M-573, Midterm Examination

II- SEMESTER-1430

Department of Mathematics (Women's Section)

College of Science, King Saud University

Time 90 Min. **Max. Marks-20**

Note: *Attempt all questions*

Q.1. (a) Show that $M = \left\{ x = (x_1, .., x_n) \in R^n - \{0\} : \|x\|^2 = 2x_1^2 \right\}$ is a smooth manifold.

(b) Explain what do you mean by an immersion, a submersion and give examples of each and an example of a submersion which is not an immersion.

Q.2. (a) Let M and N be two smooth manifolds and $f : M \to N$ be a continous function. For $p \in M$, let (U, φ) and (V, ψ) be charts around p and $f(p)$ with $f(U) \subset V$ and $y^1, .., y^n$ be local coordinates on V. If $f^i = y^i \circ f$ are smooth functions at p, $i = 1, .., n$, then show that f is smooth at p.

(b) Let $f : R^2 - \{0\} \to S^1$ be

$$f(x, y) = \left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right)$$

and $p = (1, 1)$. Find $df_p \left(\left(\frac{\partial}{\partial x} \right)_p \right)$.

Q.3. **(a)**Let M be a connected manifold and $f : M \to R$ be a non-constant smooth function. Show that there is a point $p \in M$ and a chart (U, φ) around p with local coordinates $x^1, .., x^n$ with $x^1 = f$.

(b) For the smooth function $f : R^3 \to R^3$

$$f(x, y, z) = (x^2 + y^2, xz, yz)$$

and the point $p = (1, 1, 1)$ show that there exists a neighbourhood U of p such that $f : U \to R^3$ is one to one.