Course Information:

Differential geometry is the study of curves and surfaces by means of calculus (both differential and integral). It is one of the oldest and most highly developed branches of mathematics, and remains central to modern pure mathematics as well as to much of theoretical physics. This course is a study of the curvature properties of curves and surfaces in two and three dimensions.

Outline syllabus:

(i) Curves in R^2 and R^3

(ii) Surfaces in R^3

(iii) Curves on surfaces

Aims:

(i) To demonstrate the utility of calculus and advanced calculus in the analysis and description of curves in R^3 and surfaces in R^3.

(ii) To reveal the range of curves and surfaces that can be described by means of standard calculus techniques.

(iii) To consolidate previous work on calculus and advanced calculus in a context where the importance of the applications is clear.

(iv) To demonstrate the inadequacies of a "rote" approach to elementary calculus in dealing with real geometric problems.

(v) To consolidate and apply finite-dimensional linear algebra in a context where vector spaces without preferred bases arise naturally.

(vi) To introduce the fundamental distinction between local and global properties in geometry.

(vii) To introduce the fundamental distinction between intrinsic and extrinsic properties for surfaces.

(viii) To demonstrate equally the great utility of graphical software, and the importance of theory and rigour in detecting and correcting the spurious solutions which such software often generates.

Learning outcomes:

• Calculate and work with the curvature function for curves in R^3 and deduce general features of the curve.

• Calculate and work with principal, Gaussian and mean curvatures for surfaces in R^3 and deduce general features of the surface from these functions.

• Calculate and work with Darboux frames and geodesic and normal curvatures for curves on surfaces.

Teaching methods:

Lectures, problem solving.

Assessment:

Two midterm examinations, and one formal 3 hour examination. **Format**: *Solve all questions.*

Full syllabus:

1. Curves in the plane

(About 16 lectures) Arc length and unit speed reparametrisation. Plane curves: curvature of unit speed curves from $T' = \kappa N$. Constant curvature gives a circle or a straight line. Curvature for regular curves. Invariance of κ under reparametrisation, existence of unit speed curve with given smooth function as curvature.

2. Surfaces

(About 16 lectures) First fundamental form. Weingarten map. Second fundamental form. Principal curvatures as eigenvalues and in terms of cross sections. Umbilics, hyperbolic points, et cetera. Gaussian and mean curvature and simple examples. Classical families of flat surfaces, and standard minimal surfaces. (Surfaces considered are regular smooth parametrised surfaces. Change of parameters is touched on but not treated systematically.)

3. Curves on surfaces

(About 10 lectures) Darboux frames. Significance of normal and geodesic curvature and of geodesic torsion. Extremal property of geodesics. Principal curves. Asymptotic curves.

Readling List

■R.S. Millman and G.D. Parker, “Elements of Differential Geometry”, Prentice Hall, 1977.

■M.P. do Carmo, “Differential geometry of curves and surfaces”, Prentice Hall, 1976.

■C.G. Gibson, “Elementary geometry of differentiable curves”, Cambridge University Press, 2001.

■A. Gray, "Modern differential geometry of curves and surfaces", CRC Press, 2nd edition, 1998.

■A. Pressley, "Elementary differential geometry", Springer, 2001.