Q.1. Explain what do you mean by an orientable smooth manifold. Give an example of a smooth manifold which is not orientable. Also show that an orientable smooth manifold has a differentiable structure consisting of positive charts. Is the converse true? If not under what additional condition the converse would hold?

Q.2. Consider $M = S^1$ and an open cover $\{U, V\}$, $U = S^1 \setminus \{(0, 1)\}$, $V = S^1 \setminus \{(0, -1)\}$, together with smooth functions $f, g : M \to \mathbb{R}$, $f(\theta) = \cos^2 \theta$, $g(\theta) = \sin^2 \theta$, where $\theta = (\cos \theta, \sin \theta) \in M$. Is $\{f, g\}$ a partition of unity subordinate to the locally finite open cover $\{U, V\}$? Justify your answer by explaining details.

Q.3. Let $M$ be a paracompact smooth manifold and $E \subset M$ be a closed subset. For $p \in M - E$, show that there exists a smooth function $f : M \to \mathbb{R}$ such that $f = 1$ on $E$ and $f(p) = 0$.

Q.4. (a) Let $M$ be a paracompact smooth orientable $n$-dimensional manifold and $\omega \in \Lambda^n(M)$ be a volume form on $M$. For a smooth function $f$ on $M$ with compact support, explain in detail what do you mean by the number $\int_M f \omega$. (Do not use mere $\int_M \alpha$, with $\alpha$ having compact support)

(b) Use stokes theorem to prove $\int_\gamma e^{-z} = 0$ for a closed regular curve $\gamma$ in the complex plane $\mathbb{C}$. 