Trade Allocation Modeling: Comparing the Results from Armington and Locally Regular AI Demand System Specifications of a UK Beef Import Demand Allocation Model

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Abstract

Locally regular almost ideal demand system and Armington specifications of a UK beef import demand allocation model are estimated. A Bayesian methodology is used that allows for the incorporation of prior information from microeconomic theory. The results illustrate the potential of regularity constrained locally flexible functional forms in modeling trade.

Key words: Regularity conditions, import demand, trade modeling, Bayesian, posterior density simulator, Gibbs sampling, Metropolis–Hastings algorithm.

JEL Code: Q170, F110, D120.

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1 Introduction

The continued dependence of the Irish beef industry on the UK market has been highlighted by recent supply (FMD) and demand (BSE) shocks to British meat and livestock markets. Increased dependence of the Irish beef industry on European Union (EU) markets, such as the UK, is likely to result from the more liberal agricultural trade environment that will follow future CAP reforms and the conclusion of the current WTO trade round negotiations. This paper investigates the demand in the UK for Irish beef through two alternative trade model specifications (an Armington and locally regular almost ideal demand system specification) in which beef products are differentiated by geographical source of origin, and in which demand for domestically produced beef (UK beef) is not assumed to be separable from the demand for imported substitutes.

As Alston et al. (1990) note, empirical estimates of trade demand elasticities are conditioned by the model specification chosen. Among the most commonly utilized trade model specifications, in which products are distinguished on the basis of geographic origin, is what is known as the Armington model, Armington (1969). The most common Armington specification models import demand in a multi-stage budgeting process, wherein total expenditure is allocated to some good (or aggregate), this expenditure is then divided between imports and domestically produced substitutes, and finally total imports are allocated across different source countries. Winters (1984), among others, has criticized the traditional Armington assumptions as overly restrictive. The specification assumes that import demands are homothetic and separable across import sources and that demand for imports is separable from demand for domestically produced substitutes. Following Winters, a common approach is to use a flexible functional form specification of the import expenditure allocation stage of the Armington model framework. This allows for the relaxation of the restrictive homotheticity and separability assumptions (as they apply to import demands), and means that rather than allow an inflexible model specification to largely dictate the empirical results, the import demand elasticities of interest are measured (Deaton 1986).

The accrual of the empirical advantages associated with the use of flexible functional form specifications of expenditure allocation decision also involves the foregoing of the advantages of the CES based Armington model specification. The Armington model globally satisfies microeconomic theory's regularity conditions, which are that demand functions are homogeneous, symmetric, add up and that the elasticity of substitution matrix is negative semi-definite. Regularity is important when estimated elasticities are used in policy and welfare analysis and explains the continued popularity of Armington assumptions in computable general equilibrium (CGE) and other applied trade analyses. Although flexible functional forms provide a second order approximation to any underlying utility function at a point in the data space, they do not necessarily provide a good approximation over a range of data points. The frequent regularity failures of flexible functional form demand models in the wider consumer demand literature have been repeated in their use in import demand allocation models. Shiells, Roland-Holst and Reinert (1993) for example use an AI demand system trade model specification and report positive own price elasticities of demand while Brenton (1994) finds that over a large number of import demand systems the hypothesis of a negative semi-definite substitution matrix is not supported by the data. Being able to impose (if necessary) all, but no more, of the constraints suggested by demand theory is, as Brenton (1994) argues, an important requirement of empirical trade demand systems.

In this paper we use Bayesian Markov chain Monte Carlo (MCMC) methods to impose, over the sample space, all of microeconomic theory's regularity requirements on an AI demand system specification of a UK beef import demand model in which the demand for UK produced beef is treated symmetrically with demand for import substitutes. The conditional demand elasticities associated with this locally regular AI demand system specification are compared with those obtained through the generalized least squares (GLS) estimation of a globally regular Armington specification and an AI demand system specification without all of the regularity conditions of microeconomic theory imposed.

The Bayesian methodology used is based on the accept-reject algorithm approach to the imposition of inequality restrictions suggested by Geweke (1986) and recent uses advances in Bayesian statistics and econometrics, known collectively as Markov chain Monte Carlo (MCMC) methods. MCMC methods allow for the simulation of posterior density functions that have heretofore been intractable or difficult to evaluate or simulate using traditional analytic Bayesian or Monte Carlo integration methods.[1]

Percy (1992) showed that MCMC method of Gibbs sampling could be used to simulate the posterior density function associated with Zellner's seemingly unrelated regression (SUR) model, which is intractable from an analytical Bayesian perspective. Gordon (1996) and Terrell (1996) subsequently combined Percy's MCMC Gibbs Sampling approach to the simulation of seemingly unrelated regression models with Geweke's accept-reject algorithm approach to the imposition of inequality restrictions to impose all of microeconomic theory's regularity conditions on flexible functional form factor demand models. Hanrahan (2000) has recently proposed an extension to this work in which a hybrid Gibbs sampling/ random walk Metropolis–Hastings algorithm is used to simulate the posterior density associated with seemingly unrelated nonlinear regression (Gallant 1975) consumer demand system models. The seemingly unrelated nonlinear regression model underlies the stochastic representation of all Diewert flexible functional form representations of consumer preferences. This hybrid algorithm is used to simulate the posterior density associated with a fully regular AI demand system representation of a model of UK beef import demand model. The priors used, except in so far they impose desired regularity conditions, are diffuse and noninformative and the sampling algorithm is initiated from values equal to those obtained through the GLS estimation of the AI demand system model that are reported. Full details of the algorithm are available in Hanrahan, Jensen and Westhoff (2001), Hanrahan (2000) or directly from the authors.

The empirical results obtained illustrate that for the data set used the classically

[1]For introductions to Markov chain Monte Carlo methods, with special reference to their application in econometric contexts, the reader is directed to Albert and Chib (1996) and Chib and Greenberg (1996).

estimated AI demand system is locally (and globally) irregular. The Bayesian MCMC methods used offer a means of incorporating regularity conditions in the empirical estimation of trade allocation models The approach utilized in this paper may represent a middle way between the inflexible but globally regular Armington trade model specification and flexible but frequently irregular flexible functional form trade model specifications.

The policy implications of the result that the UK demand for Irish beef is inelastic are that attempts to move the large volumes of beef are currently exported with subsidy to non-EU markets to a market similar to the UK will require large cuts in prices and or a reorientation of the Irish beef industry from being a commodity producing sector to one that produces something that appeals to EU consumer's non-price concerns.

2 Functional specification

The Armington trade model, wherein imports are distinguished on the basis of their geographic origin is one of the most commonly used trade model specifications. The Armington model is based on a constant elasticity of substitution (CES) functional form. The CES functional form implies that demands are homothetic and separable across sources of supply. In most Armington models demand for products from "foreign" sources of supply are treated as separable from the demand for the domestically produced substitute. As Winters notes, nothing in Armington's original specification suggests that this need necessarily be the case. In this paper we relax this assumption in the trade model specifications estimated.

The Armington model is based on a two-step budgeting process where total demand for the good concerned is determined in the first stage, and in the second stage this demand is allocated across alternative (geographically differentiated) sources of supply. In this paper we concentrate on the second stage demand for the good and assume that total demand is predetermined, thus, all of the elasticities are conditional and should be interpreted as such. The full details of the derivation of Armington model's second stage demand functions can be found in Armington. Armington's demand functions have the following form

$$q_{ij} = b_{ij}^{\sigma_i} Q_i \left(\frac{p_{ij}}{P_i}\right)^{-\sigma_i} \qquad j = 1, \ldots, 4. \tag{1}$$

where q_{ij} is the quantity of the good demanded in country i from the j-th source, p_{ij} is the price of the product from the j-th source, Q_i and P_i are CES quantity and price indices respectively, and b_{ij} and σ_i are unknown parameters. In this paper we approximate the CES quantity and price indices by Divisia quantity and price indices. Treating Q_i as the simple sum of the quantities of the product q_{ij} demanded from different geographic locations implies that unknown parameters (b_{ij} and σ_i) are known, and as noted by Davis and Kruse (1993), that the source differentiated products (q_{ij}) are perfect substitutes.

Rather than estimate the Armington model in the form of equation (1), the following log linear rearrangement of the demand functions are estimated.

$$\log\left(\frac{q_{ij}}{Q_i}\right) = \alpha_{ij} - \sigma_i \log\left(\frac{p_{ij}}{P_i}\right) \quad j = 1, \ldots, 4. \tag{2}$$

where $\alpha_{ij} = \sigma_i \ln b_{ij}$, and all else are as in equation (1). With m alternative sources of supply this amounts to an m equation seemingly unrelated regression system. Cross equation restrictions impose the single CES assumption across the estimated SUR equations.

The CES based Armington model specification while globally regular is inflexible. The specification's inflexibility, the separability of the demands for substitute products from different locations implied by the functional form, and the homotheticity of the Armington demand functions led Winters' to advocate the use of flexible functional forms in modeling import demand. The AI demand system, following Winters, has dominated in applications of flexible functional form models of import demand, see Alston, Carter, Green and Pick (1990), de Gorter and Meilke (1987), Heien and Pick (1991), Yang and Koo (1994), Brenton (1989a, 1989b, 1994), Brenton and Winters (1992), and Shiells, Roland-Holst, and Reinert (1993), among other for examples.[2] The AI demand system,

[2] In addition to specifications that utilize the AI demand system specification in modeling trade shares

which is a flexible functional form, does not restrict the substitution elasticities nor is it necessarily homothetic or separable across goods, three of the principal criticisms of the Armington model assumptions. Most applications of the AI demand system and other flexible functional form based demand models in modeling import demand have retained the assumption that the demand for imports is separable from the demand for the home produced product, exceptions include de Gorter and Meilke (1987), Brenton (1989a) and Shiells, Roland-Holst, and Reinert (1993). Criticism of this particular separability assumption was arguably the most important of those made by Winters. In the AI demand system model specification used here demand for UK produced beef is treated in a symmetrical manner to demand in the UK for beef imports.

The full details of the AI demand system model's derivation may be found in Deaton and Muellbauer (1980). With the AI demand system trade model specification the expenditure share of the products from the i-th geographic source (s_i) is given by

$$s_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln \frac{Y}{P^*}, \tag{3}$$

where $P^* = \alpha_0 + \sum_i \alpha_i \ln p_i + \frac{1}{2} \sum_i \sum_j \gamma^*_{ij} \ln(p_i) \ln(p_j)$, p_j is the price of the product originating from the j-th source, and Y total expenditure on all beef products, where $Y = \sum_i p_i q_i$.

The AI demand system, unlike the CES system, does not satisfy the regularity conditions associated with the theory of consumer demand; however, Slutsky symmetry is imposed by setting $\gamma_{ij} = \gamma_{ji}$ in estimation. Similarly adding up and homogeneity are imposed as follows. [3]

$$\begin{aligned} \text{Adding up} \quad &: \quad \sum_{i=1}^{m} \alpha_i = 1; \quad \sum_{i=1}^{m} \gamma_{ij} = 0; \quad \text{and} \quad \sum_{i=1}^{m} \beta_i = 0 \quad \text{and} \\ \text{Homogeneity} \quad &: \quad \sum_{j=1}^{m} \gamma_{ij} = 0. \end{aligned}$$

other flexible functional forms have also been used, such as the Rotterdam model used by Seale, Sparks and Buxton (1992) and the NBR model used by Weatherspoon and Seale (1995).

[3] Symmetry and adding up restrictions are in and of themselves sufficient to guarantee that homogeneity holds, since $\gamma_{ij} = \gamma_{ji}$ and $\sum_{i=1}^{m} \gamma_{ij} = 0 \Rightarrow \sum_{j=1}^{m} \gamma_{ij} = 0$.

Imposing the curvature requirement in the AI demand system is more difficult. The negativity condition, i.e. that the expenditure function for the AI demand system be concave, is satisfied if the matrix C, whose elements c_{ij} are defined as

$$c_{ij} = \gamma_{ij} + \beta_i \beta_j \log(Y/P^*) - s_i \delta_{ij} + s_i s_j,$$

is negative semi-definite, where δ_{ij} is the Kronecker delta that equals 1 if $i = j$ and 0 otherwise. The curvature condition for the AI demand system (concave expenditure function) can be equivalently expressed in terms of the system's AUES matrix. the necessary and sufficient conditions are that the principal minors of the AUES matrix alternate in sign. To derive the AUES measure for the AI demand system we use the following identity between the ij-th Allen-Uzawa elasticity of substitution (AUES) and the more familiar Marshallian price η_{ij} and expenditure elasticity η_i concepts

$$\sigma_{ij} = s_j^{-1} \eta_{ij} + \eta_{i0}. \tag{4}$$

Since the Marshallian price elasticities of demand for the AI demand system are

$$\eta_{ij} = -\delta_{ij} + \frac{1}{s_i}\left[\gamma_{ij} - \beta_i\left(\alpha_j + \sum_{k=1}^{m} \gamma_{jk} \log p_k\right)\right], \tag{5}$$

and the income elasticities of demand are

$$\eta_{i0} = 1 + \frac{\beta_i}{s_i}, \tag{6}$$

by substituting the expressions for the Marshallian price and income elasticities of demand into (4) we can derive the AUES elasticity of substitution for the AI demand system. After some rearrangement, the AUES for the AI demand system is

$$\sigma_{ij} = 1 - \frac{\delta_{ij}}{s_j} + \frac{\gamma_{ij}}{s_i s_j} + \frac{\beta_i}{s_i}\left[1 - \frac{\alpha_j + \sum_{k=1}^{m} \gamma_{jk} \log p_k}{s_j}\right]. \tag{7}$$

The monotonicity condition associated with the AI demand system is that the first derivative of the expenditure function associated with the AI demand system be positive. Since the first derivative of the AI demand system with respect to the log of the i-th price is the share equation (3), this monotonicity condition is equivalent to the restriction that these expenditure shares are positive.

To impose all of the regularity conditions associated with the theory of consumer demand a Bayesian Markov chain Monte Carlo (MCMC) approach is used. Bayesian econometrics is based on Bayes' Theorem, which states that

$$\pi(\theta|y) \quad = \quad \frac{\pi(\theta)\mathcal{L}(\theta|y)}{\pi(y)} \quad \propto \quad \pi(\theta)\mathcal{L}(\theta|y), \tag{8}$$

where θ is a vector of parameters; y a vector or matrix of data; $\pi(\theta)$ the prior density; $\mathcal{L}(\theta|y)$ the likelihood function; and $\pi(\theta|y)$ the posterior density function. The posterior density summarizes all of the information (prior and sample) available about the parameter vector θ, Zellner (1971). Inferences concerning θ are obtained from the posterior, for example, the posterior mean of θ, which is frequently used a Bayesian point estimator of θ, is $E(\theta|y) = \int \theta\pi(\theta|y)\mathrm{d}y$. In incorporating theoretical information derived from microeconomic theory, such as the inequality based restrictions that demand functions be monotonic and that the associated elasticity of substitution matrix is negative semidefinite, we modify the prior density, $\pi(\theta)$, so as to reflect prior information derived from economic theory. Other prior beliefs derived from economic theory, such as symmetry adding up, and homogeneity are introduced via the specification of the functional form used so that they are, in terms of Bayes' Theorem, reflected in the model's likelihood function, $\mathcal{L}(\theta|y)$.

For some standard statistical models, analytic formulae exist that allow for the straightforward evaluation of integrals, such as $E(\theta|y) = \int \theta\pi(\theta|y)\mathrm{d}y$, however, such analytic solutions often do not exist. MCMC methods provide a means with which such integrals can evaluated. A MCMC method produces a simulated sample $\{\theta^{(1)}, \ldots \theta^{(G)}\}$ from the posterior density $\pi(\theta|y)$, and from this simulated sample it is possible to compute inferential summaries. Thus, the posterior mean $\int h(\theta)\pi(\theta|y)\mathrm{d}y$ can be estimated by the average of the simulated values

$$E(\theta|y) = G^{-1}\sum_{j=1}^{G} h(\theta^{(j)}), \tag{9}$$

where $h(\theta) = \theta$.

The posterior density function associated with the seemingly unrelated regression (SUR) model and the seemingly unrelated nonlinear regression (SUR-NL) model are simulated using MCMC methods by partitioning the model's parameter vector, θ, into blocks. In the SUR and SUR-NL models the model's parameter vector can be partitioned as $\theta = (\beta, \Omega)$, so that the conditional posterior densities of the model's location parameters (β) and its precision matrix (Ω), denoted as $\pi(\beta|\Omega, y)$ and $\pi(\Omega|\beta, y)$, are available in forms that are amenable to sampling. Percy (1992) and Chib and Greenberg (1995a, 1996) have shown that the posterior density function associated with Zellner's (1962) SUR model is amenable to simulation using a MCMC method known as Gibbs sampling. The posterior density function associated with seemingly unrelated nonlinear regression model, the statistical model representation of the AI demand system, may be similarly partitioned. However, one of the resulting conditional posterior densities, $\pi(\beta|\Omega, y)$, is of a nonstandard and seemingly intractable form. To simulate this conditional posterior density the Gibbs sampling MCMC approach proposed by Percy (1992), and used by Chib and Greenberg (1995a), Gordon (1996), Terrell (1996) and Deschamps (2000), is extended through the use of a hybrid Gibbs/random walk Metropolis–Hastings sampling algorithm.

In hybrid Gibbs/random walk Metropolis–Hastings sampling algorithm draws from the conditional posterior density associated with the model's precision matrix are made from a Wishart distribution using a Gibbs sampling step while draws from the intractable and nonstandard conditional posterior density function, $\pi(\beta|\Omega, y)$, are made using a random walk Metropolis–Hastings step. [4] Fuller details of the posterior density associated with the seemingly unrelated nonlinear regression and the hybrid MCMC posterior den-

[4]For an intuitive introduction to the Gibbs sampler the reader is directed to Casella and George (1992), while Chib and Greenberg (1995b) is a good introduction to the Metropolis–Hastings algorithm. The usefulness of the Bayesian paradigm in econometrics in general is argued for in Poirier (1988), while Koop (1994) summarizes in a non-technical way many of the recent innovations in Bayesian econometrics.

sity simulator are available in Hanrahan et al. (2001).

In the absence of the regularity restrictions, diffuse, non-informative and independent priors are specified, $\pi_0(\beta)$ and $\pi_0(\Omega)$. To redefine the prior density, $\pi_0(\beta)$, so as to incorporate the monotonicity and curvature regularity conditions an indicator function $I(\beta)$ is defined such that $I(\beta) = 1$ if the curvature and monotonicity restrictions hold for all prices and expenditures in the sample space and set equal to 0 otherwise. With this indicator function, an informative prior for the seemingly unrelated nonlinear regression model, $\pi_1(\beta, \Omega)$, is defined as

$$\pi_1(\beta, \Omega) = \pi_1(\beta) \, I(\beta) \, \pi_0(\Omega).$$

The product of this theoretically informed prior and the likelihood function associated with the AI demand system is, from Bayes' Theorem, proportional to the posterior density function of the locally regular AI demand system,

$$\pi(\beta, \Omega|y) \propto \pi_1(\beta, \Omega) \, \mathcal{L}(\beta, \Omega|y). \tag{10}$$

The mean of the draws made from the from the conditional posterior density function $\pi(\beta|\Omega, y)$, is used as a Bayesian point estimate of the parameters of the locally regular AI demand system specification of a UK beef import demand model. Table 1 presents some pseudo code for the hybrid Gibbs/ random walk Metropolis–Hastings sampling algorithm used.

The priors, $\pi_0(\beta)$ and $\pi_0(\Omega)$, that are used in simulating the posterior density function associated with the locally regular AI demand system model of UK beef imports, in addition to the prior information that is represented by the indicator function $\mathbf{I}(\beta)$, are independent, noninformative, and diffuse.[5] These priors are centered at zero with a large variance. The hyperparameters associated with these priors are

$$\begin{aligned} \mathcal{N}_k\left[\underline{\beta}, \underline{\Omega}_{\beta}\right] &= \mathcal{N}_k\left[\mathbf{0}, 10^6 \times \mathbf{I}_k\right], \text{ and,} \\ \mathcal{W}_m\left[\underline{\mathbf{S}}, \underline{\nu}\right] &= \mathcal{W}_m\left[10^6 \times \mathbf{I}_k, 13\right], \end{aligned}$$

[5]The priors are also improper. While this does not affect the calculation of the moments of the posterior density function it does affect the use of the results in Bayesian hypothesis tests.

Table 1: Hybrid MCMC algorithm pseudo–code

```
Initialize Ω_1^(0) and β_2^(0); set i = 0
At the i-th iteration of the Monte Carlo process, for example
Sample Ω_1^(i)|β_2^(i-1) using the Gibbs sampler, store the draw on Ω_1^(i)
Sample β_2^(i)|Ω_1^(i) using the Metropolis--Hastings algorithm
Store β_2^(i) if it is regular and accepted, else store β_2^(i-1) as the draw.
Sample Ω_1^(i+1)|β_2^(i) using the Gibbs sampler, store Ω_1^(i+1)
Sample β_2^(i+1)|Ω_1^(i+1) using the Metropolis--Hastings algorithm
Store β_2^(i+1) if it is regular and accepted, else store β_2^(i) as the draw.
⋮
Sample Ω_1^(N)|β_2^(N-1) using the Gibbs sampler, store Ω_1^(N)
Sample β_2^(N)|Ω_1^(N) using the Metropolis--Hastings algorithm
Store β_2^(N) if it is regular and accepted, else store β_2^(N-1) as the draw.
```

where k is the dimension of the AI demand system's parameter vector.

The hybrid Gibbs/random walk Metropolis–Hastings posterior density simulator is initialized using the AI demand system parameter estimates and associated variance–covariance matrix that are obtained through generalized least squares (GLS) estimation. A total of $6,000$ draws are made in simulating the posterior density function associated with the locally regular AI demand system specification of the UK beef import demand model. The first $1,000$ of these draws discarded, the so called "burn–in phase", so that the results presented are based on the remaining $5,000$ draws. The discarding of the "burn–in phase" draws ensures that the remaining draws are not conditioned by the starting point of the Markov chain, see Albert and Chib (1996). As noted by Chib and Greenberg (1996), the question of the convergence of the draws from a given Markov chain Monte Carlo run of some finite length to draws from the invariant posterior density is one that requires "considerable care". Due to the serially correlated nature of the draws from the conditional posterior densities involved in MCMC algorithms, standard methods of calculating the variance and standard error of the draws cannot be used. The numerical standard errors and the related diagnostic, that Chib, Nardari and Shephard (1999) refer to as an "inefficiency measure" are calculated and reported.

3 Data and nonparametric tests

In the UK beef import demand model specifications estimated, the bundle of goods modeled are UK consumption of UK produced beef, UK consumption of beef imported from Ireland, UK consumption of beef imported from EU member states other than Ireland and UK consumption of beef imported from non-EU countries.

SITC Revision 3 trade data (volume and value) at the 5 digit level on UK imports of beef for the period 1977 to 1998 were obtained from Eurostat (Eurostat 2000b), the statistical agency of the EU. These data were aggregated, after converting bone–in volumes and values to boneless equivalents, to form a set of 3 UK beef import series (volume and value) series: UK imports of beef from Ireland (IRL), from other EU member states (REU), and UK beef imports from non-EU member states (ROW).[6] Figure 1(a) presents the data on UK beef imports.[7]

The aggregated beef import value and volume data are used to construct unit values. These unit values are used as the demand prices of the associated imports in the absence of other bilateral trade prices. The use of unit values has been criticized on the grounds that with aggregated trade data unit values are associated with bundles of very heterogeneous goods (Kravis and Lipsey 1974). This heterogeneity means that changes in unit values may arise due to changes in the composition of the aggregates concerned rather than changes in the actual prices of the individual goods. The data used here, given the nature

[6] The coefficient used to convert bone-in beef volumes and values to boneless equivalent were taken from the UK Meat and Livestock Commission's European Handbook. The coefficient is constant over the time period examined. This implicitly assumes that the cut–out weight of meat from bone–in beef did not change over the period 1977 to 1998. This assumption is unlikely to be valid.

[7] The disaggregated trade data are available from the authors.

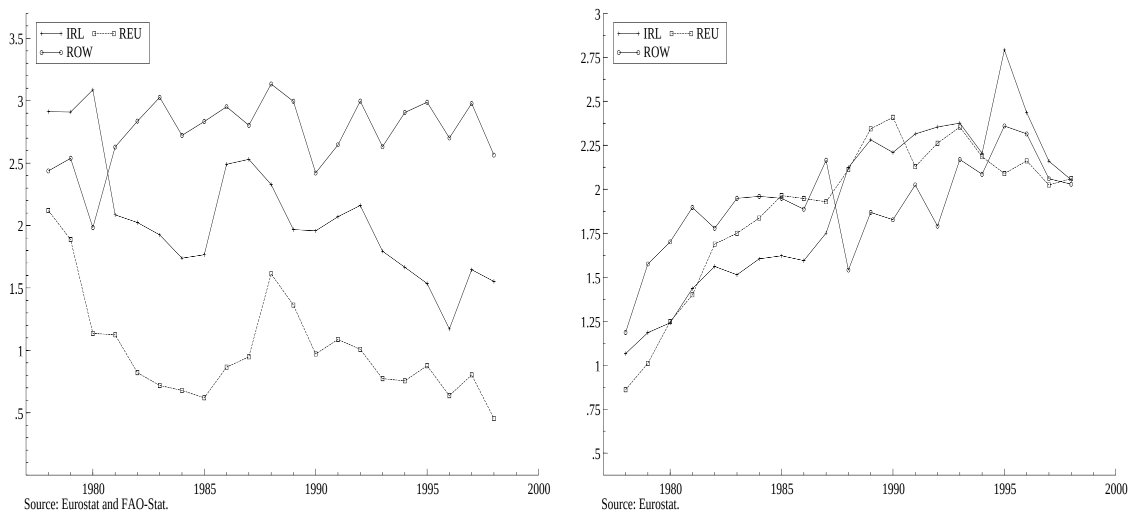


Figure 1: (a) UK beef imports (Kg/p.c.) (b) UK beef import unit values: (£/Kg)

of the aggregation made across cuts of beef imported into the UK, may be subject to this criticism. Research by Shiells (1991) suggests that the use of disaggregated import unit value data does not greatly affect estimates of import demand elasticities when compared to those estimated with actual import prices. The calculated unit values associated with the UK imports of beef, in terms of pounds sterling per kilogram, are graphed in Figure 1(b).[8]

The unit values graphed in Figure 1(b), with the exception of one data point, do not give rise to any immediate concerns. The 1995 unit value for UK imports of Irish beef is problematic. There appears to be either an error in the data set received from Eurostat or an accounting mistake made at the customs and excise level (where these data ultimately originate). In 1995 the volume of imports into the UK from Ireland of fresh bone–in beef was 16,037 Mt, this volume is equivalent to an increase of 5367 Mt over the level of imports of this code in 1994, the value of the imports in 1995, however, increased over the value in 1994 by 185% (from approximately 28 million ECU to 79 million ECU). Attempts to elicit an explanation of these curious data from Eurostat and

[8] An exchange rate series between the pound sterling and the Euro/ECU was obtained from Eurostat (2000a) and was used to translate the unit value series from Euro/ECU per kilogram to pounds sterling per kilogram.

HM Customs and Excise in the UK failed. Irish trade data on exports of fresh bone–in beef to the UK for the same period indicate that there is a mistake in the Eurostat data on the value of these exports.

The quantity of UK produced beef consumed in the UK is derived from a supply and utilization identity. The data used are from the *FAO-Stat* database (FAO 2000) and are approximately consistent with the carcass weight equivalent SITC UK beef import data.[9] The standard supply and utilization identity (at time t) is

$$PROD_{uk_t} + BS_{uk_t} + M_{uk_t} \equiv CON_{uk_t} + ES_{uk_t} + X_{uk_t}, \tag{11}$$

where in time period t, $PROD_{uk}$ is UK beef production, BS_{uk} is UK beginning stocks of beef, M_{uk} is total imports of beef into the UK, CON_{uk} is UK consumption of beef, ES_{uk} is UK ending stocks of beef and X_{uk} is UK beef exports. Rearranging (11) leads to the following equivalent expression

$$\begin{aligned} PROD_{uk_t} - [ES_{uk_t} - BS_{uk_t}] + M_{uk_t} - X_{uk_t} &\equiv CON_{uk_t} \\ PROD_{uk_t} - \Delta STK_{uk_t} + M_{uk_t} - X_{uk_t} &\equiv CON_{uk_t}, \end{aligned} \tag{12}$$

where $\Delta STK_{uk_t} = [ES_{uk} - BS_{uk}]_t$ is equal to the change in stocks in year t.

We make the following assumptions; (i) all imports of beef into the UK are consumed in the UK and (ii) that UK aggregate consumption can be divided into consumption of beef produced in the UK (CON_{uk}^{UK}) and consumption of beef imported into the UK (CON_{uk}^{M}), i.e., that

$$CON_{uk} \equiv CON_{uk}^{UK} + CON_{uk}^{M}.$$

These assumptions allows us to rearrange identity (12) as follows

$$PROD_{uk} - \Delta STK_{uk} + M_{uk} - X_{uk} \equiv CON_{uk}^{UK} + CON_{uk}^{M}. \tag{13}$$

[9]Over the sample period (1977–1998) the average discrepancy between the supply and utilization data on aggregate imports of beef (carcass weight equivalent) and the aggregate level of beef imports from the SITC data set amounted to 2.5% of the total supply and utilization import total from FAO-Stat.

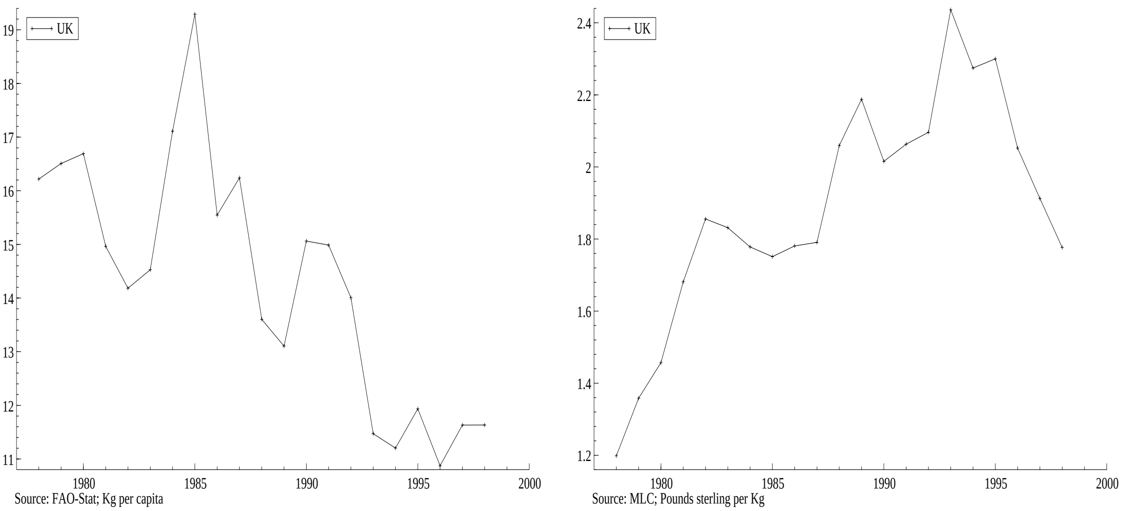


Figure 2: (a) UK consumption of UK beef and (b) UK wholesale beef price

Assumptions (i) and (ii) then imply that

$$\begin{aligned} CON_{uk}^{UK} &\equiv PROD_{uk} - \Delta STK_{uk} + M_{uk} - CON_{uk}^{M} - X_{uk} \\ CON_{uk}^{UK} &\equiv PROD_{uk} - \Delta STK_{uk} - X_{uk}. \end{aligned} \tag{14}$$

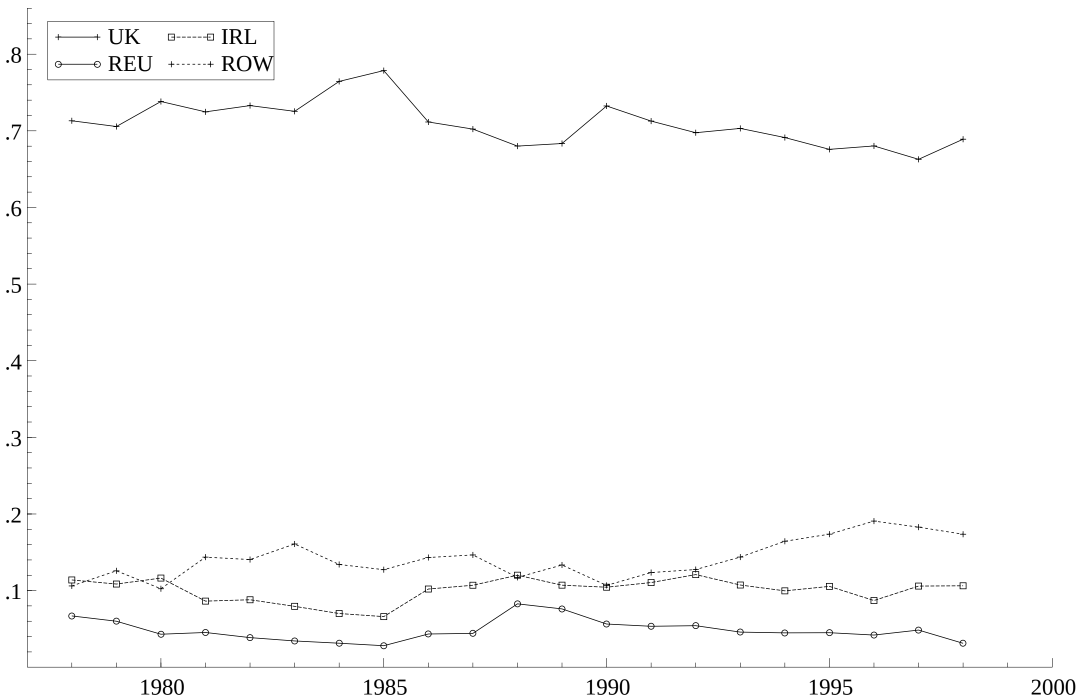
The price associated with the demand for UK beef is a wholesale carcass price obtained from the UK Meat and Livestock Commission (MLC). The absence of origin specific retail price data precludes the use of retail data in the import demand model. The use of wholesale prices as a proxy for the retail price of the domestically produced product and of unit values as proxies for the demand prices of imported products is standard in the empirical trade literature, see Winters, Brenton (1989a) and Yang and Koo (1994). The per capita quantities of UK produced beef consumed in the UK (in carcass weight equivalent), and the UK wholesale beef price in pounds sterling per kilogram of carcass weight beef are graphed in Figure 2(a) and 2(b).

All of the quantity data were divided by total UK population to give per capita demands for the different beef products. Together with the unit values and the wholesale price of UK beef, a set of expenditure shares was derived for beef produced in the UK, beef imported into the UK from Ireland, from other EU members and from non–EU origins, these are presented in Figure 3. The resulting beef product demand data set comprising of quantities demanded and prices of beef products from the UK, Ireland, other EU member states and non-EU countries that were consumed in the UK over the period 1977 to 1998, was examined for its consistency with utility maximizing behavior using Varian's nonparametric testing software NONPAR. [10] The nonparametric tests conducted examined the consistency of the full beef product demand data set, and subsets of the data set, with the existence of a regular utility (and sub-utility) function. Varian's (1982) generalized axiom of revealed preference (GARP), if satisfied, indicates that there exists a regular utility (or sub-utility) function that rationalizes the observed price and quantity data. The satisfaction of the GARP by the demand data set consisting of price and quantity data for all four beef products is equivalent to the satisfaction of a necessary condition for the weak separability of demand for these beef products from consumer demands for other goods. The violations of the GARP that occur with the sub-set of the beef product demand data set that excludes UK demand for UK produced beef implies that the commonly assumed separability of demand for imported products from demand for "home produced" product is not supported by the data. Similarly, when one of the three alternative beef import sources is omitted, the resulting subset (consisting of the three remaining beef product prices and quantities demanded) also led to GARP violations. This provides support for the hypothesis that the Armington model's separability structures are also inappropriate.

4 Empirical results and discussion

The results from the estimation of the Armington model specification, as given in equation (2), are presented in Table 2. The elasticity of substitution, σ_i, is the same across all of the estimated equations, this reflects the single CES assumption of the Armington model. The Armington model seems to provide a reasonable good fit for the data, with all of the equation's R^2 values in excess of 0.95, with the obvious exception of that for UK beef

[10] The NONPAR program was obtained from Hal Varian, at UC Berkeley.



Source: Own calculations, Eurostat, FAO-Stat and MLC.

Figure 3: Shares of UK beef consumption: By product origin

imports from EU countries other than Ireland. The own price elasticity of demand, η_{ii}, is equal to $-\sigma_i$, this, and the absence of other cross price effects means that the Armington model's results satisfy the curvature requirements of microeconomic theory, i.e. that the principal minors of the elasticity of substitution matrix alternate in sign.

The elasticities (Marshallian and Allen-Uzawa elasticities of substitution) associated with the GLS parameter estimates of the AI demand system specification of the UK beef import demand are presented in Table 3. In contrast to the Armington model, cross price and expenditure elasticity measures are available.[11] Though the generalized least squares AI demand system estimates satisfy the necessary conditions, i.e. all of the own AUES are negative, the necessary and sufficient conditions for the satisfaction of the curvature requirements, that the principal minors of the AUES matrix alternate in sign, are only satisfied at six points in the sample space (the total sample size is twenty two).

[11] The GLS parameter estimates, and Hicksian elasticities are available on request from the corresponding author, and are omitted here in the interests of brevity.

Table 2: Armington system elasticities (SUR)

	Elasticity of substitution (σ)	R^2
UK Beef	0.840^{**}	0.999
Irish Beef	0.840^{**}	0.969
REU Beef	0.840^{**}	0.333
ROW Beef	0.840^{**}	0.979

** indicates significance at the 1% level.

The monotonicity requirement, that all expenditure shares are positive, is satisfied at all points in the sample space.

The principal purpose of the GLS parameter estimates presented in Table 3 is to illustrate (i) the irregularity of the GLS parameter estimates of the AI demand system and (ii) to obtain the parameter values and the variance-covariance matrix with which to initialize the MCMC posterior density simulator that is used to estimate the locally regular AI demand system specification. The complementary relationships implied by the Marshallian cross price elasticities of demand are contrary to priors that would contend that while beef products from different geographic origins may be imperfect substitutes they are, nevertheless, substitutes in consumption. The consistent pattern of Marshallian demand elasticities which imply complementary relationships may indicate, at least in terms of gross substitution relationships, that this does not hold for the UK beef import demand data set used. Such negative Marshallian cross price elasticities have the counterintuitive implication that an increase in, for example, the price of Irish beef on the UK market would lead to a fall in the demand for UK produced beef if the AI demand specification of the UK meat demand model is accepted as an adequate representation of UK preferences across different beef products. The Allen-Uzawa elasticities reported in Table 3 are more consistent with priors relating to the demand relationships between different beef products. In the AI demand system specification of the UK beef import

Table 3: AI demand system elasticities (GLS)

Marshallian Elasticities

	UK Beef	Irish Beef	REU Beef	ROW Beef	Expenditure
UK Beef	−1.098	−0.109	0.065	0.002	1.140
Irish Beef	−0.925	−0.503	0.222	−0.125	1.327
REU Beef	0.552	0.439	−1.447	−1.338	1.787
ROW Beef	0.975	0.062	−0.349	−0.475	−0.215

Allen-Uzawa Elasticities of Substitution

	UK Beef	Irish Beef	REU Beef	ROW Beef
UK Beef	−0.409	0.030	2.565	1.153
Irish Beef	0.030	−3.650	6.443	0.411
REU Beef	2.565	6.443	−31.241	−7.808
ROW Beef	1.153	0.411	−7.808	−3.538

demand model all goods are net substitutes with the exception of REU and ROW beef, which are implied to be both gross and net substitutes.

The magnitude of the cross price elasticities implies that shocks to prices in the Irish market would have little impact on the UK market or prices of beef products. These results indicate that the hypothesis of perfect substitution does not hold, at least for imports of beef into the UK. The fining that, for the AI demand system model specification of the UK beef import demand model, the own AUES for UK produced beef in the UK is smaller than the AUES for the other imported beef product imports is in accord with a prior belief that UK beef consumers have some degree of “loyalty” to UK produced beef.

Relative to the elasticity of substitution found with the Armington model specifi-

Table 4: MCMC parameter estimates: Fully regular AI demand system

	[1]	[2]	[3]		[1]	[2]	[3]
α_1	-2.874	1.866	1.081	γ_{11}	-0.396	0.027	0.811
α_2	-1.001	0.298	0.821	γ_{12}	-0.149	0.002	0.771
α_3	-1.271	0.480	0.906	γ_{13}	-0.059	0.004	1.160
β_1	0.096	$4.28E-04$	0.742	γ_{22}	-0.011	0.001	0.917
β_2	0.030	$5.41E-05$	0.948	γ_{23}	-0.025	$5.74E-04$	0.794
β_3	0.035	$3.97E-05$	0.685	γ_{33}	-0.080	0.001	0.832
α_0	-34.442	279.16	0.980				

[1]: Mean of draws from the posterior; [2] Numerical Standard Error; [3] Inefficiency Statistic.

cation, the values associated with the (irregular) AI demand system specification are generally larger. This finding echoes that of Alston et al. (1990) who found that when prices of substitute goods are omitted from the demand function specification (as in the Armington model) the own price parameter estimate is biased positively with the concomitant result that the own price elasticities are underestimated.

When all of the regularity conditions of microeconomic theory (symmetry, adding up, homogeneity, monotonicity and curvature) are imposed, the Bayesian parameter estimates (presented in Table 4) of a locally regular AI demand system specification of a UK beef import demand model are obtained. The inefficiency statistics associated with each of the sequences of simulated parameter values reported in Table 4 imply that the MCMC algorithm's draws have converged to draws from the posterior densities associated with the AI demand system specification of the UK beef import demand model. Autocorrelation functions associated with the draws over each of parameters of the AI demand system specification of the UK beef import demand models decay rapidly, this implies that the draws being made after the initial 1,000 draws are discarded are close to *i.i.d.* draws from the associated posterior density functions.[12] A non–decaying autocorrelation function would be indicative of convergence problems in the Markov

[12]Due to space constraints the correlograms representing these autocorrelation functions are not reported. They are available on request from the authors.

Table 5: Fully regular AI demand systems: Elasticities

Marshallian

	UK beef	Irish Beef	REU Beef	ROW Beef	Expenditure
UK Beef	−1.171	−0.074	0.088	0.022	1.136
Irish Beef	−0.661	−0.814	0.132	0.031	1.303
REU Beef	0.919	0.239	−1.791	−1.146	1.786
ROW Beef	1.046	0.168	−0.287	−0.753	−0.174

Allen-Uzawa Elasticities of Substitution

	UK Beef	Irish Beef	REU Beef	ROW Beef
UK Beef	−0.517	0.376	3.079	1.294
Irish Beef	0.376	−6.952	4.329	1.521
REU Beef	3.079	4.329	−39.456	−6.433
ROW Beef	1.294	1.521	−6.433	−5.594

chain.

The elasticities associated with the fully regular AI demand system specification of the UK beef import demand model are presented in Table 5. The imposition of regularity at all points in the sample space leads to some changes in the magnitudes of the associated Marshallian and Allen-Uzawa elasticities. The complementary relationship implied by the negative cross price elasticity of demand between the demand for Irish beef imports in the UK and the price of beef from non-EU countries is reversed. Overall, the changes in the elasticity measures between the unconstrained AI demand system specification (i.e. where monotonicity or curvature are not imposed) and the locally regular AI demand system specification of the UK beef import demand model are not very great.

The results presented in Table 5 for the fully regular AI demand system imply that there remain product pairs that are gross complements (UK and Irish beef products, and other EU beef products and beef products from non-EU countries). However, as with the locally irregular AI demand system's results (Table 3), the elasticity measures associated with the fully regular AI demand system indicate that only one product pair can be considered net complements. The sign pattern of AUES for both the irregular and regular AI demand system specifications is the same. As noted above, the negative Marshallian cross price elasticity associated with the ROW beef price in the Irish beef product expenditure share equation in irregular model switches sign. In terms of the magnitude of the own price elasticities, these are greater in the regularity constrained specification than in the unconstrained AI demand system specification. This result echoes that of Chalfant, Gray and White (1991), who, in an AI demand system model of Canadian meat demand, imposed regularity at one point in the sample space using Monte Carlo integration methods and found that the own price elasticities of demand increased in magnitude relative to an unconstrained AI demand system specification.

In comparison to the regular Armington model specification, the Marshallian own price elasticities of demand associated with the regular AI demand system specification are larger for UK and REU beef products, while the own price elasticities associated with the Irish beef and beef products from non-EU countries are slightly smaller. The AUES obtained with the locally regular AI demand system specification indicate that all of the beef product pairs, with one exception, are Allen substitutes as they are in the highly restricted Armington specification. The magnitude of the substitution elasticities, however, indicates that the goods while net substitutes do not approach the magnitudes that we would expect to find for products that are perfect, or very close, substitutes. The largest substitution relationships are found between the REU beef product and the other beef products. The complementary relationship indicated by the negative AUES between the REU and ROW beef products does not fit with priors concerning the demand relationships between products such as these. This counter-intuitive result may be due to the degree of aggregation involved in each of these products.

The own price Allen-Uzawa elasticities indicate that the compensated demand response to a change in own price is smallest for UK beef and that in general UK demand for UK beef responds the least to changes in the prices of alternative beef products. These results fit with the prior belief that UK consumers have a preference for UK produced beef. The results obtained with the regular AI demand system specification indicate that the Irish own price elasticity of demand is relatively inelastic. This result if it were replicated across other EU beef markets, would have important implications for the Irish beef industry.

The methodology used to impose the monotonicity and regularity conditions on the AI demand system could also impose a prior belief that all of the beef products are Allen or Marshallian substitutes. To do this however would further restrict the AI demand system specification's flexibility. The relatively small nature of the changes in the sign and magnitude of the elasticity measure obtained when the AI demand system is constrained to be regular over the sample space indicates that the losses in flexibility that arise from imposing regularity condition relating to monotonicity and curvature are not large.

5 Conclusions

The importance of theoretically regular trade model specifications is illustrated by the continued use and importance of what are known as the Armington assumption in trade modeling. The separability assumptions inherent in the Armington model specification and its inflexibility have lead to the use of flexible functional form based demand system specifications. The frequent regularity failures of flexible functional form based consumer demand model have been replicated in their import demand modeling applications. Brenton (1994), among others has argued for the need for a modeling approach that combines the regularity advantages of the CES based Armington model with the flexibility advantages that are associated with flexible functional form based model specifications such as the AI demand system. The Bayesian methods used in this paper impose only those restrictions that theory suggests should hold. Though a trade off exists between the flexibility of the specification and the size of the space over which regularity conditions are imposed, see Terrell (1996), the methods used here, given the similarity of the elasticity measures derived from the locally regular and irregular AI demand system specifications imply that flexibility characteristics of the AI demand system specification are not entirely negated. The Bayesian approach used in this paper appears to offer a conceptually simple, though admittedly computationally intense, solution to the problem of imposing regularity on flexible functional form specifications of import demand models.

The policy implications for the Irish beef industry of the results obtained are also important. It is expected that the greater geographic differentiation of beef products required by recent EU legislation on product origin labeling will increase the impact of national preference in EU consumer beef purchases. Such a development, other things equal, would be expected to make the own price elasticity of demand for UK produced beef even more inelastic with respect to price and make the demands for imported beef product more inelastic in an own price sense. The increased importance of market outlets for beef produced in Ireland and the concomitant decline in importance of policy driven demand means that the Irish beef industry will have to either price Irish beef onto EU beef markets or market Irish beef in such a way as to appeal to EU consumer's non-price concerns. For an industry that has traditionally been very reliant on non-market determined demand, the latter strategy is unlikely to provide outlets for the bulk of Irish beef production. The successfulness of a strategy of pricing Irish beef onto other EU markets depends on a number of factors, among others these include the following. The price sensitivity of EU consumers' demand for beef vis á vis other meats, the extent to which national preference factors emerge in EU beef markets that would be expected to reduce the own price elasticity of demand for domestically produced beef, and the price sensitivity of demand for Irish beef on these markets. The results presented in this paper indicate that the elasticity of demand for Irish beef in the UK is inelastic. Thus, at least on the UK market, any attempt to increase market share through lower prices would not be able to dramatically increase the demand for Irish beef. If the elasticity of demand for Irish beef in continental EU beef markets is similarly inelastic the outlook for the Irish beef industry as a commodity beef producer is not bright

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