M-570 MIDTERM EXAMINATION

I-Semester, 1428-1429

Department of Mathematics (Women's Section)

College of Science, King Saud University

Time 2 Hours **Max. Marks-20**

Note: *Attempt all questions*

Q.1. Show that the union of connected subsets of topological space having a common point is connected. Use this to show that R^2 is connected (Do not use the product $R^2 = R \times R$).

Q.2. Explain what do you mean by connected component of a topological space. Show that a connected component of a topological space is a closed subset and give an example to show that a component need not be an open subset.

Q.3. (a) Show that S^1 is not homomorphic to S^2.

(b) For a locally connected space X, and $x \in X$, show that $P_x = C_x$. Is converse of this reasult true? (Support your answer by a proof in case it is true, or a counter example in case it is not true)

Q.4. (a) Let X be a topological space and $x \in X$. Show that the quasi component Q_x of x is intersection of those subsets of X containing x, which are both open and closed.

(b) Show that $[a, b)$ is both open and closed in R_l and use this to show that $Q_0 = \{0\}$. Does $P_x = C_x = Q_x$ for each $x \in R_l$ imply R_l is locally path connected?

Q.5. Show that

(i) A locally compact dense subspace of a Hausdorff space X is open subset of X.

(ii) one-point compactification of the space R is $S^1 \subset R^2$.