coefficient test is considered one of the easiest of such tests, that is because it is only needs special tables introduce from Monte Carlo simulations. The correlation coefficient test introduced by Filliben (1975) first for normal distributions and updated later by Looney and Gulledge (1985) is one of such tests that requires only tables introduced by Monte Carlo simulations. Among others, Kinnison (1985, 1989) used the correlation coefficient method to present tables for testing goodness-of-fit to the extreme-value Type-I (Gumbel) and the extreme-value distribution, respectively. Recently, Sultan (2001) has developed the correlation goodness-of-fit to the logarithmically-decreasing survival distribution.

Let $X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{n:n}$ be the order statistics from the GED given in (1.3). Then, the pdf of the rth order statistics is given by

$$f_{r:n}(x) = C_{r:n}\{F(x)\}^{r-1}\{1 - F(x)\}^{n-r}f(x), \quad 0 < x < \infty, \tag{1.4}$$

where $C_{r:n} = \frac{n!}{(r-1)!(n-r)!}$. For more details see, David (1981), David and Nagaraja (2003) and Arnold et al. (1992). Raqab and Ahsanullah (2001) have derived the first single moments of the rth order statistics from the GED as

$$\mu_{r:n} = C_{r:n}\sum_{i=0}^{n-r}\frac{(-1)^i\binom{n-r}{i}}{i+r}[\psi(1 + [i + r]\alpha) - \Psi(1)], \tag{1.5}$$

where $\Psi(\cdot)$ is the diagamma function defined by $\Psi(x) = d/dx(\ln\Gamma(x)), x \neq 0, -1, -2, \ldots$ (see Abramowitz and Stegun, 1972).

In Sec. 2, we develop the goodness of fit tests of the two-parameter and three-parameter GED. Also, we calculate the power of the tests based on some alternative distributions. In Sec. 3, we use the BLUEs to construct confidence intervals for the location and scale parameters of the GED. Finally, in Sec. 4, we apply the procedures developed in the article to real data set.

2. Goodness-of-Fit Test

In this section, we use the moments of order statistics given in (1.5) to develop the correlation goodness-of-fit test for both two-parameter and three-parameter GED. Also, we calculate the power of the test. In addition, we discuss some numerical examples.

2.1. *Test for the Two-Parameter Case*

Let $X_{1:n}, X_{2:n}, \ldots, X_{n-r:n}$ denote a Type-II right-censored sample from the GED given in (1.2), and let $Z_{i:n} = X_{i:n}/\sigma$, $i = 1, 2, \ldots, n-r$, be the corresponding order statistics from the one-parameter GED given in (1.3). Let us denote $E(Z_{i:n})$ by $\mu_{i:n}$, then $E(X_{i:n}) = \sigma\mu_{i:n}$, $i = 1, 2, \ldots, n-r$. The correlation-type goodness-of-fit test in this case can be formed as

H_0 : F is correct, that is $X_1, X_2, \ldots, X_n$ have $GE(0, \sigma, \alpha)$ given in (1.2) versus,

H_1 : F is not correct, that is $X_1, X_2, \ldots, X_n$ have another pdf,

and the statistic used to run the test is given by

$$T_1 = \sum_{i=1}^{n-r} X_{i:n}\mu_{i:n} \Bigg/ \sqrt{\sum_{i=1}^{n-r} X_{i:n}^2 \sum_{i=1}^{n-r} \mu_{i:n}^2}, \tag{2.1}$$

this statistic represents the correlation between $X_{i:n}$ and $\mu_{i:n}$, $i = 1, 2, \ldots, n-r$. By using the moments $\mu_{i:n}$, $i = 1, 2, \ldots, n-r$ given in (1.5), the statistic T_1 is simulated through Monte Carlo method based on 10,000 simulations. Table 1 represents the percentage points of T_1 for sample sizes $n = 10, 20, 25$, different censoring ratios $p = \frac{n-r}{n}$ and $\alpha = 0.5, 2$.

As we can see from Table 1, the percentage points of T_1 increase as the sample size increases as well as the significance level increases for censoring ratios $p = 1.0, 0.8$.

2.2. *Test for the Three-Parameter Case*

Let $X_{1:n}, X_{2:n}, \ldots, X_{n-r:n}$ denote a Type-II right-censored sample from the distribution in (1.1), and let $Z_i = X_{i+1:n} - X_{1:n}$ and $v_i = \mu_{i+1:n} - \mu_{1:n}$, $i = 1, 2, \ldots, n-r-1$, where $\mu_{i:n}$ be the corresponding moments of order statistics from the one-parameter GED given in (1.3). The correlation-type goodness-of-fit test in this case can be formed as

H_0 : F is correct, that is $X_1, X_2, \ldots, X_n$ have $GE(\mu, \sigma, \alpha)$ given in (1.1) versus,

H_1 : F is not correct, that is $X_1, X_2, \ldots, X_n$ have another pdf,

Table 1
Percentage points of T_1

α	p	n	0.5%	1%	2%	2.5%	5%	10%	20%	30%	40%	50%
0.5	1.0	10	.8667	.8805	.8980	.9038	.9210	.9384	.9547	.9637	.9702	.9756
		20	.8877	.8997	.9165	.9217	.9368	.9510	.9650	.9723	.9775	.9814
		25	.8857	.9082	.9244	.9299	.9447	.9571	.9688	.9753	.9799	.9834
	0.8	10	.8494	.8743	.8967	.9034	.9237	.9409	.9578	.9669	.9734	.9784
		20	.9040	.9200	.9336	.9394	.9527	.9643	.9746	.9801	.9839	.9867
		25	.9192	.9331	.9464	.9497	.9609	.9707	.9789	.9833	.9863	.9886
2.0	1.0	10	.9028	.9207	.9353	.9408	.9538	.9645	.9741	.9796	.9833	.9861
		20	.9318	.9432	.9544	.9584	.9676	.9758	.9826	.9861	.9887	.9906
		25	.9362	.9460	.9593	.9627	.9718	.9787	.9847	.9879	.9900	.9916
	0.8	10	.9272	.9411	.9530	.9563	.9656	.9740	.9808	.9847	.9874	.9895
		20	.9641	.9695	.9743	.9756	.9808	.9852	.9891	.9912	.9927	.9938
		25	.9718	.9751	.9793	.9805	.9844	.9879	.9911	.9928	.9940	.9950

and the statistic used to run the test is given by

$$T_2 = \sum_{i=1}^{n-r-1} (Z_i)(v_i) \Bigg/ \sqrt{\sum_{i=1}^{n-r-1} Z_i^2 \sum_{i=1}^{n-r-1} v_i^2}, \tag{2.2}$$

this statistic represents the correlation between Z_i and v_i, $i = 1, 2, \ldots, n-r$.

Once again by using the moments $\mu_{i:n}$, $i = 1, 2, \ldots, n-r$ given in (1.5), the statistic T_2 is simulated through Monte Carlo method based on 10,000 simulations. Table 2 represents the percentage points of T_2 for sample sizes $n = 10, 20, 25$ and different censoring ratios $p = 1.0, 0.8$.

Table 2
Percentage points of T_2

α	p	n	0.5%	1%	2%	2.5%	5%	10%	20%	30%	40%	50%
0.5	1.0	10	.8706	.8851	.8971	.9015	.9188	.9359	.9532	.9635	.9702	.9756
		20	.8782	.8976	.9151	.9213	.9368	.9514	.9646	.9722	.9773	.9814
		25	.8905	.9063	.9229	.9278	.9433	.9568	.9689	.9752	.9797	.9832
	0.8	10	.8439	.8694	.8948	.9019	.9229	.9419	.9586	.9673	.9735	.9783
		20	.8924	.9149	.9318	.9360	.9498	.9627	.9740	.9796	.9836	.9865
		25	.9181	.9299	.9442	.9483	.9614	.9703	.9789	.9837	.9866	.9889
2.0	1.0	10	.8972	.9101	.9259	.9311	.9452	.9578	.9691	.9755	.9800	.9833
		20	.9129	.9292	.9450	.9496	.9617	.9714	.9793	.9836	.9865	.9888
		25	.9210	.9376	.9527	.9566	.9668	.9748	.9821	.9859	.9885	.9903
	0.8	10	.9019	.9186	.9340	.9384	.9505	.9628	.9736	.9790	.9831	.9860
		20	.9491	.9571	.9650	.9676	.9743	.9807	.9860	.9890	.9907	.9923
		25	.9639	.9688	.9733	.9752	.9801	.9847	.9888	.9911	.9926	.9938

From Table 2, we see that, the percentage points of T_2 increase as the sample size increases as well as the significance level increases for censoring ratios $p = 1.0, 0.8$.

2.3. *Power of the Test*

We calculate the power of the test by replacing the $GE(\mu, \sigma, \alpha)$ random variates generator in the simulation program with generators from the alternative distributions including: normal, lognormal, Cauchy, Weibull, and gamma. Based on different sample sizes, different censoring ratios and 10,000 simulations, the power is calculated to be

$$\text{Power} = \frac{\#\text{ of rejection of } H_0}{10,000},$$

where H_0 is rejected if $T_1(T_2) \geq$ the corresponding percentage points given in Table 1 (Table 2), and $T_1(T_2)$ is evaluated from the alternative distributions.

Tables 3 and 4 represent the power of the test for the two-parameter and three-parameter cases, respectively. The different alternative distributions considered are: (i) normal distribution $N(\mu, \sigma)$; (ii) lognormal $Ln(\mu, \sigma)$, (iii) Weibull distribution with shape a, scale parameter σ, and location parameter μ, $W(\mu, \sigma, a)$; (iv) gamma distribution with shape parameter k, scale parameter σ, and location parameter μ, $G(\mu, \sigma, k)$; and (v) Cauchy distribution with scale parameter σ and location parameter μ, $C(\mu, \sigma)$.

Tables 3 and 4 indicate that the correlation test has good power to reject sample from the chosen alternative distributions. Also, the power increases as the sample sizes increase for all given censoring ratios $p = 1.0$ and 0.8 as well as the significance level increases.

Table 3
Power of the test of the two-parameter case ($\sigma = 1$)

			$N(0, 1)$		$W(0, 1, 3)$		$G(0, 1, 7)$	
α	p	n	5%	10%	5%	10%	5%	10%
0.5	1.0	10	.9580	.9748	.9875	.9977	.9647	.9897
		20	.9972	.9989	1.0000	1.0000	.9997	1.0000
		25	.9995	.9998	1.0000	1.0000	1.0000	1.0000
	0.8	10	.9877	.9921	.9267	.9800	.9520	.9894
		20	.9999	1.0000	.9999	1.0000	1.0000	1.0000
		25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.0	1.0	10	.9923	.9958	.3903	.6218	.3149	.5225
		20	.9999	1.0000	.8241	.9470	.6717	.8609
		25	1.0000	1.0000	.9307	.9849	.8065	.9311
	0.8	10	.9987	.9993	.2128	.4033	.2482	.4587
		20	1.0000	1.0000	.6321	.7952	.7170	.8661
		25	1.0000	1.0000	.7775	.8929	.8721	.9502

Table 4
Power of the test of the three-parameter case $\mu = 0.0, \sigma = 1.0$

			$LN(1, 5)$		$W(0, 1, 6)$		$C(0, 1)$	
α	p	n	5%	10%	5%	10%	5%	10%
0.5	1.0	10	.7530	.8280	.7588	.8650	.6569	.7327
		20	.9567	.9785	.9909	.9975	.8997	.9392
		25	.9827	.9914	.9987	.9995	.9427	.9712
	0.8	10	.6001	.6962	.4455	.6357	.7174	.8059
		20	.8964	.9338	.9361	.9748	.9787	.9881
		25	.9521	.9728	.9857	.9949	.9943	.9969
2.0	1.0	10	.9768	.9872	.2860	.4549	.5864	.6772
		20	1.0000	1.0000	.7241	.8550	.8587	.9117
		25	1.0000	1.0000	.8272	.9195	.9172	.9523
	0.8	10	.9362	.9626	.1036	.2076	.4897	.5928
		20	.9993	.9998	.4363	.5742	.8760	.9155
		25	1.0000	1.0000	.5712	.6961	.9442	.9630

2.4. *Numerical Examples*

In order to show the performance of the test of GED in both cases (two-parameter and three-parameter), we simulate four sets of order statistics each of size 25; they are:

1. Sample from $GE(0, 1, 2)$: two-parameter case of the GED with scale parameter is equal to 1 and shape parameter is equal to 2.
2. Sample from $GE(1, 1, 2)$: three-parameter case of the GED with location parameter is equal to 1, scale parameter is equal to 1 and shape parameter is equal to 2.
3. Sample from $G(0, 2, 10)$: gamma distribution with location parameter is equal to 0 and scale parameter is equal to 2 and shape parameter is equal to 10.
4. Sample from $G(2, 2, 10)$: gamma distribution with location parameter is equal to 2, scale parameter is equal to 2 and shape parameter is equal to 10.

The above four order statistics samples with the analogous moments of order statistics from $GE(0, 1, 2)$ are used to run the test. The results of the test at 5% significance level and at $\alpha = 2$ (whether accept (A) or reject (R) H_0) are given for different values of censoring ratios in the following table

	Decision			
p	GE(0,1,2)	GE(1,1,2)	G(0,2,10)	G(2,2,10)
1.0	A	A	R	R
.8	A	A	R	R

3. Approximate Inference

By using the BLUE of σ given in Balakrishnan and Cohen (1991), we construct confidence intervals for the scale parameter σ of the two-parameter GED through the pivotal quantity

$$P = \frac{\sigma^*}{\sigma\sqrt{V}}, \tag{3.1}$$

where, σ^* is the BLUE of the scale parameter σ and $\sigma^2 V$ is the corresponding variance.

Once again, by using the BLUEs of the location and scale parameters of the three-parameter GED given in Balakrishnan and Cohen (1991), we calculate confidence intervals of the location and scale parameters of the three-parameter GED by using the following pivotal quantities

$$R_1 = \frac{\mu^* - \mu}{\sigma\sqrt{V_1}}, \quad R_2 = \frac{\sigma^* - \sigma}{\sigma\sqrt{V_2}}, \quad \text{and } R_3 = \frac{\mu^* - \mu}{\sigma^*\sqrt{V_1}}, \tag{3.2}$$

where μ^* and σ^* are the BLUE's of μ and σ with variances $\sigma^2 V_1$ and $\sigma^2 V_2$, respectively. R_1 can be used to draw inference for μ when σ is known, while R_3 can be used to draw inference for μ when σ is unknown. Similarly, R_2 can be used to draw inference for σ when μ is unknown.

Tables (5–7) show the lower and upper 1%, 2.5%, 5% and 10% percentage points of R1 and R2 through Monte Carlo Simulations (based on 10,000 runs). The performance of the developed inference can be shown from the simulated average width of the confidence intervals given in Table 8. The following example shows the usefulness of the approximate inference in this section.

1. In this example, we calculate the BLUEs and their variances of the location and scale parameters of the three-parameter GED when $\alpha = 2.0$, $r = 0$, $n = 10, 20, 25$. Then by using the percentage points presented in Table 5, we construct 95% confidence intervals for μ (when σ is known to be 1 and $\alpha = 2.0$) as follows:

$$P(\mu^* - \sigma\sqrt{V_1}(R_1)_{1-\alpha/2} \le \mu \le \mu^* - \sigma\sqrt{V_1}R_1)_{1-\alpha/2} = 1 - \alpha,$$

where σV_1 is the variance of R_1

n	10	20	25
C.I	(−.538, .389)	(−.327, .228)	(−.278, .197)

Table 5
Simulated values of the distribution of R_1 when $\mu = 0.0$ and $\sigma = 1.0$

α	n	1%	2.5%	5%	10%	90%	95%	97.5%	99%
0.5	10	−1.591	−1.061	−.854	−.679	.886	1.729	2.764	4.163
	20	−1.861	−1.111	−.833	−.653	.853	1.671	2.834	4.133
	25	−2.635	−1.404	−1.043	−.738	.807	1.638	2.644	4.268
2.0	10	−2.103	−1.650	−1.448	−1.174	1.317	1.832	2.292	2.863
	20	−2.006	−1.629	−1.415	−1.167	1.349	1.837	2.290	2.808
	25	−1.998	−1.624	−1.425	−1.188	1.340	1.839	2.297	2.852

Table 6
Simulated values of the distribution of R_2 when $\mu = 0.0$ and $\sigma = 1.0$

α	n	1%	2.5%	5%	10%	90%	95%	97.5%	99%
0.5	10	−1.695	−1.519	−1.363	−1.151	1.340	1.815	2.306	2.955
	20	−1.868	−1.655	−1.461	−1.199	1.325	1.777	2.210	2.682
	25	−1.925	−1.676	−1.475	−1.212	1.306	1.786	2.234	2.763
2.0	10	−1.896	−1.698	−1.472	−1.187	1.345	1.775	2.234	2.701
	20	−2.047	−1.784	−1.535	−1.233	1.322	1.719	2.084	2.578
	25	−2.090	−1.799	−1.528	−1.240	1.318	1.748	2.154	2.627

Table 7
Simulated values of the distribution of R_3 when $\mu = 0.0$ and $\sigma = 1.0$

α	n	1%	2.5%	5%	10%	90%	95%	97.5%	99%
0.5	10	−.984	−.831	−.713	−.591	.966	1.967	3.076	4.851
	20	−1.341	−1.024	−.806	−.616	.876	1.767	2.797	4.384
	25	−1.922	−1.333	−1.010	−.730	.832	1.700	2.808	4.419
2.0	10	−1.312	−1.219	−1.125	−.985	1.711	2.595	3.416	4.620
	20	−1.462	−1.351	−1.220	−1.049	1.560	2.239	2.884	3.702
	25	−1.524	−1.387	−1.247	−1.077	1.521	2.180	2.780	3.674

Table 8
Average width of the C.I.'s when $\mu = 0.0$ and $\sigma = 1.0$

		R_1		R_2		R_3	
α	n	90%	95%	90%	95%	90%	95%
0.5	10	0.760	0.112	1.914	2.473	0.079	0.115
	20	0.021	0.033	1.169	1.440	0.022	0.032
	25	0.015	0.022	1.010	1.232	0.015	0.023
2.0	10	0.771	0.927	1.079	1.355	0.879	1.095
	20	0.460	0.555	0.676	0.820	0.490	0.600
	25	0.395	0.475	0.588	0.719	0.416	0.505

2. Similarly, by using the BLUEs and their variances given the above example, and the percentage points of R_2 given in Table 6, we construct 95% confidence intervals for σ, through the formula

$$P\left(\frac{\sigma^*}{1+\sqrt{V_2}(R_2)_{1-\alpha/2}} \leq \sigma \leq \frac{\sigma^*}{1+\sqrt{V_2}(R_2)_{\alpha/2}}\right) = 1-\alpha,$$

and they are as follows:

n	10	20	25
C.I	(.607, 1.955)	(.711, 1.529)	(.729, 1.447)

3. In the case when σ in unknown, we replace it by σ^* and by using the BLUEs given in the above two examples, we determined the 95% confidence intervals for μ through the following formula

$$P(\mu^* - \sigma^*\sqrt{V_1}(R_3)_{1-\alpha/2} \leq \mu \leq \mu^* - \sigma^*\sqrt{V_1}(R_3)_{\alpha/2}) = 1 - \alpha.$$

n	10	20	25
C.I	(−.802, .287)	(−.410, .188)	(−.336, .169)

4. Applications

In this section, we discuss two applications, the first is goodness-of-fit test of real data set from (Lawless, 1982, p. 28). The second application is to obtain the BLUE of the scale parameter of the GED and use it to construct confidence interval.

1. The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in life test and they are: 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84,127.92, 128.04, 173.40. Gupta and Kundu (2004) have obtained the MLEs of the shape and scale parameter as $\hat{\alpha} = 5.2589$ and $\hat{\sigma} = 31.85$. For the purpose of using our test to the above lifetime data, we need to the moments of order statistics $\mu_{i:23}$, $i = 1, 2, \ldots, 23$ from the GE. These moments are calculated from (1.5) when $\alpha = 5.2589$ as: 0.7273, 0.9408, 1.9500, 1.2255, 1.3437, 1.4551, 1.5628, 1.6689, 1.7750, 1.8826, 1.9928, 2.1072, 2.2271, 2.3544, 2.4913, 2.6407, 2.8068, 2.9956, 3.2168, 3.4871, 3.8399, 4.3587, 5.376. Then by using the above moments and data life, we calculate T_1 and T_2 as follows:

Statistic	Calculated	Simulated
T_1	0.9984	0.9817
T_2	0.9975	0.9695

As we can see, the GE distribution fits the data at 5%. So, we recommend the two-parameter GED for the given data.

2. The coefficients of the BLUE of the scale parameter σ, when $\sigma = 1$ and $\alpha = 5.2589$ are: calculated to be:

0.03028, 0.02846, 0.02730, 0.02636, 0.02554, 0.02479, 0.02408, 0.02341, 0.02276, 0.02212, 0.02149, 0.02087, 0.02024, 0.01961, 0.01897, 0.01831, 0.01762, 0.01692, 0.01617, 0.01537, 0.01449, 0.01350, 0.01226. Then the BLUE of σ is calculated to be $\sigma^* = 30.8658$ with variance $Var(\sigma^*) = 0.01147$.

Next, by using the BLUE $\sigma^* = 30.8658$ and $Var(\sigma^*) = 0.01147 = V$, we can obtain a confidence interval for σ by simulating the percentage points of the pivotal quantity (3.1) (10,000 runs when $n = 23$ and $\sigma = 1$) to get

1%	2.5%	5%	10%	90%	95%	97.5%	99%
7.2580	7.5249	7.7767	8.1002	10.6344	11.0228	11.3205	11.8116

and 95% confidence intervals of σ is calculated as

$$\left(\frac{\sigma^*}{11.3205\sqrt{V}}, \quad \frac{\sigma^*}{7.5249\sqrt{V}}\right) = (25.4583, 38.2997).$$

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