King Saud University	Stat 621 (Statistical Inference-I)
College of Science	Assignment 4
Department of Statistics and OR	After Eid

Answer the Following Questions:

1. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from
$$f(x;\theta) = 3\theta x^2 \exp\left(-\theta x^3\right), \quad x \geq 0,\ \theta > 0.$$
   (a) Show that $2\theta \sum_{i=1}^{n} x_i^3$ is a pivotal function.

   (b) Construct a 90% equal-tailed confidence interval for θ based on $X_1 X_2, \ldots, X_{10}$.

2. Let $\bar{X}$ be the mean of a random sample of size n from an exponential distribution $\exp(1/\theta)$. Independently form the first sample, a second sample of size m and mean $\bar{Y}$ is taken from $\exp(1/\lambda)$. Find the distribution of $\frac{\lambda \bar{X}}{\mu \bar{Y}}$ and use it to obtain a $100(1-\alpha)\%$ equal-tailed confidence interval for λ/μ.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from Poisson distribution with mean θ, truncated at 0. Show that the MLE $\hat{\theta}$ must satisfy the equation
$$\bar{X} = \hat{\theta}\left(1 - e^{-\hat{\theta}}\right)^{-1}.$$
What is the asymptotic distribution of $\hat{\theta}$

4. Let Y denote the smallest observation in a random sample of size n from the distribution with pdf
$$f(x) = \frac{\theta}{x^2},\ x > 0,\ \theta > 0.$$
Use the pivotal function $Z = Y/\theta$ to find a 95% equal tailed CI for θ if the sample is: 3.7, 2.6, 2.0, 2.9.

5. Let Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from Poisson distribution with mean μ. Find $100(1-\alpha)\%$ equal tailed large sample CI for μ.