

**Answer the Following Questions**

1. Let  $X_1, \dots, X_n$  is a random sample from

$$f(x; \alpha, c) = \frac{c}{\alpha} \left(\frac{x}{\alpha}\right)^{c-1} \exp\left[-\left(\frac{x}{\alpha}\right)^c\right], \quad x > 0, \quad \alpha, c > 0.$$

- (a) Find the MMEs and MLEs of both  $\alpha$  and  $c$ .  
(b) Discuss the efficiency of the obtained estimates.
2. Let  $X_1, \dots, X_n$  represent a random sample from a gamma( $\alpha = 3, \beta = \theta$ ) distribution,  $\theta > 0$ . Determine the MLE of  $\theta$ .
3. Let  $X_{1:n} \leq \dots \leq X_{n:n}$  be the order statistics of from

$$f(x) = 1, \quad \theta - \frac{1}{2} \leq x \leq \theta + \frac{1}{2}.$$

Find the MLE of  $\theta$

4. Suppose  $X_1, \dots, X_n$  are iid with pdf  $f(x) = \frac{2x}{\theta^2}$ ,  $0 < x \leq \theta$ . Find
- The MLE of  $\theta$
  - The constant  $c$  so that  $E(c\hat{\theta}) = \theta$ .
  - The MLE of the median of the distribution
5. Let  $X$  have a gamma distribution with  $\alpha = 4$  and  $\beta = \theta > 0$ .
- Find Fisher information  $I(\theta)$ .
  - Let  $X_1, \dots, X_n$  is a random sample from this distribution, show that the MLE of  $\theta$  is UMVUE.
  - What is the asymptotic distribution of  $\sqrt{n}(\hat{\theta} - \theta)$ ?

6. Let  $X$  is a random variable whose pdf is given by

$$f(x, \theta) = a(\theta)b(x) \exp[c(\theta)d(x)]$$

Show that

$$E(d(X)) = \frac{-a'(\theta)}{a(\theta)c'(\theta)}$$

$$I(\theta) = \left[ \frac{a'(\theta)}{a(\theta)} \right]^2 - \frac{a''(\theta)}{a(\theta)} + \frac{c''(\theta)a'(\theta)}{c'(\theta)a(\theta)}$$

$$\text{Var}(d(X)) = \frac{I(\theta)}{[c'(\theta)]^2}.$$