CHAPTER SEVEN

An ALGEBRIC APPROACH TO THE TARGETING OF MASS-EXCHANGE NETWORKS

As mentioned in Chapter Six, algebraic approaches have key advantages over graphical tools. While graphs provide visualization insights, they may become cumbersome as the problem size increases or when scaling problems become an issue. On the other hand, algebraic techniques can be effective in handling large problems as they can be conveniently carried out using spreadsheets or calculators. Additionally, major differences in composition scales do not constitute a computational problem. Finally, algebraic techniques can be readily integrated with other design tools including simulators. In this chapter, we present an algebraic approach to the targeting of mass-exchange network that yields results equivalent to those provided by the graphical pinch analysis presented in Chapter Four. In particular, we focus on the identification of minimum load to be removed using external MSAs, excess capacity of process MSAs, and maximum removable load using process MSAs. For constructing the actual network of mass exchangers, the reader is referred to El-Halwagi and Manousiouthakis (1989a) and El-Halwagi (1997).

7.1 . The Composition-Interval Diagram

In order to insure thermodynamic feasibility of mass exchange, we construct the composition-interval diagram (CID). On this diagram, $N_{sp} + 1$ corresponding composition axes are generated. First, a composition axis, *y*, is established for the rich streams. This axis does not have to be drawn to scale. Each rich stream is represented as a vertical arrow whose tail corresponds to its supply composition and head corresponds to its target composition. Then, Eq. (4.17) is employed to each MSA construct N_{sp} corresponding composition scales for the process MSAs. Each process MSA is represented versus its composition axis as a vertical arrow extending between supply and target compositions. Next, horizontal lines are drawn at the heads and tails of the arrows. The vertical distance between two consecutive horizontal lines is referred to as a *composition interval*. The number of intervals is related to the number of process streams via

$$N_{int} \leq 2\,(N_R + N_{SP}) - 1 \qquad (7.1)$$

with the equality applying in cases where no heads on tails coincide. An index k is used to designated composition intervals with k = 1 being the top interval and k = N_{int} being the bottom interval. Figure 7.1 provides a schematic representation of the CID. Mass exchange is thermodynamically (and practically) feasible from rich streams to MSAs within the same interval. Additionally, it is also feasible to transfer mass from a rich stream to lean stream that lies in an interval below it.

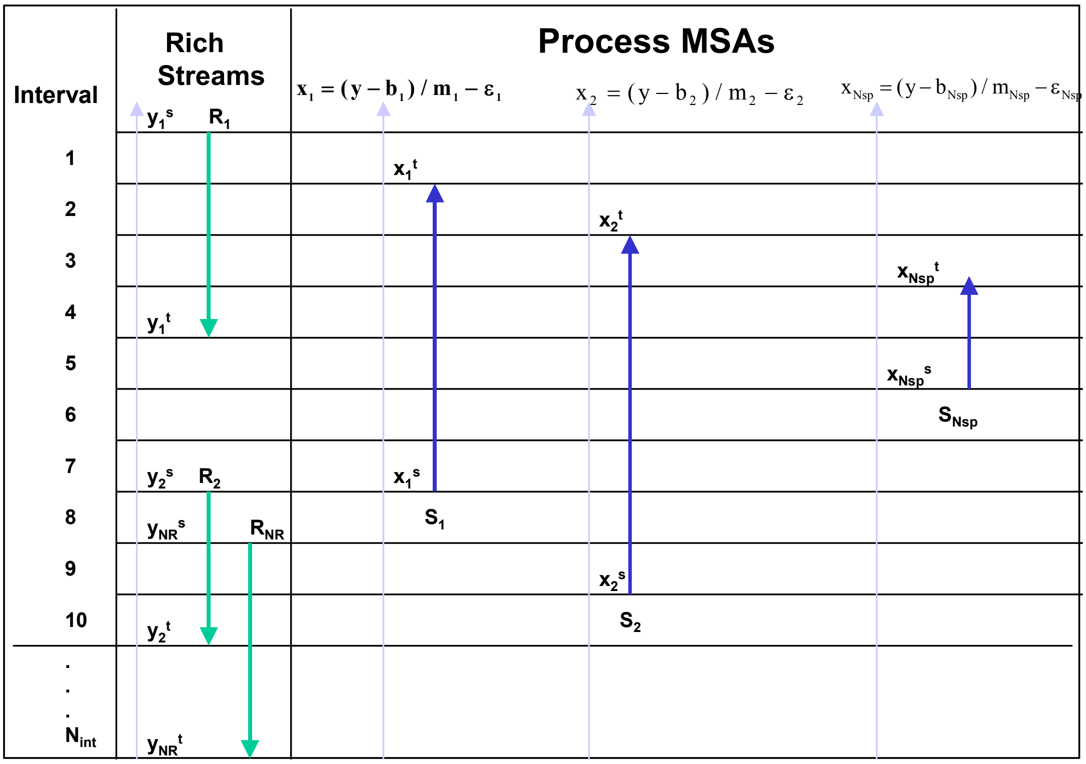

Fig. 7.1. Composition Interval Diagram (El-Halwagi, 1997)

7.2. Table of Exchangeable Loads

In order to determine the exchangeable loads among the rich and lean streams in each interval, we construct the table of exchangeable loads (TEL). The exchangeable load of the ith rich stream that passes through the k th interval can be calculated from the following expression:

$$W_{i,k}^R = G_i \, (y_{k-1} - y_k), \tag{7.2}$$

where y_{k-1} and y_k are the rich-axis compositions of the transferable species that respectively correspond to the top and the bottom lines defining the k^{th} interval. Similarly, maximum load that may be gained by the j^{th} process MSA within interval k can be determined through the following equation:

$$W_{j,k}^{S} = L_j^C (x_{j,k-1} - x_{j,k}), \tag{7.3}$$

where $x_{j,k-1}$ and $x_{j,k}$ are the compositions on the j^{th} lean-composition axis which respectively correspond to the higher and lower horizontal lines bounding the k^{th} interval. Naturally, if a stream does not pass through an interval, its load within that interval is zero.

Once the individual loads of all rich and process MSAs have been determined for all composition intervals, one can also obtain the collective loads of the waste and the lean streams. The collective load of the rich streams within the k^{th} interval is calculated by summing the individual loads of the waste streams that pass through that interval, i.e.

$$W_k^R = \sum_{i \text{ passes through interval } k} W_{i,k}^R . \tag{7.4}$$

Similarly, the collective load of the lean streams within the *k*th interval is evaluated as follows:

$$W_k^S = \sum_{j \text{ passes through interval } k} W_{j,k}^S . \tag{7.5}$$

With the collective loads calculated, the next step is to exchange loads among rich and lean streams. This is accomplished through feasible mass balances as described in the next section.

7.3. Mass-Exchange Cascade Diagram

In constructing the CID, composition axes were mapped such that it is feasible to transfer mass from the rich streams to the lean streams within the same interval or at any lower interval.

Therefore, the following component material balance for the targeted species can be written for the k^th interval:

$$W_k^R + \delta_{k-1} - W_k^S = \delta_k \qquad (7.6)$$

where $?_{k-1}$ and $?_k$ are the residual masses of the targeted species entering and leaving the k^{th} interval with the residual mass entering the first interval (δ_0) being zero. Figure 7.2 illustrates the component material balance for the targeted species around the k^{th} composition interval.

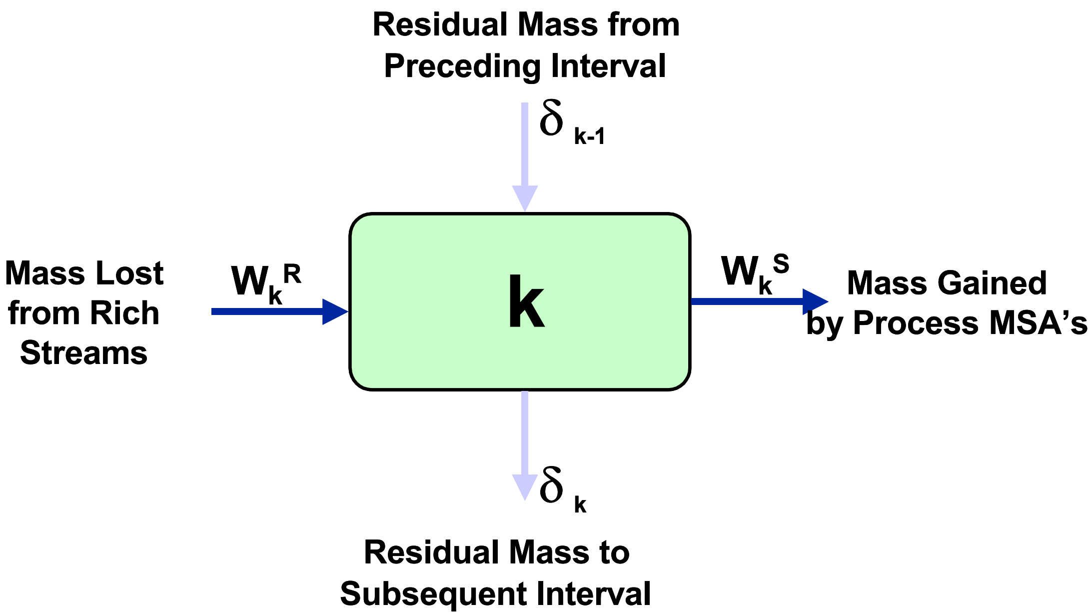

Fig. 7.2. Component Material Balance around a Composition Interval

It is worth pointing out that δ_0 is zero because no waste streams exist above the first interval. In addition, thermodynamic feasibility is ensured when all the δ_k's are nonnegative. Hence, a negative δ_k indicates that the capacity of the process lean streams at that level is greater than the load of the waste streams. The most negative δ_k corresponds to the excess capacity of the process MSAs in removing the pollutant. Therefore, this excess capacity of process MSAs should be reduced by lowering the flowrate and/or the outlet composition of one or more of the MSAs. After removing the excess MSA capacity, one can construct a revised TEL in which the flowrates and/or outlet compositions of the process MSAs have been adjusted. Consequently a revised cascade diagram can be generated. On the revised cascade diagram the location at which the residual mass is zero corresponds to the mass-exchange pinch composition. As expected, this location is the same as that with the most negative residual on the original cascade diagram. Since an overall material balance for the network must be realized, the residual mass leaving the lowest composition interval of the revised cascade diagram must be removed by external MSAs.

7.4. Example on Cleaning of Aqueous Wastes

An organic pollutant is to be removed from two aqueous wastes. The data for the rich streams are given in Table 7.1.

Table 7.1. Data of Rich Streams for the Wastewater Cleaning Example

Stream	Flowrate G_1 kg/s	Supply composition of pollutant (mass fraction) y_i^s	Target composition of pollutant (mass fraction) y_i^t
R_1	2.0	0.030	0.005

Stream	Flowrate G_1 kg/s	Supply composition of pollutant (mass fraction) y_i^s	Target composition of pollutant (mass fraction) y_i^t
R_2	3.0	0.010	0.001

Two process MSAs are considered for separation. The stream data for S_1 and S_2 are given in Table 7.2. The equilibrium data for the pollutant in the two process MSAs are given by:

$$y = 0.25x_1 \tag{7.7}$$

and

$$y = 0.50x_2, \tag{7.8}$$

The minimum allowable composition difference for both process MSAs is taken as 0.001 (mass fraction of pollutant).

Determine the minimum load to be removed using an external MSA, the pinch location, and excess capacity of the process MSAs.

Table 7.2. Data of Process Lean Streams for the Wastewater Cleaning Example

Stream	Upper bound on flowrate L_j^C kg/s	Supply composition of pollutant (mass fraction) x_j^s	Target composition of pollutant (mass fraction) x_j^t
S_1	17.0	0.007	0.009
S_2	1.0	0.005	0.015

Solution:

The CID for the problem is constructed as shown in Fig. 7.3. Then, the TEL is developed as shown in Table 7.3. Next, the mass-exchange cascade diagram is generated. As can be seen in Fig. 7.4, the most negative residual mass is -0.006 kg/s. This value corresponds to the excess capacity of process MSAs. Such excess capacity can be removed by reducing the flowrates and/or outlet compositions of the process MSAs. If we decide to eliminate this excess by decreasing the flowrate of S_1, the actual flowrate of S_1 can be calculated as follows:

$$L_1^C = L_1^{actual} + L_1^{excess} \quad (7.9a)$$

Multiplying both sides by the composition range for S_1, we get

$$L_1^C * (0.009 - 0.007) = L_1^{actual} * (0.009 - 0.007) + Excess\,Load \quad (7.9b)$$

i.e.,

$$17.0 * (0.009 - 0.007) = L_1^{actual} * (0.009 - 0.007) + 0.006 \quad (7.9c)$$

Hence,

$$L_1^{actual} = 17.0 - \frac{0.006}{0.009 - 0.007} = 14.0\ kg/s \quad (7.9d)$$

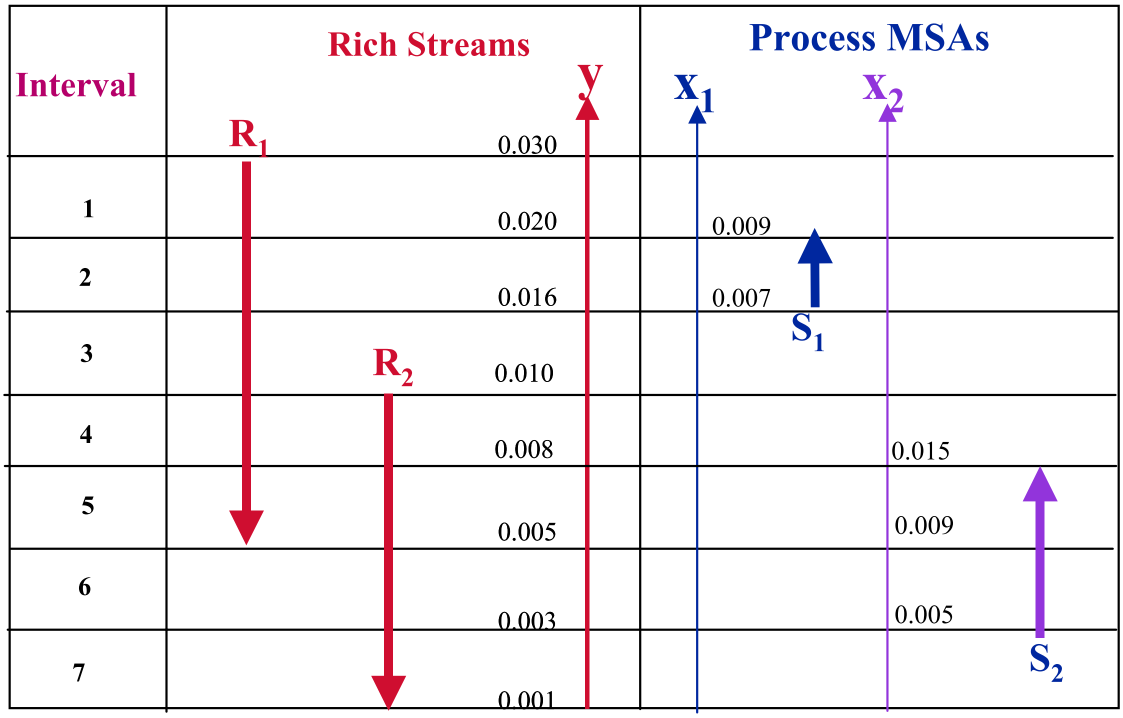

Fig. 7.3. CID for the Wastewater Cleaning Example

Using the adjusted flowrate of S_1, we can now construct the revised cascade diagram as depicted by Fig. 7.4. Hence, the revised mass-exchange cascade diagram is generated as shown in Fig. 5.5. On this diagram, the residual mass leaving the second interval is zero. Therefore, the mass- exchange pinch is located on the line separating the second and the third intervals. As can be seen in Fig. 7.3, this location corresponds to a y composition of 0.016 (which is equivalent to an x_1 composition of 0.007). Furthermore, Fig. 7.4 shows the residual mass leaving the bottom interval as 0.039 kg/s. This value is the amount of pollutant to be removed by external MSAs.

Table 7.3. TEL for Wastewater Cleaning Example

Interval	Load of R_1 kg/s	Load of R_2 kg/s	Load of $R_1 + R_2$ kg/s	Load of S_1 kg/s	Load of S_2 kg/s	Load of $S_1 + S_2$ kg/s
1	0.020	-	0.020	-	-	-
2	0.008	-	0.008	0.034	-	0.034
3	0.012	-	0.012	-	-	-
4	0.004	0.006	0.010	-	-	-
5	0.006	0.009	0.015	-	0.006	0.006
6	-	0.006	0.006	-	0.004	0.004
7	-	0.006	0.006	-	-	-

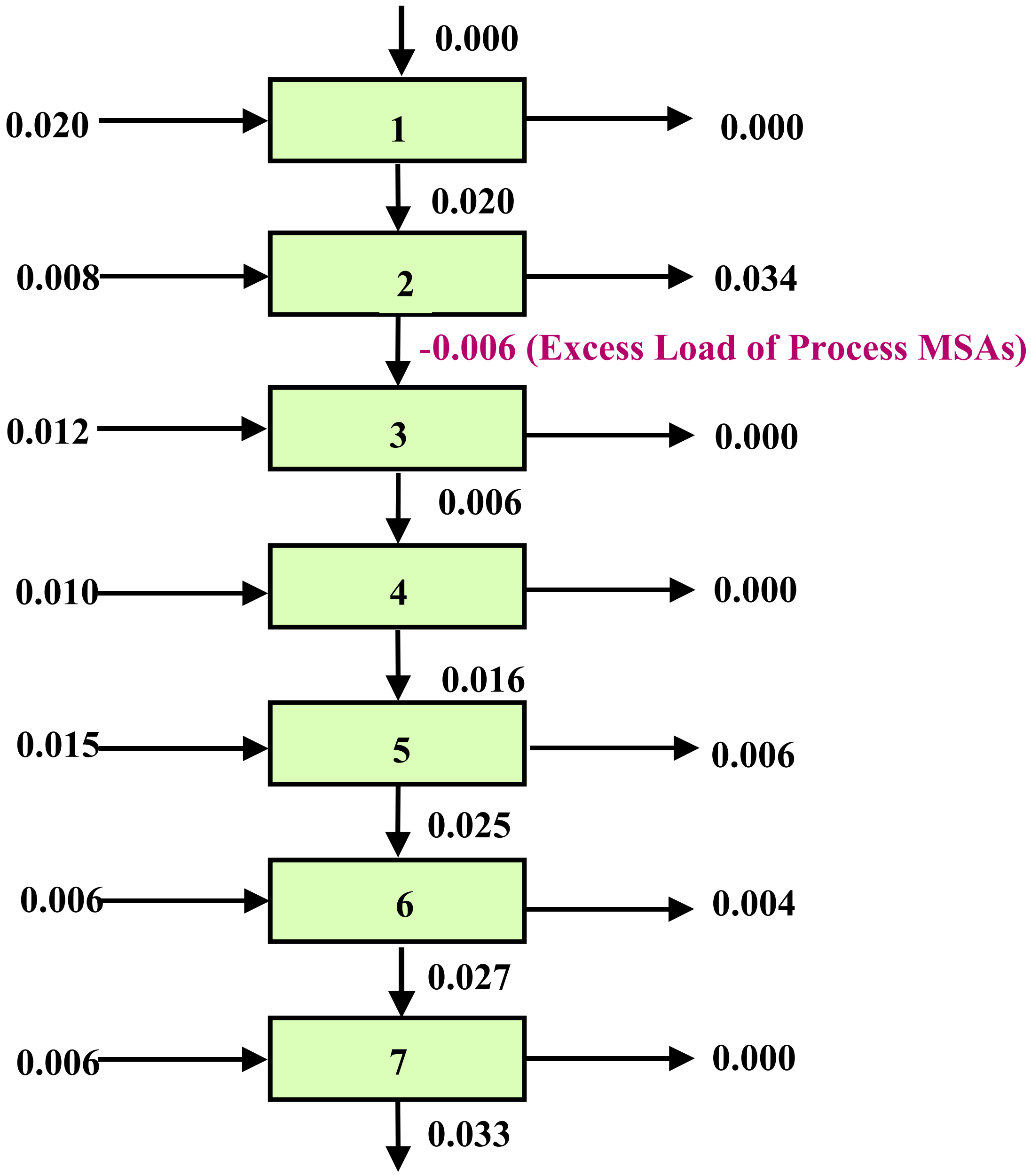

Fig. 5.5 .Cascade Diagram for the Wastewater Cleaning Example.

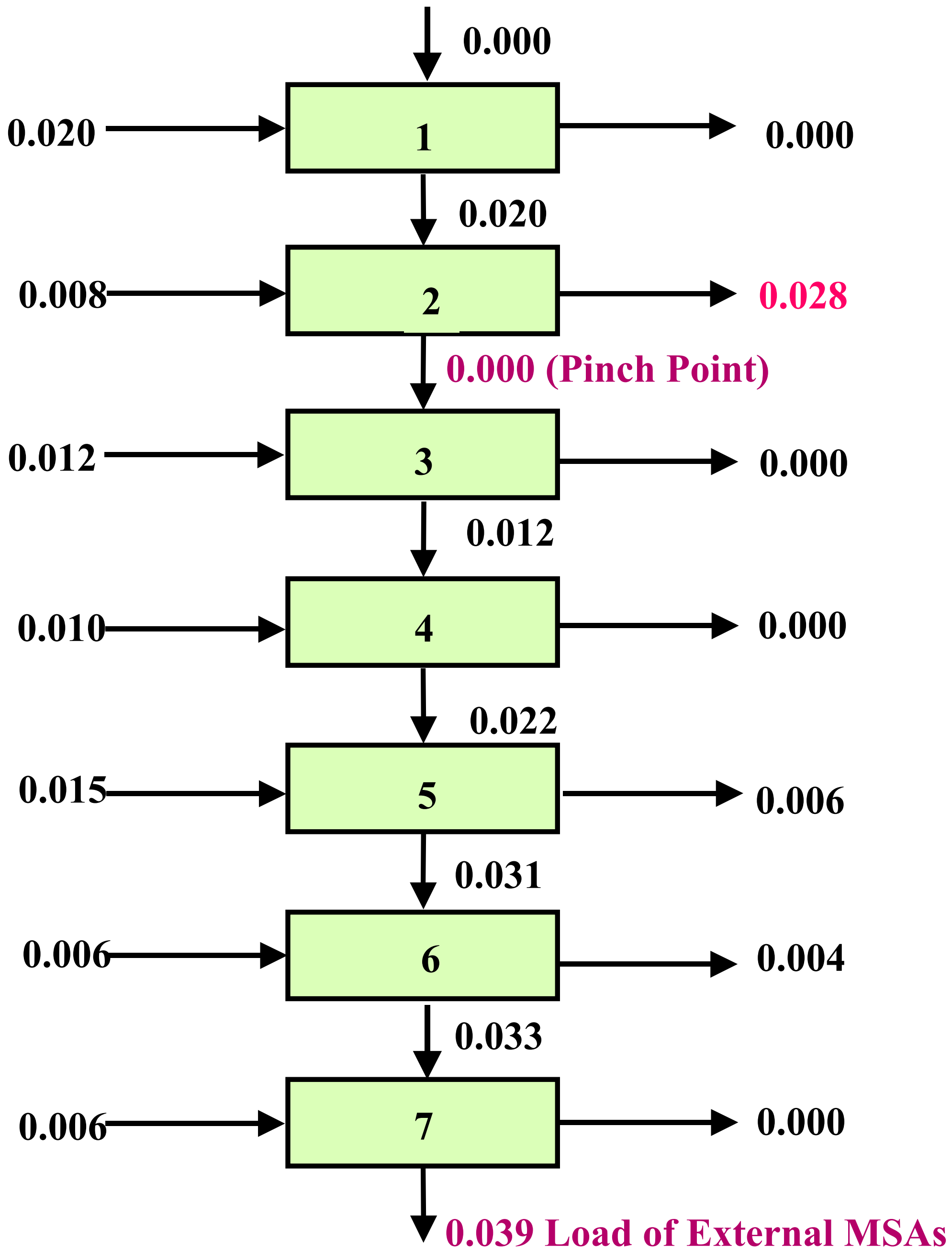

Fig. 5.5 .Revised Cascade Diagram for the Wastewater Cleaning Example.

PROBLEMS

7.1. Using an algebraic procedure, synthesize an optimal MEN for the benzene recovery example described in Example 4.1

7.2. Use the algebraic technique to solve the dephenolization case study described in problem 4.8.

7.3. Resolve the dephenolization case study described in problem 4.8 when the two waste streams are allowed to mix.

7.4. The techniques presented in this chapter can be generalized to tackle MENs with multiple pollutants. Consider the COG sweetening process addressed by the previous problem. Carbon dioxide often exists in COG in relatively large concentrations. Therefore, partial removal of CO_2 is sometimes desirable to improve the heating value of COG, and almost complete removal of CO_2 is required for gases undergoing low-temperature processing. The data for the CO_2-laden rich and lean streams are given in Table 7.4.

Table 7.4. Stream Data for COG-Sweetening Problem

Rich streams				**MSAs**						
	G_i				L_j^c					c_j

Stream	(kg/s)	Supply mass fraction of CO_2	Target mass fraction of CO_2	Stream	kg/s	Supply mass fraction of CO_2	Target mass fraction of CO_2	m_j for CO_2	b_j for CO_2	$/kg
R_1	0.90	0.0600	0.0050	S_1	2.3	0.0100	?	0.35	0.000	0.00
R_2	0.10	0.1150	0.0100	S_2	?	0.0003	?	0.58	0.000	0.10

Synthesize an MOC MEN which removes hydrogen sulfide and carbon dioxide simultaneously.

Hint: See El-Halwagi and Manousiouthakis (1989a,b).

SYMBOLS

b_j intercept of equilibrium line of the jth MSA

G_i flowrate of the ith waste stream

i index of waste streams

j index of MSAs

k index of composition intervals

L_j flowrate of the j*th* MSA

L_j^C upper bound on available flowrate of the *j*th MSA

m_j slope of equilibrium line for the jth MSA

N_{int} number of composition intervals

N_{SP} number of process MSAs

R_i the *i*th waste stream

S_j the *j*th lean stream

$W_{i,k}^R$ the exchangeable lead of the *i*th rich stream which passes through the *k*th interval

$W_{j,k}^S$ the exchangeable load of the *j*th MSA passing through the *k*th interval

W_k^R the collective exchangeable load of the rich streams in interval *k*

W_k^S the collective exchangeable load of the MSAs in interval *k*

$x_{j,k-1}$ composition of key component in the *j*th MSA at the upper horizontal line defining the *k*th interval

$x_{j,k}$ composition of key component in the *j*th MSA at the lower horizontal line defining the *k*th interval

y_{k-1} composition of key component in the *i*th rich stream at the upper horizontal line defining the *k*th interval

y_k composition of key component in the *i*th rich stream at the lower horizontal line defining the *k*th interval

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El-Halwagi, M. M., "Pollution Prevention through Process Integration: Systematic Design Tools", Academic Press, San Diego (1997).

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