King Saud University
College of Science
Department of Statistics and OR

Stat 559 (Theory of Statistics –II)
Assignment-1
Second semester, 1428-1429
Due date: 17-3-1429H

Answer the Following Questions

1. Suppose that $X_1,\ldots,X_n$ is a random sample from a symmetric population with mean μ and variance σ^2
   (a) Show that the sample median is an unbiased estimator of μ.
   (b) Show that $\dfrac{\sum_{i=r+1}^{n-r} X_{i:n}}{n-2r}$ is also an unbiased estimator of μ, where $X_{i:n}$ is the i^{th} order statistics from the sample.

2. Suppose that $X_1,\ldots,X_n$ is a random sample from Uniform$(0,\theta)$ population, where θ is unknown parameter
   (a) Show that $\bar{X}$ and $\dfrac{(n+1)X_{n:n}}{n}$ are both unbiased estimators of θ
   (b) Compare their variance and comment which parameter is more precise and what the relative efficiency is?

3. Suppose $X_1,\ldots,X_n$ is a random sample from the exponential distribution with mean θ
   (a) Prove that $\dfrac{\bar{X}+X_1}{2}$ represents an unbiased estimate for θ .
   (b) Prove that $\dfrac{n\bar{X}^2}{n+1}$ represents an unbiased estimate for θ^2.

4. Let $X_1,\ldots,X_n$ is a random sample from

$$f(x;\alpha,c)=\frac{c}{\alpha}\left(\frac{x}{\alpha}\right)^{c-1}\exp\left[-\left(\frac{x}{\alpha}\right)^{c}\right],\ x>0,\ \alpha,c>0.$$

   (a) Find the MMEs and MLEs of both α and c .
   (b) Discuss the efficiency of the obtained estimates.