Q.1(a) Let $X$ be a topological space and $x \in X$. Explain the terms
(i) A path component $P_x$ of $x$
(ii) A connected component $C_x$ of $x$
(iii) A quasi component $Q_x$ of $x$
and show that $P_x \subseteq C_x \subseteq Q_x$. [4]

(b) For a locally path connected space show that $P_x = C_x = Q_x$. [4]

Q.2 Show that each connected component of a locally connected space is both open and closed. Is the converse true? [4]

Q.3 Show that the following pairs of spaces are not homeomorphic
(i) $\mathbb{R}$, $\mathbb{R}^n$, $n > 1$
(ii) $S^1$, $S^n$, $n > 1$ [4]

Q.4 Show that $R_f$ (the real line with finite complement topology) is connected. Is $R_f$ locally connected? (Use “Components of open subsets being open” criteria) [4]