M-690 MIDTERM EXAMINATION

II-Semester, 1427-1428

Department of Mathematics (Women's Section)

College of Science, King Saud University

Time 2 Hours **Max. Marks-20**

Note: *Attempt all questions*

Q.1. Explain what do you mean by an orientable smooth manifold. Give an example of a smooth manifold which is not orientable. Also show that an orientable smooth manifold has a differntiable structure consisting of positive charts. Is the converse true? If not under what additional condition the converse would hold?

Q.2. Consider $M = S^1$ and an open cover $\{U, V\}$, $U = S^1 - \{(0,1)\}$, $V = S^1 - \{(0,-1)\}$, together with smooth functions $f, g : M \to R$, $f(\theta) = \cos^2\theta$, $g(\theta) = \sin^2\theta$, where $\theta = (\cos\theta, \sin\theta) \in M$. Is $\{f, g\}$ a partition of unity subordinate to the locally finite open cover $\{U, V\}$? Justify your answer by explaining details.

Q.3. Let M be a paracompact smooth manifold and $E \subset M$ be a closed subset. For $p \in M - E$, show that there exists a smooth function $f : M \to R$ such that $f = 1$ on E and $f(p) = 0$.

Q.4. (a) Let M be a paracompact smooth orientable n-dimensional manifold and $\omega \in \Lambda^n(M)$ be a volume form on M. For a smooth function f on M with compact support, explain in detail what do you mean by the number $\int_M f\omega$. (Do not use mere $\int_M \alpha$, with α having compact support)

(b) Use stokes theorem to prove $\int_\gamma e^{-z} = 0$ for a closed regular curve γ in the complex plane $\boldsymbol{C}$.