King Saud University	Stat 559 (Theory of Statistics –II) Midterm Exam
College of Science	Second semester, 1428-1429
Department of Statistics and OR	Time: 2 Hours

Answer the Following Questions

1. Let $X_1,\ldots,X_n$ is a random sample from population with pdf

$$f(x;\theta)=\frac{3\theta^3}{(x+\theta)^4},\quad x>0,\ \theta>0.$$

(a) Show that $T=2\bar{X}$ is an unbiased estimator of θ.
(b) Determine the efficiency of T.

2. Let X is a random variable whose pdf is given by

$$f(x,\theta)=a(\theta)b(x)\exp[c(\theta)d(x)]$$

(a) Show that

$$E(d(X))=\frac{-a'(\theta)}{a(\theta)c'(\theta)}$$

$$I(\theta)=\left[\frac{a'(\theta)}{a(\theta)}\right]^2-\frac{a''(\theta)}{a(\theta)}+\frac{c''(\theta)a'(\theta)}{c'(\theta)a(\theta)}$$

$$Var(d(X))=\frac{I(\theta)}{\left[c'(\theta)\right]^2}.$$

(b) Apply part (a) when X is distributed as $N(\mu,\sigma^2)$, σ is known.

3. (a) State and prove the Rao-Blackwell theorem.
(b) Suppose $X_1,\ldots,X_n$ is a random sample from Poisson distribution with mean θ.
(i) Find the CRLB for the variances of unbiased estimators of θ.
(j) State why the estimator $\hat{\theta}=X_1$ is a bad estimator for θ in spite of unbiased.
(k) Based on $\hat{\theta}=X_1$, find an improved estimator for θ using the Rao-Blackwell theorem.
(l) Is this improved estimator the MVUE?

4. Let $\bar{X}$ denote the mean of a random sample of size n from a distribution that has mean μ and variance $\sigma^2 = 10$. Find n so that the probability is approximately 0.9954 that the random interval $(\bar{X} - 0.5, \bar{X} + 0.5)$ include μ.