M-374, FINAL EXAMINATION
DEPARTMENT OF MATHEMATICS
COLLEGE OF SCIENCE (WOMEN'S SECTION)
KING SAUD UNIVERSITY
SEMESTER-I, 1426

Time: 3 Hours Max. Marks-50

Q.1 Let $\alpha(t) = \left(\frac{t}{\sqrt{1+t^2}}, 1, \frac{1}{\sqrt{1+t^2}}\right)$ be a regular curve and $\beta(s)$ be its reparametrization by arc-length.

(a) Find $\beta(s)$. [4]

(b) Find the torsion $\tau(s)$ of $\beta(s)$. [4]

(c) Find the equation of the normal plane to $\beta(s)$ at any point and conclude that $\alpha(t)$ is a sphere curve. [4]

Q.2 (a) For a regular curve $\beta(t)$ show that its torsion is given by [4]

$$\tau = \frac{\left[\dot{\beta}, \ddot{\beta}, \dddot{\beta}\right]}{\left\|\dot{\beta} \times \ddot{\beta}\right\|^2}$$

(b) Let $\alpha(s)$ be a unit speed curve with $\kappa \neq 0$, $\tau \neq 0$ and $\beta(s) = B(s)$ be its binormal spherical image. If $\overline{\tau}$ is the torsion of the curve $\beta(s)$, then use (a) to show that [4]

$$\overline{\tau} = \frac{\dot{\kappa}\tau - \kappa\dot{\tau}}{\tau\left(\tau^2 + \kappa^2\right)}$$

(c) Use (b) to show that the unit speed curve $\alpha(s)$, $\kappa \neq 0$, $\tau \neq 0$, is a helix if and only if the binormal spherical image is an arc of a circle. [4]

Q.3 For the simple surface $x(u, v) = (u, v, uv)$ answer the following

(a) For $P = x(1, 1)$ and $X = (1, 1, 2) \in R^3$, show that $X \in T_P$. [3]

(b) For a neighbourhood $N \subset x(U)$ of P and $f : N \to R$, $f(x, y, z) = x^2 - y + z$, find the directional derivative $X(f)$ for X in (a). [4]

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(c) Find the geodesic curvature κ_g of the curve $\alpha(t) = x\left(\frac{t}{\sqrt{2}}, \frac{t}{\sqrt{2}}\right)$. [4]

Q.4 For the simple surface $x(u,v) = \left(\sqrt{1-u^2-v^2}, u, v\right)$ answer the following questions

(a) Show that $\overrightarrow{n}(u,v) = x(u,v)$. [3]

(b) Use (a) to find $L(x_1)$ and $L(x_2)$. [4]

(c) Use $II(X,Y) = \langle L(X), Y \rangle$, $X, Y \in T_P$, to find $II(T,T)$, where T is the tangent to the u^1 curve $\alpha(u) = x(u,0)$. [4]

(d) Show that the curve $\alpha(u)$ in (c) is a geodesic. [4]