King Saud University
College of Science
Department of Statistics and OR

Stat 621 (Statistical Inference-I)
First semester, 1431-1432: Assignment 3
Due date: After Eid

Answer the Following Questions:

1. Let $X_1, X_2, \ldots, X_n$ is a random sample from Poisson distribution with mean θ and T_1 be an unbiased estimator of θ. Find MVUE of θ.

2. Let $X_1, X_2, \ldots, X_n$ is a random sample from the exponential distribution

$$f(x) = \frac{1}{\theta}\exp\{-x/\theta\}, \; x > 0, \; \theta > 0.$$

   Use Lehman-Scheffe Theorem to obtain MVUE of θ and $1/\theta$.

3. Let $X_1, X_2, \ldots, X_n$ is a random sample of size n drawn from gamma distribution with parameters $\alpha = 4$ and θ, where

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1}\exp\left(-x/\theta\right), \; x > 0, \; \theta > 0.$$

   (a) Show that the MLE of θ is a efficient estimator.

   (b) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$?.

4. Let $X_1, X_2, \ldots, X_n$ is a random sample of size n drawn from

$$f(x) = \frac{3\theta^3}{(x+\theta)^4}, \quad x > 0, \; \theta > 0,$$

   show that $2\bar{X}$ is an unbiased estimator of θ and determine its efficiency.

5. Let $X_1, X_2, \ldots, X_n$ is a random sample of size n drawn from Bernoulli(p), where $n \geq 2$ and $0 < p < 1$ is the unknown parameter

   - Derive the UMVUE of $e^2(p(1-p))$.
   - Derive the CRLB of $e^2(p(1-p))$.

6. Exercises (6.5) page 300 in Casella and Berger (2002) problems: (6.25), (6.27), (6.37),(6.43).

7. Exercises (7.4) page 355 in Casella and Berger (2002) problems: (7.6), (7.7), (7.8), (7.11), (7.12), (7.18), (7.44), (7.45), (7.51), (7.57).