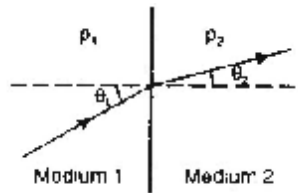


## REFRACTION OF ELECTRICAL RESISTIVITY

### A. Distortion of Current flow

At the boundary between two media of different resistivities the potential remains continuous and the current lines are refracted according to the law of tangents.

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\rho_1}{\rho_2}$$

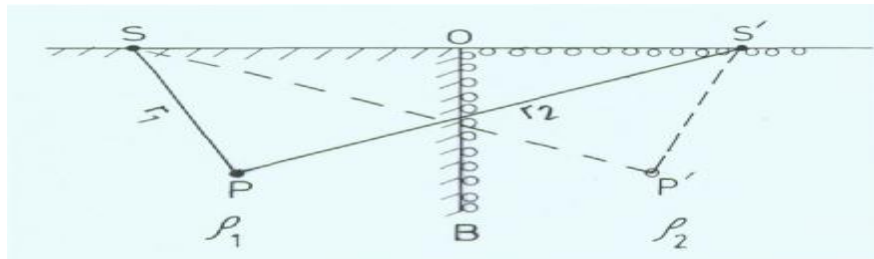


$$\rho_1 \tan \theta_1 = \rho_2 \tan \theta_2$$

If  $\rho_2 < \rho_1$ , The current lines will be refracted away from the Normal. The line of flow are moved downward because the lower resistivity below the interface results in an easier path for the current within the deeper zone.

### B. Distortion of Potential

Consider a source of current  $I$  at the point  $S$  in the first layers  $\rho_1$  of Semi infinite extent. The potential at any point  $P$  would be that from  $S$  plus the amount reflected by the layer  $\rho_2$  as if the reflected amount were coming from the image  $S'$



$$V_1(P) = i \rho_1 / 2\pi [ (1 / r_1) + (K / r_2) ]$$

$$K = \text{Reflection coefficient} = \rho_2 - \rho_1 / \rho_2 + \rho_1$$

In the case where P lies in the second medium  $\rho_2$ , then transmitting light coming from S. Since only  $1 - K$  is transmitted through the boundary. The Potential in the second medium is

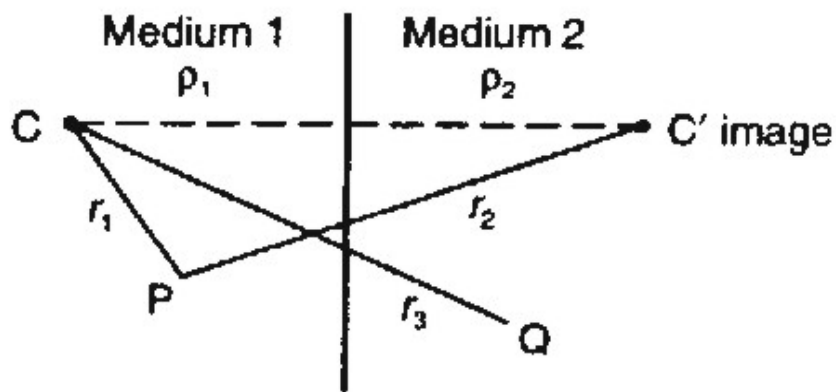
$$V_2(P) = I \rho_2 / 2\pi [ (1 / r_1) - (K / r_1) ]$$

Continuity of the potential requires that the boundary where  $r_1 = r_2$ ,  $V_1(p)$  must be equal to  $V_2 ( P)$ .

At the interface  $r_1 = r_2$ ,  $V_1 = V_2$

### Method of Images

Potential at point close to a boundary can be found using "Method of Images" from optics.



### In optics:

Two media separated by semi transparent mirror of reflection and transmission coefficients  $k$  and  $1-k$ , with light source in medium 1. Intensity at a point in medium 1 is due to source and its reflection, considered as image source in second medium, i.e source scaled by reflection coefficient  $k$ . Intensity at point in medium 2 is due only to source scaled by transmission coefficient  $1-k$  as light passed through boundary.

## Electrical Reflection Coefficient

Consider point current source and find expression for current potentials in medium 1 and medium 2: Use potential from point source, but  $4\pi$  as shell is spherical:

Potential at point P in medium 1:

$$V_P = \frac{I, \rho_1}{4 \pi} \left[ \frac{1}{r_1} + \frac{k}{r_2} \right]$$

Potential at point Q in medium 2:

$$V_Q = \frac{I, \rho_2}{4 \pi} \frac{[1 - k]}{r_3}$$

At point on boundary mid-way between source and its image:

$r_1=r_2=r_3=r$  say. Setting  $V_p = V_q$ , and cancelling we get:

$$\frac{\rho_1}{\rho_2} = \frac{1 - k}{1 + k} \qquad k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$k$  is electrical reflection coefficient and used in interpretation

The value of the diming factor ,  $K$  always lies between  $\pm 1$

If the second layer is a pure insulator

(  $\rho_2 = \infty$  ) then  $K = + 1$

If The second layer is a perfect conductor

(  $\rho_2 = 0$  ) then  $K = - 1$

When  $\rho_1 = \rho_2$  then No electrical boundary Exists and  $K = 0$  .

