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AN ITERATIVE LEAST-SQUARES MINIMIZATION APPROACH TO DEPTH DETERMINATION FROM GRAVITY ANOMALIES

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ABSTRACT

A new least-squares approach is developed to determine the depth to a buried structure from a residual gravity anomaly. By defining the anomaly value g(max) at the origin, the problem of depth determination has been transformed into the problem of finding a solution of a nonlinear equation of the form $z = f(z)$. Formulas have been derived for a sphere, a two-dimensional (2-D) horizontal cylinder, a vertical cylinder and the first horizontal derivative of the gravity from a 2-D thin faulted layer. The method is applied to synthetic data with and without random errors. The validity and accuracy of the method is tested on two field examples from Sweden and Senegal, the depth obtained by the present method compares favorable with other methods given in the literature.

INTRODUCTION

The sphere, the horizontal cylinder, the vertical cylinder and the thin faulted layer models are very useful in determining the depth to a buried structure from its residual gravity anomaly. The models may not be geologically realistic, but usually an approximate equivalence between the calculated gravity effect and the observed gravity data is sufficient to justify a simple geometric interpretation (Abdelrahman and El -Araby, 1993).

Several graphical and non-least-squares numerical methods have been developed to find the depth of most geological structures from the residual gravity anomaly. Examples are given by Nedelkov and Burnev (1962), Odegard and Berg (1965), Nettleton (1976), Mohan et. al.. (1986), Bowin et.al. (1986) and Shaw and Agrawal (1990). On the other hand, several least-squares approaches have been developed to determine depth of burial from gravity data, such as those given by Gupta (1983), Lines and Treitel (1984), Abdelrahman (1990), Abdelrahman et. al. (1991), Abdelrahman and El-Araby (1993) and Abdelrahman and Sharafeldin (1995).

The aim of the present paper is to develop a new least-squares approach to depth determination from the residual gravity anomaly. The validity of the method is tested on synthetic data with and without random errors and also on two field examples from Karrbo, Vastmanland, Sweden and Louga, Senegal, West Africa.

THE LEAST-SQUARES METHOD

The gravity effects of the sphere, the horizontal cylinder, and the vertical cylinder (semiinfinite vertical line-element approximation) are given by Gupta (1983) as

$$\Delta g(x_i, z) = \frac{A z^m}{(x_i^2 + z^2)^q}, \qquad i = 1,2,3,\ldots N. \qquad (1)$$

In equation (1), z is the unknown depth to center of the geometric object (or depth to top of vertical cylinder), q is the shape factor, A is the amplitude coefficient, and x is a position coordinate. The gravity effect of a thin faulted layer was given in Nettleton (1976) as $\Delta g(x_i, z) = A(\pi/2 + \tan^{-1} x_i/z)$

The first horizontal derivative (FHD) of the gravity effect of this structure is given by equation (1) for the horizontal cylinder, where $A = 2 G \sigma t$ and t is the thickness of the fault (Mohan et al., 1986). The geometries are shown in **Figure 1**. **Table 1** shows the definition of **m, A,** and **q.**

At the origin (x = 0), equation (1) gives

$$g(\max) = A z^{m-2q} \qquad (2)$$

Using equation (2), equation (1) can be written in a logarithmic normalized form.

$$\ln\left[\frac{g(x_i, z)}{g(\max)}\right] = q[\ln z^2 - \ln(x_i^2 + z^2)] \qquad (3)$$

Table 1. Definition of m, A, and q used in equation (1). G is the universal gravitational constant; σ is the density contrast between geometric object and the host rock; R is the radius of the buried structure, and t is the thickness of the fault. (after Gupta, 1983).

Model	m	A	q
Sphere	1	$4/3\ \pi\ G\ \sigma\ R^3$	3/2
Horizontal cylinder	1	$2\ \pi\ G\ \sigma\ R^2$	1
Vertical cylinder	0	$\pi\ G\ \sigma\ R^2$	1/2
Vertical fault (FHD)	1	$2\ G\ \sigma\ t$	1

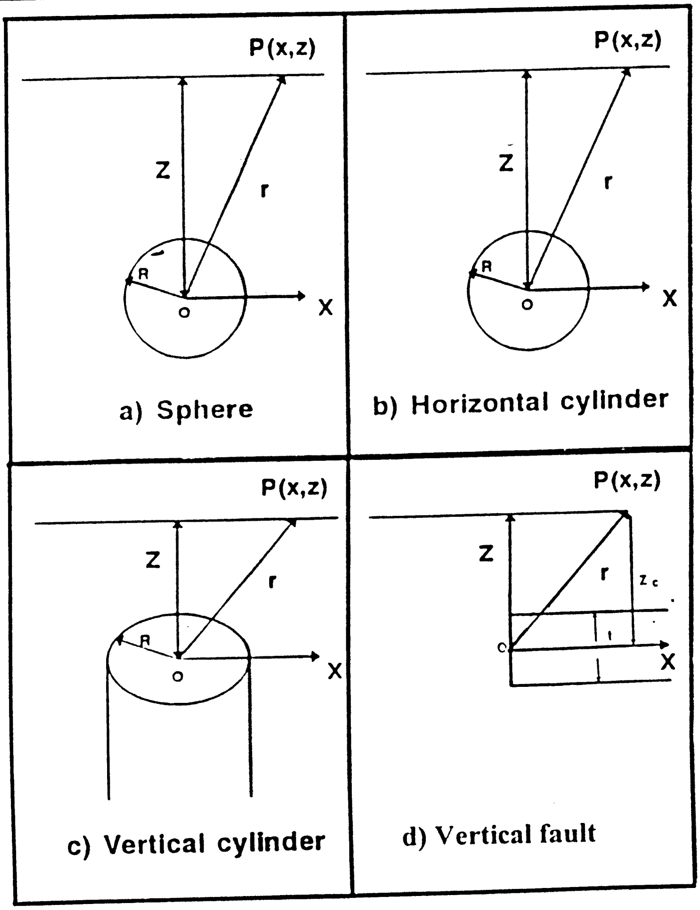

Figure 1. Diagrams for simple geometrical structures.

Let $f(x_i, z) = \ln\left[\dfrac{g(x_i, z)}{g(max)}\right]$, where $f(x_i, z)$ are the logarithm of the normalized residual gravity values. Then the unknown z can be obtained by minimizing

$$\varphi(z) = \sum_{i=1}^{N}\left[f(x_i, z) - q\ \ln z^2 + q\ \ln(x_i^2 + z^2)\right]^2 \qquad (4)$$

with respect to z. This yields

$$Z = \sqrt{\exp\left[\frac{\sum_{i=1}^{N}\left[f(x_i, z) - q\ \ln z^2 + q\ \ln(x_i^2 + z^2)\right]\dfrac{x_i^2}{x_i^2 + z^2}}{\sum_{i=1}^{N}\dfrac{x_i^2}{x_i^2 + z^2}}\right]} \qquad (5)$$

Equation (5) can be solved for z using standard methods for solving systems of nonlinear equations. We use the simple iteration method described by Demidovich and Moran (1973). Numerical results obtained for various cases are shown in Table 2. Any initial guess for z works well for the various cases.

Once z and g(max) are known, the amplitude coefficient A can be determined from equation (2). If the density contrast σ is assumed to be known, the radius R of the sphere or the cylinder can be obtained from A and the relations given in Table 1.

DISCUSSION OF THE RESULTS

Solution of equation (5) yields the exact value of depth (z) when using synthetic data (Table 2). Adding ± 10 percent random error in the synthetic data results in maximum uncertainties of ± 4 percent for the depth parameter and ± 2 percent for the amplitude coefficient.

The depth error obtained by Gupta (1983) after adding only ± 5 percent random error to the synthetic data, was ± 5 percent in the case of the sphere. Thus our method gives better results than Gupta' s method . The small errors in depth determination using our method (Table 2) are due to the fact that the residual gravity anomalies are first normalized and then taking the logarithm of the normalized values.

For synthetic data with no errors, it was found numerically that only a few points near g(max) are sufficient to determine the exact value of z . However, for data with random errors more points around g(max) are required to obtain reliable results.

Table 2. Theoretical results for a profile length of 20 units with a station separation of 1 unit.

		The horizontal cylinder (q=1)			The sphere (q=3/2)		
Model depth	Using synthetic data	Using data with random errors of 10%	% of error in depth	% of error in ampl. coeff.	Using data with random errors of 10%	% of error in depth	% of error in ampl. coeff.
1	1	1.01	1.27	-0.16	1.01	0.64	-0.85
2	2	1.96	-1.82	1.31	2.02	1.10.	1.13
3	3	2.90	-3.19	1.03	2.97	-1.02	-0.37
4	4	4.03	0.73	-0.24	4.01	0.14	0.62
5	5	4.87	-2.59	0.95	5.11	2.21	1.61
6	6	6.16	2.73	-0.76	5.84	-2.61	-0.79
7	7	6.75	-3.55	0.10	7.11	1.57	0.51

FIELD EXAMPLES

To examine the applicability and stability of the present method, the following two field examples are presented.

The Karrbo anomaly

A residual gravity profile of 25.6 m length over the two-dimensional Pyrrhotite ore, Karrbo, Vastmanland, Sweden (Hedstrom, 1940; Shaw and Agarwal, 1990) is shown in Figure 2. The gravity anomaly was digitized at an interval of 1.6 m. Equations (5) and (2) were used to determine the depth and amplitude coefficient, assuming a horizontal cylinder model for the causative ore body. The results were **z = 5.5 m**, and **A = 6.16 mGal*m** (Figure 2). The pattern established by the Walsh spectrum of the anomaly under study suggested a horizontal cylinder buried at a depth of 5.8 m (Shaw and Agarwal, 1990). Thus the two results are in good agreement.

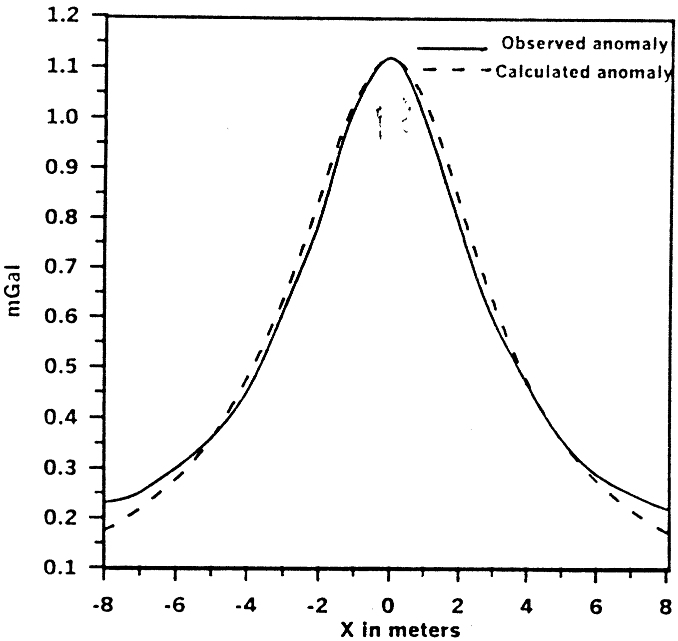

Figure 2. The Karrbo gravity anomaly profile, Vastmanland, Sweden.

The Louga anomaly

Figure 3 shows profile of the gravity anomaly of an area on the west coast of Senegal in West Africa. The gravity anomaly was digitized at an interval of 2.5 km along this 25 km long north-south profile. Equations (5) and (2) were used to determine the depth and amplitude coefficient, assuming a spherical target. The results were **z = 9.23 km**, and **A = 7241.39 mGal*km^2**. The depth according to Nettleton (1962, 1976) is 9.3 km for a heavy spherical body.

For both field examples, individual depth solution errors do not exceed 5 percent.

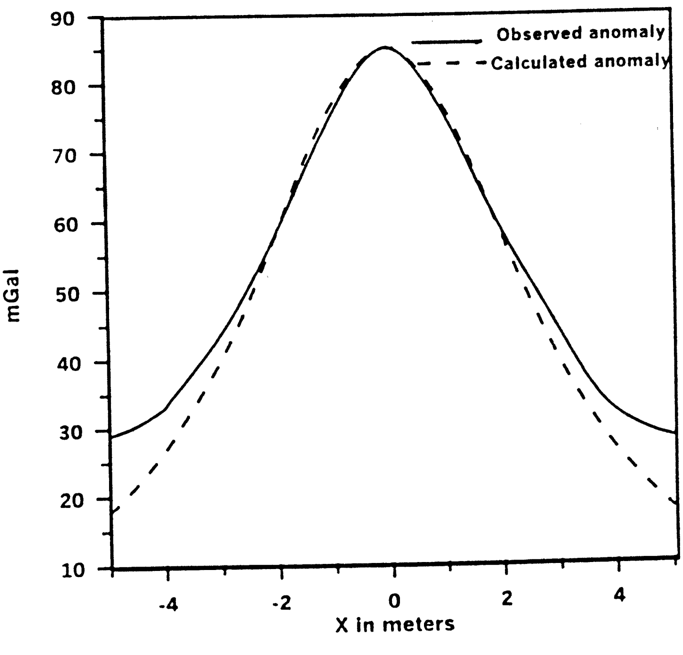

Figure 3. The Louga gravity anomaly profile, Senegal, West Africa.

CONCLUSIONS

The depth determination problem, assuming a simple buried structure, from a residual gravity anomaly has been transformed into the problem of finding a solution of a nonlinear equation of the form $z = f(z)$. Good results are obtained by using a simple iterative least-squares algorithm, particularly for depth estimation, which is a primary concern in gravity prospecting and other geological work.

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طريقه تكراريه لتقدير العمق من البيانات التثاقليه على أساس الحد الأدنى لأقل المربعات

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تم تطوير طريقه جديده لتعين العمق من الشاذات المحليه التثاقليه للتراكيب الجيولوجيه التحت سطحيه باستخدام طريقه أقل المربعات، وذلك بتحديد القيمه العظمى للشاذات التثاقليه مباشره فوق منتصف التركيب الجيولوجى ثم تحويل مشكله ايجاد العمق الى ايجاد حل لمعادله غير خطيه فى الصوره $z=f(z)$

تم استنتاج المعادلات الخاصه بالأشكال الهندسيه البسيطه الممثله للتراكيب الجيولوجيه وهى الكره والأسطوانه الأفقيه والرأسيه والصدع الرأسى.

أختبرت الطريقه باستخدام معلومات نظريه تخليقيه فى كلتا الحالتين باحتوانها وعدم احتوانها على أخطاء عشوانيه. كذلك اختبرت صلاحيه ودقه الطريقه المقترحه على مثالين حقليين من السويد والسنغال وكان العمق المعين بهذه الطريقه متوافق مع ذلك المعين بطريقه شاو وأجروال ١٩٩٠ وكذلك نيتلتون ١٩٧٦ عل التوالى.