King Saud University	Stat 559 (Theory of Statistics –II) Assignment-3
College of Science	Second semester, 1428-1429
Department of Statistics and OR	Due date:

Answer the Following Questions

1. Let $X_1,\ldots,X_n$ is a random sample from $\Gamma(2,\theta)$ distribution, where θ is unknown. Let $Y = \sum_{i=1}^{n} X_i$.
   (a) Find the distribution Y and determine the constant c so that cY is an unbiased estimator of θ.
   (b) If $n = 5$, show that $P(5.59 < \frac{2Y}{\theta} < 34.2) = 0.95$.
   (c) Using part (b), show that if y is the value of Y once the sample is drawn, and then the interval $\left(\frac{2y}{34.2}, \frac{2y}{9.59}\right)$ is a 95% confidence interval for θ.
   (d) Suppose the sample results in the values 44.8079, 1.5215, 12.1929, 12.5734, 43.2305. Based on these data, obtain the point estimate of θ as determined in part (a) and the computed 95% confidence interval in part (c). What does the confidence interval mean?

2. Let $X_1,\ldots,X_n$ represents a random sample from a exponential($\frac{1}{\theta}$), if $T = \frac{(n-1)}{n\bar{X}}$ an unbiased estimate for θ. Then find the efficiency of T

3. Suppose $X_1,\ldots,X_n$ is a random sample from Poisson distribution with mean θ and let X_1 is an unbiased estimate of θ. Find UMVUE of θ.

4. Let x be a single observation from beta(θ,1) pdf

(a) Let $Y = -(\log X)^{-1}$. Evaluate the confidence coefficient of the set [Y/2, Y].

(b) Use the pivotal quantity X^{θ} to set up a confidence interval having the same confidence coefficient as the interval in part (a).

(c) Compare the two confidence intervals in parts (a) and (b).

5. Find a $(1-\alpha)$ confidence interval for θ, given $X_1,\ldots,X_n$ iid with pdf

(a) $f(x,\theta) = 1,\ \theta - 1/2 < x < \theta + 1/2.$

(b) $f(x,\theta) = \dfrac{2x}{\theta^2},\ 0 < x < \theta,\ \theta > 0.$