KING SAUD UNIVERSITY
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION, SEM. II, 1427-28
M 107

TIME: 3 **HOURS** **MARKS:** 50

Question #1 [**Marks: 3+3=6**]

(a) Find the values of λ for the system

$$\begin{cases} x + \lambda y & = 1 \\ x + (1-\lambda) y & = 1 \end{cases}$$

to have (i) infinitely many solutions (ii) unique solution.
(b) If

$$A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix},$$

find the 2×3 matrix B that satisfies $\left(AB + \begin{pmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}\right)^T = \begin{pmatrix} -1 & 2 \\ -2 & 1 \\ 1 & 0 \end{pmatrix}$.

Question #2 [**Marks: 3+3=6**]

(a) Let A be a 4×4 matrix and $\det A = 6$.
Find $\det[(2A).(A^{-1})^3]$.

(b) Use Cramer's Rule to Solve the following system of linear equations for x_2 only:

$$\begin{cases} x_1 + 2x_2 + x_3 & = 7 \\ 2x_1 - x_2 + 3x_3 & = 0 \\ 5x_1 + 4x_2 - 2x_3 & = 1. \end{cases}$$

Question #3 [**Marks: 2+2+4=8**]

(a) Find the value of m such that the two vectors $\mathbf{u} =< 4, 2, m >$, $\mathbf{v} =< 1, 22, -3m >$ are orthogonal.

(b) Determine whether the four points $A(1, 3, -2), B(3, 4, 1), C(2, 0, -2)$ and $D(4, 8, 9)$ are in the same plane.

(c) Show that the lines L_1 and L_2 are not parallel and do not intersect:
$L_1 : x - 2 = -t, y - 1 = 2t$ and $z - 5 = 2t$;
$L_2 : x - 1 = s, y - 2 = -s$ and $z - 1 = 3s$.

Question #4 [**Marks: 3+4+3=10**]

(a) Find the equation for the plane through the point $(1, 4, -5)$ and parallel to the plane $2x - 5y + 7z = 12$.

(b) Find the unit tangent vector, normal component of acceleration and curvature of the curve C given by $\mathbf{r}(t) =< 2\sin t, 2\cos t, 4t >$.

(c) Identify and sketch the graph of $x^2 + z^2 - y^2 + 1 = 0$.

Question #5 [Marks: 3+3+4=10]

(a) Show that the $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^3+y^3}$ does not exist.

(b) For the function $f(x,y) = e^{xy}$, show that $f_{xx} + f_{yy} = e^{xy}(x^2 + y^2)$.

(c) Use linear approximation formula to approximate

$$(8.1)^{\frac{1}{3}}(3.98)^{\frac{1}{2}}.$$

Question #6 [Marks: 5+5=10]

(a) Find the directional derivative of $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$ at $A(-2,3,1)$ in the direction from $A(-2,3,1)$ to $B(0,-5,4)$.

(b) Find the local maxima and minima for the function $f(x,y) = 4x + x^2 - xy^2$.