Q.1(a) Consider $R$ and $A = (0, 1) \subset R$ and the topology

$$\tau_A = \{ U \subset R : A \subset U \} \cup \{ \emptyset \}$$

Is the topological space $(R, \tau_A)$ connected? Is the subset $A$ connected? Is $(R, \tau_A)$ locally connected? [3]

(b) Show that the topological space $R_f$ is path connected. Is $R_f$ locally connected? What are the quasi components of $R_f$? [4]

Q.2(a) Suppose $(X, \tau)$ be a connected space and $\tau_1, \tau_2$ are two topologies on $X$ satisfying $\tau_1 \subset \tau \subset \tau_2$. Are the spaces $(X, \tau_1)$, $(X, \tau_2)$ connected? Support your answer by either giving a proof or by giving an example. [3]

(b) Consider the subspace

$$X = \{(x, 0) \in \mathbb{R}^2 : x < 0 \} \cup \{(x, y) \in \mathbb{R}^2 : x \geq 0 \}$$

of $\mathbb{R}^2$. Is the space $X$ connected? Is the space $X$ path connected? what are the path components of $X$? [4]

Q.3 Show that the pathcomponents, components and quasicomponents of a topological space $X$ containing a point $x \in X$ satisfy

$$P_x = C_x = Q_x$$

Use this to show that a connected locally path connected space is path connected. [4]

Q.4 Let $X = [0, 1] \times [0, 1]$ and $Y = [0, 2]$ be subspaces of $\mathbb{R}^2$ and $R$ respectively. Show that the map $f : X \to Y$, defined by $f(x, y) = x + y$ is a quotient map. [2]