King Saud University
College of Science
Department of Statistics and OR

Stat 621 (Statistical Inference-I)
First semester, 1431-1432: Assignment 1-b
Due date:

Answer the Following Questions:

1. Let $X_1X_2, \ldots, X_n$ be a random sample of size n drawn from $N(\mu, \sigma^2)$. Find the mean squared errors of the following estimators:

$$(i) \quad \hat{\mu} = \bar{X} \quad (ii) \quad \hat{\sigma^2} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 \quad (iii) \quad \hat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

2. Let $x_1, X_2, \ldots, X_n$ be a random sample of size n drawn from the exponential distribution with mean θ.
   (a) Prove that $T_1 = \frac{1}{2}(\bar{X} + X_1)$ represents an unbiased estimate for θ,
   (b) Prove that $T_2 = \frac{n}{n+1}\bar{X}^2$ represents an unbiased estimate for θ^2,
   (c) Find the constant c such that $T_3 = \frac{c}{\bar{X}}$ represents an unbiased estimate for $\tau(\theta) = \frac{1}{\theta}$.

3. Suppose that $x_1, X_2, \ldots, X_n$ is a random sample drawn from a population whose density function is uniform on the interval $(\mu - \sqrt{3}\sigma\ ,\ \mu + \sqrt{3}\sigma)$, where both of μ and σ are unknown. Find the MMEs and MLEs of μ and σ. In your judgement, which method gives a better efficiency?.

4. Let $x_1, X_2, \ldots, X_n$ is a random sample drawn from Weibull

$$f(x; \alpha, c) = \frac{c}{\alpha}\left(\frac{x}{\alpha}\right)^{c-1} \exp\left[-\left(\frac{x}{\alpha}\right)^{c}\right], \quad x > 0,\ \alpha, c > 0.$$

   Find the MMEs and MLEs of α and c. Discuss the efficiency of the obtained estimates.