# CHAPTER 2

# PLANNING AND SCHEDULING

Planning is the most important of the management processes. Planning construction projects involves the determination of what must be done, how it is to be performed and the sequential order in which it will be carried out. The planning process aims at giving every opportunity for the work to proceed on certain directed lines and within estimated costs. The whole basis of planning is to gain an understanding of the constraints and priorities of the work, to identify potential problems and to think of ways of overcoming these problems before they occur.

A construction schedule is a time-phased plan to perform the work that is necessary to complete a construction project. The schedule indicates the planned starting and ending dates for each of the work elements that make up the total amount of construction work to be performed.

The chapter starts now with demonstration of the planning process. Various scheduling techniques used in construction are given next.

## 2.1 The Planning Process

The proper and successful running of a project will be very much a matter of chance without planning. Time is the unit which is most frequently planned. The project plan is usually prepared and adjusted during the different stages of the project. It usually takes one of the following forms:

1. Pre-tender plan that is prepared for submission of a tender. It is usually produced using tender documents.
2. Pre-contract plan that is prepared between the tender acceptance and start of work on site. Contract documents and any information gained during this period are used to adjust the pre-tender plan.
3. Contract plan or site plan that is used for construction on site. Now each of the activities contained in the pre-contract plan is divided into sub-activities for easier control of construction.

The plan of a project is obtained by examining all possible alternative construction methods and selecting the cheapest feasible solution. This may depend on detailed calculations. Therefore, the planner needs a thorough knowledge of the construction procedures that will be followed. He can consult the estimator and field engineers who will be involved with the project. However, construction planning consists of the following steps:

1. Determination of the project activities.
2. Calculation of the duration of these activities.
3. Establishment of the relationship among the activities and specificity of the overlap (if any) between the activities.

Each of these steps will be discussed in detail next.

## 2.2 Determination of Project Activities

This section will demonstrate the process followed by a planner in establishing a satisfactory set of activities. The creation of the activity list is an important step in project planning. It follows that if the list is not complete, the resulting computations will not be realistic.

### Work Breakdown

Each work element represented in a plan is called an activity. The objective in the work breakdown process is to divide the total amount of work into enough activities so that the relationship between work activities can be specified. All the work that make up a project must be included in only one of the defined activities.

Production activities are those that can be taken directly from drawings and specifications. They are the most obvious and consume the greatest amount of time for completion of the project. Some examples are: excavating, setting reinforcements, placing concrete, and so on. The omission of one of such activities from the activity list might result in failure to fulfill the contract.

Other kinds of activities may be classified as procurement and management activities. Examples include:

- Jobsite mobilization such as building a field office, supplying electricity, water and telephone and getting employees from surrounding regions.
- Equipment delivery to the site, and assembly and subsequently disassembly of equipment items such as tower cranes.

- Clearing and grading of the site.
- Resurvey of the site where the designer's survey is inadequate.
- Preparation of shop drawings.
- Receipt of completed project design documents.
- The design and testing of plant mixed materials.
- Determining the length of the piles that must be driven to attain a specified bearing capacity.
- Delivery of critical materials.
- Engineering approvals and inspections.
- The contractor must clean the site, remove field office and his equipment when the construction work is completed.

## Grouping of Work Elements

The principles for grouping of work elements together to form activities are as follows:

- Activities should be defined so that their durations can be predictable. If there is any doubt about the duration of an activity it should be divided.
- Breakdown into several activities work elements if different subcontractors or specific resources will be required to perform these elements. For example; do not include as one activity these work elements which require drilling rigs and excavators.
- An activity should contain only work elements that have common constraints or serve as common constraints. For example "Concrete foundations and slabs" must be defined as two activities.
- Activities should be defined so that their included work can be performed without being interrupted by another activity. For example "Concrete floor slab and place the flooring on the slab" must not be one activity; erecting interior partitions is in between.

## Appropriate Level of Detail

The appropriate level of detail of a plan should satisfy the varying needs of users. The pre-tender plan prepared by the contractor for bidding will have less detail than plans prepared by section engineers during work on site. For example: an activity called "Construct Foundation" contained in a pre-tender plan will be broken by the section engineer into the following activities: "fix reinforcement", "erect side shutters", "concrete", "strike shutters", and "cure".

**Example 2.1**

Figure 2.1 shows a double span bridge. Break the construction works of the bridge down into activities. The plan will be used for bidding purpose.

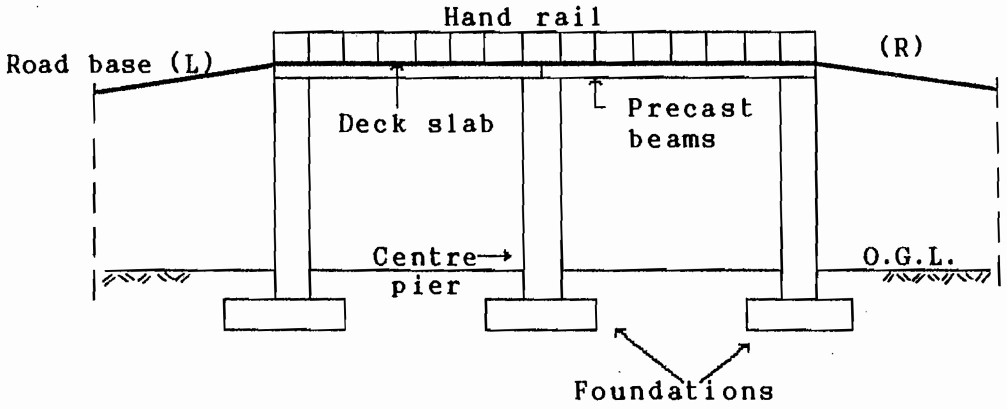


**Figure 2.1 Double Span Bridge**

**Table 2.1 Example of Work Breakdown**

| No. | Activity |
|---|---|
| 10 | Set up site |
| 14 | Procure reinforcement |
| 16 | Procure deck beams |
| 20 | Excavate foundation Left |
| 30 | Excavate foundation Right |
| 40 | Excavate foundation Centre |
| 50 | Construct foundation Left |
| 60 | Construct foundation Right |
| 70 | Construct foundation Centre |
| 80 | Construct abutment Left |
| 90 | Construct abutment Right |
| 100 | Construct Centre pier |
| 110 | Place deck beams Left |
| 120 | Place deck beams Right |
| 140 | Fill embankment Left |
| 150 | Fill embankment Right |
| 155 | Construct R.C. deck slab |
| 160 | Road base Left |
| 170 | Road base Right |
| 180 | Surface road |
| 190 | Erect bridge railing |
| 200 | Clear site |

**Solution**

See Table 2.1. The project total work has been broken down into activities. Each activity has been given a number.

## 2.3 Estimating Activity Duration

Proper time duration for each activity is to be established if success is to be achieved. At times these durations are easy to establish but more often their formulation is time consuming.

The time interval to be used as a basic measure must be selected. Convenient intervals are days or weeks. The information required to establish the durations may be obtained from direct interviews with the field forces that are actually to perform the work. On the other hand, durations of activities such as "establish site" and "clear site on completion" can be estimated by consulting similar contracts to avoid under-estimation.

There will usually be a single critical resource for each activity, the number and output of that resource determine the time required to complete the activity. This critical resource may be a type of craftsman or a type of equipment. If the other resources taking part in performing the activity are not available in adequate amounts to keep up with the critical resource, one of them may be the critical resource. The objective is to assign a balanced mix of resources to an activity as long as that is practical.

If an activity contains one kind of work, then the time required to complete the activity will be the activity duration, or:

$$\text{activity duration} = \frac{\text{quantity of work}}{\text{no. of resources} \times \text{resource output}} \tag{2.1}$$

If several kinds of work will be done, activity duration will be computed according to the critical resource. The following examples demonstrate the above principles.

**Example 2.2**

Calculate the duration of driving sheetpiles required for the construction of a 3 km culvert, giving that:

- Construction will proceed concurrently on three faces.
- 14 m long sheetpiles will be used. They weigh 185.1 $kg/m^2$.
- Output of a sheetpiling gang consisting of a ganger, 3 labourers, a welder, a crane and a hammer equals 72 tonnes/week.

**Solution**

total quantity of sheetpiling to be driven =
3000 m × 2 sides × 14 m long × 0.1851 t/$m^2$ = 15550 t
use 3 gangs for the three faces, then

$$\text{duration} = \frac{15550}{3 \times 72} = 72 \text{ weeks}$$

**Example 2.3**

The construction of a reinforced concrete wall involves placing 660 $m^3$ concrete, fixing 50 t of steel reinforcement and fixing and subsequently striking 790 $m^2$ of formwork. A batching plant will be used for mixing concrete. Dumpers will be used to transport concrete to the site. The required quantity of steel has already been cut and bent. Formworks have also been prepared. The following information belongs to the jobs involved in this activity:

- A 6 man concrete gang can place 16 $m^3$ of concrete / day.
- One steelfixer and one assistant can fix 0.5t of reinforcement/day
- One carpenter and one assistant can fix and strike 16 $m^2$ / day.

Calculate the duration of the activity considering the steelfixers as the critical resource.

**Solution**

- use one steelfixer, then:
  duration = 50 / 0.5 = 100 days

- 0.42 concrete gang can keep pace with the work:
  output = 0.42 (100) 16 = 670 $m^3$

- 0.5 carpenter can keep pace with the work
  output = 0.5 (100) 16 = 800 $m^2$

- then use 2 steelfixers, 1 concrete gang and 1 carpenter
  Activity duration = 50 days or 9 weeks for a 6-day working week.

**Example 2.4**

It is required to construct a 696 000 $m^3$ embankment. Fill will be placed in layers of 15 cm thickness. The following information pertains to the operations involved in this activity:

1. Fill will be loaded using an excavator with 2.1 $m^3$ bucket.
2. 14 $m^3$ dump trucks will be used for hauling fill. Haul distance equals 1.75 km with gradient 1 : 8 upwards to the quarry.
3. Fill will be spread by a bulldozer; output = 300 $m^3$ / hour.
4. Soil moisture content will be increased by 5% using 1500 gallon water distributed trucks with a discharge rate = 100 gallon / min. The soil's dry density = 1.6 t/$m^3$. It will take 15 min. to travel from the job site to the reservoir to fill the tank and to return to site.
5. Fill will be compacted through six passes of a sheepsfoot roller which has an effective width of 1.83 m and speed = 3.2 km/hour.

Given that 1 $m^3$ of the undisturbed soil ( bank measure ) equals 1.17 $m^3$ of the loose soil and 1 $m^3$ of the compacted soil equals 1.47 $m^3$ of the loose soil, determine a balanced mix of resources to perform this activity. Calculate the required duration to construct the embankment assuming a 60-hour working week.

Solution

Excavation and loading

assuming the fill factor = 85%, operator efficiency = 85% and job cycle time = 30 seconds,

then max. production = 2.1 × .85 × .85 × 60 × 60 × 60 / 30
= 10 924 $m^3$/week (undisturbed soil)
= 12 780 $m^3$/week (loose soil)

Hauling

travelling time: loaded - downhill (max. safe speed = 24 km/hr)
= 1.75 (60) / 24 = 4.38 min.

travelling time: unloaded - uphill (max. safe speed = 20 km/hr)
= 1.75 (60) / 20 = 5.25 min.

total travelling time = 9.63 min.

loading time = 14 (60 × 60) / 12780 = 3.94 min.

unloading time (to position and dump truck) = 1.5 min.

total time required = 15 min. approx.

each truck will carry 14 × 60 × 60 / 15 = 3360 $m^3$/week (loose soil)

Spreading

output = 300 $m^3$/hr = 18 000 $m^3$/week (loose soil)

Distributing water

water required = 0.05 (1.6) = 0.08 t/$m^3$ = 20.6 gallon/$m^3$.

time required to fill the distributor, discharge its content and return to reservoir for refilling = 15 + (1500/100) = 30 min. = 0.5 hour.

output = 1500 / 0.5 = 3 000 gallon / hour = 180 000 gallon / week.
this will satisfy 180000 / 20.6 = 8 738 $m^3$/week (loose soil)

Compacting

the roller will cover 3.2 × 1000 × 1.83 = 5856 $m^2$/hr (one pass)
= 976 $m^2$/hr (six passes)

output = 976 × 0.15 × 60 = 8784 $m^3$/week (compacted soil)
= 12912 $m^3$/week (loose soil)

Grouping outputs

| Equipment | Output $m^3$/ wk (loose soil) |
|---|---|
| Excavator | 12 780 |
| Truck | 3 360 |
| Bulldozer | 18 000 |
| Water Distributor | 8 738 |
| Roller | 12 912 |

- Assuming the major item of plant is the excavator and the other items will be assigned to keep up with it:

then the balanced team will comprise :

| | |
|---|---|
| 1 | excavator |
| 4 | trucks |
| 1 | bulldozer |
| 1.5 | water distributor |
| 1 | roller |

the output of this team is 12780 $m^3$/wk (loose soil) or 8694 $m^3$/wk compacted soil.
for 696 000 $m^3$ embankment (compacted) use 2 teams for 40 weeks; as this activity should be carried out in the dry season of the year.

## 2.4 Logical Relationships

The next step in project planning is to determine the logic of the job. It refers to the determined order in which the activities are to be done. It may have its basis in technology or result from physical considerations or be derived from managerial decisions. The relationships among the activities can be established by asking the following questions:

- which activities must be complete before this activity starts?
- which activities cannot start until this activity is complete?
- which activities have no logical relationships with this activity and can take place at the same time?

In this way each activity is examined and the necessary sequences of activities are determined. Predecessor activities mean coming before, while successor activities mean coming after. If x is a Predecessor to y, then y is a Successor to x. Therefore the basic logical relationship between two activities can be specified by indicating the predecessor of the two activities. If x is a predecessor to y, and y is a predecessor to z, then x is a predecessor to z.

**Example 2.5**

Determine the relationships between activities of the project studied in Example 2.1.

**Solution**

Table 2.2 specifies the relationships between project activities.

**Table 2.2 Double Span Bridge: Dependencies of the activities**

| No. | Activity | Dependency |
|---|---|---|
| 10 | Set up site | - |
| 14 | Procure reinforcement | - |
| 16 | Procure deck beams | - |
| 20 | Excavate foundation Left | 10 |
| 30 | Excavate foundation Right | 10 |
| 40 | Excavate foundation Centre | 10 |
| 50 | Construct foundation Left | 20, 14 |
| 60 | Construct foundation Right | 30, 14 |
| 70 | Construct foundation Centre | 40, 14 |
| 80 | Construct abutment Left | 50 |
| 90 | Construct abutment Right | 60 |
| 100 | Construct Centre pier | 70 |
| 110 | Place deck beams Left | 80, 100, 16 |
| 120 | Place deck beams Right | 90, 100, 16 |
| 140 | Fill embankment Left | 80 |
| 150 | Fill embankment Right | 90 |
| 155 | Construct R.C. deck slab | 110, 120 |
| 160 | Road base Left | 140 |
| 170 | Road base Right | 150 |
| 180 | Surface road | 155, 160, 170 |
| 190 | Erect bridge railing | 155 |
| 200 | Clear site | 180, 190 |

**Overlaps**

The planner should choose the overlap between the activities. That is how much of a particular activity must be completed before a succeeding activity may start. The absence of an overlap implies that the first activity must be complete before the second may start. A negative overlap means a delay is required between the two activities. The overlap between any two activities should be less than or equal to the duration of the predecessor.

For example, a positive overlap may be specified between the following two activities: "Excavate foundation", and "Erect foundation shutters" where the excavation process is carried out for a long pipeline trench. A negative overlap may be specified between the following two activities: "Pour concrete in slabs", and "Strip slab formwork."

**Example 2.6**

The sequence of operation envisaged for the construction of a mass concrete seawall 1200 m long is given in Table 2.3. The output of one gang on each operation in terms of bays 8 m long is also listed. The number of men in each gang is given in Table 2.4. In accordance with the specifications, one week is to be elapse between concreting and stripping shutters. Determine the duration of each activity, its predecessor and any specified overlap, taken into consideration achieving minimum construction cost. Assume a 6-shift week.

**Table 2.3 Data for Example 2.6**

| Operation | Gang | Output bays/shift |
|---|---|---|
| Excavate | Excavation | 3.0 |
| Pump out and erect foundation shutters | Pumping & Shuttering | 2.0 |
| Pump out and erect concrete foundation | Pumping & Concreting | 1.0 |
| Strip shutters | Shuttering | 3.0 |
| Erect wall shutters | Shuttering | 1.5 |
| Concrete wall | Concreting | 1.0 |
| Strip wall shutters | Shuttering | 2.0 |
| Mixing concrete | Batching Plant | 3.0 |

**Table 2.4 Data for Example 2.6**

| Type of gang | Labour content (men) |
|---|---|
| Excavation | 8 |
| Pumping | 4 |
| Shuttering | 6 |
| Concreting | 8 |
| Batching Plant | 4 |

Solution

No. of bays = 150. No. of shifts / week = 6. Then, the volume of work in each activity can be determined as given in Table 2.5.

Minimum construction cost can be achieved by using one batching plant and the minimum number of sets of steel shutters. The planner should also make full use of each gang of labours.

One shuttering gang can strip 3 bays of foundation shutters per shift. This determines number of gangs required for activities C and B. The cycle time for activities B, C and D can be determined using Figure 2.2. Consequently a set of shutters can be reused after 9 shifts.

∴ No. of sets of foundation or wall shutters =
3 bays x 9 shifts = 27 set each

The duration of each activity is given in Table 2.6.

## 2.5 Project Scheduling

A project schedule is a timetable of project activities that enables managers to execute the project in a timely manner. The uses of construction project schedules are:

- The construction manager can make arrangements to have different resources available on site in the correct time.
- There may be financial penalties for late completion of a project. Therefore, if the schedule indicates that the plan will result in late completion, the construction manager should employ more resources, work overtime or perform activities in a different sequence.
- The rate of progress on the project affects the rate at which the contractor will receive payments. He may need to borrow money to cope with his projects.
- To give the client a timely notice of the likely consequences of changes in the project scope, indecision or delay.
- If construction is delayed by the client, the contractor should be given an extension of time for the completion of the works and he should be reimbursed to cover the cost incurred.
- A properly updated schedule may be a valuable source of cost and scheduling data that estimators and schedulers can use for future projects.
- In some cases the client has a contractual right to be provided with a copy of the contractor's schedule.

**Table 2.5 Volume of Work in Each Activity**

| Code | Activity | Volume of work (gang.shift) |
|---|---|---|
| A | Excavate | 50 |
| B | Pump & erect shutters | 75 |
| C | Pump & concreting foundations | 150 |
| D | Strip shutters | 50 |
| E | Erect wall shutters | 100 |
| F | Concrete wall | 150 |
| G | Strip wall shutters | 75 |
| H1 | Batching plant (foundations) | 50 |
| H2 | Batching plant (walls) | 50 |

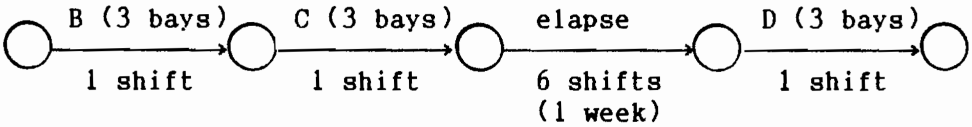


Figure 2.2 Cycle Time for Using Foundation Shutters.

**Table 2.6 Planning for the Seawall**

| Activity | No. of gangs | No. of men | Duration (shifts) | Predecessors | Overlap (shifts) |
|---|---|---|---|---|---|
| A | 1 | 8 | 50 | - | |
| B | 2 | 20 | 50 | A | 49 |
| C | 3 | 36 | 50 | B, H1 | 49 with B<br>50 with H1 |
| D | 1 | 6 | 50 | C | 43 |
| E | 2 | 12 | 50 | D | 6 |
| F | 3 | 24 | 50 | E, H2 | 49 with E<br>50 with H2 |
| G | 2 | 12 | 50 | F | 43 |
| H1 | 1 | 4 | 50 | B | 49 |
| H2 | 1 | 4 | 50 | H1 | |

Scheduling is the determination of the timing of the activities comprising the project. The critical path method ( CPM ) is a powerful tool for the scheduling and management of most types of construction projects. It is the representation of a project plan by a network and the manipulation of this network in determining the best overall schedule of activities. It permits the comparison of alternative work programs and construction methods. During construction, it provides the precise information on the effects of each variation or delay in the adopted schedule.

The CPM can be represented by arrow networks, precedence networks and time-scaled diagrams. These methods will be discussed in the following sections.

## 2.6 Activity-On-Arrow Networks

### Networks (Logic Diagrams)

A network is a graphic representation of a project plan. The activities that make up the project and the logical relationships that exist between these activities are represented by a group of arrows and nodes. In an arrow network the arrows represent activities and the nodes represent events, see Figure 2.3.

An event is a point in time at which something or some things happen. The first event in a project schedule is the start of the project. The last event is the end of the project.

The nodes are generally drawn as circles large enough in diameter to allow for the node number to be written within. The node at the tail of an arrow is termed the i-node. The node at the head of an arrow is termed the j-node. Each activity is defined by its i and j nodes. If a node is the j-node for activity A and it is also the i-node for activity B then A is a predecessor to B. The completion of A is a constraint to the initiation of B.

The arrows may be vertical, horizontal or sloping straight line segments. They originate at the right side of a node and terminate at the left side of a node. Scaling the arrows to activities durations is not recommended because it results in a wide network. However, the arrow should be long enough to allow space for the description of the activity to be written above it. The duration of the activity is written below the arrow.

All logical relationships must be shown on the network. If a logical relationship is ignored, the resulting schedule will be incorrect.

## Conventions

These are rules to make a diagram easier to read and to reduce the chances of errors. Figure 2.4 demonstrates these conventions.

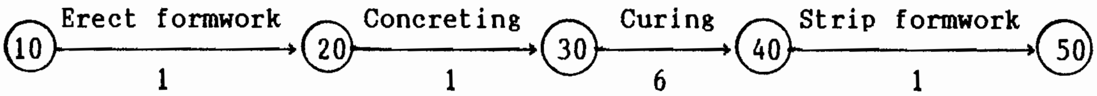


Figure 2.3 Arrow Networks Notation

1. Nonconsecutive node numbers to allow adding nodes with appropriate number.
2. j-node number greater than i-node number.
3. Arrows point right, up or down but not to left to avoid creating a loop.
4. Use symbols to indicate crossovers to avoid misunderstanding.

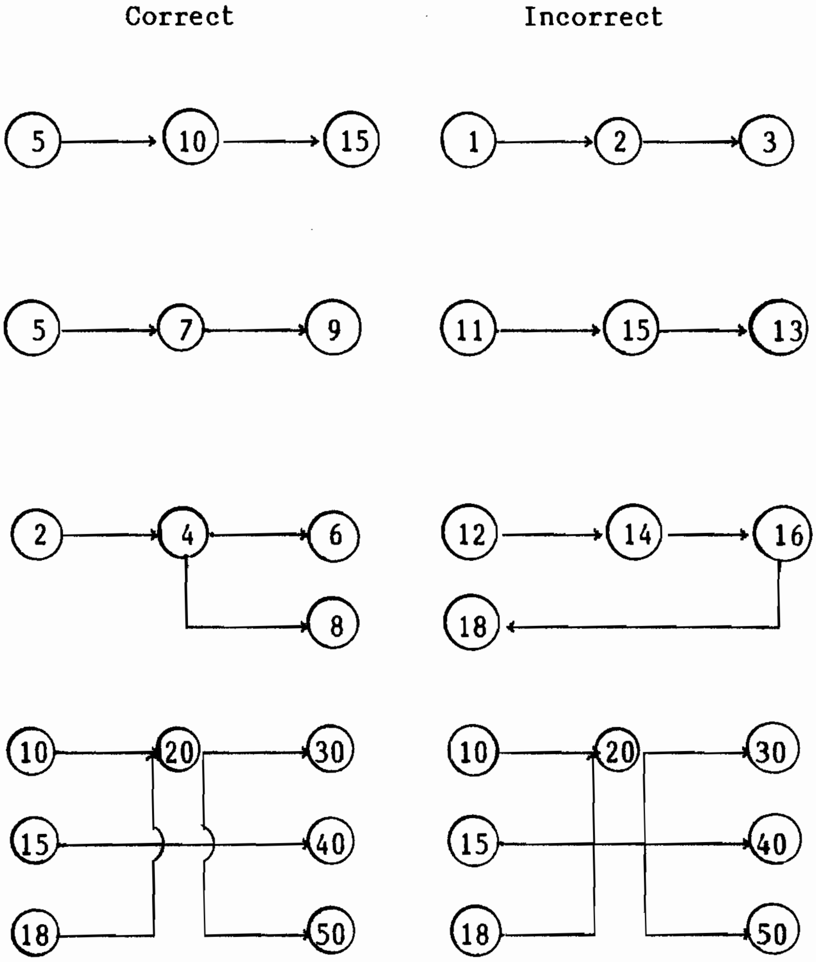


Figure 2.4 Conventions for Arrow Networks

### Dummy Activities

Activities must have unique pair of node numbers. Therefore, when more than one arrow leave the same event and arrive at another event, dummy activities must be used. They have zero durations and are drawn as dashed lines. Dummy activities can also be used as logical constraints. In Figure 2.5, activity F can not start before activity A ends.

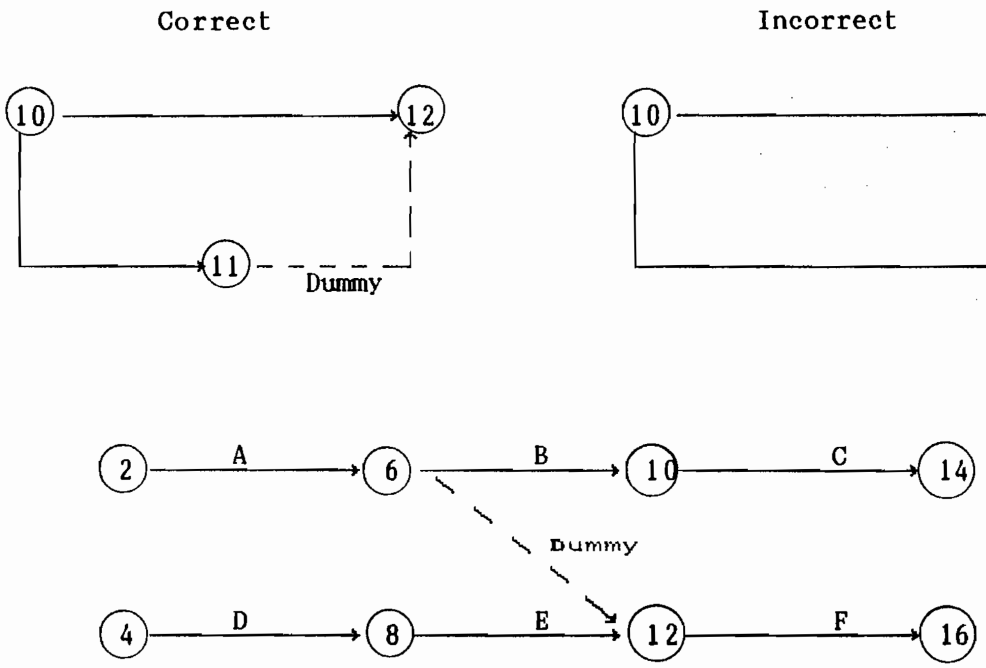


Figure 2.5 Use of Dummy Activities

### Example 2.7

The activities comprising a simple contract are given in Table 2.7. Draw an arrow network for this contract.

### Solution

The required arrow diagram is shown in Figure 2.6.

### Network Analysis

The objective of arrow network analysis is to compute for each event in the network its early and late timings of occurrence. These are defined as follows:

Table 2.7 Data for Example 2.7

| Activity | Constraints |
|---|---|
| A | — |
| B | A |
| C | B |
| D | C |
| E | C |
| F | C |
| G | E |
| H | G |
| I | H |
| J | E, F, I |
| K | I |
| L | I |
| M | H |
| N | M |
| O | D, J, K, L, N |

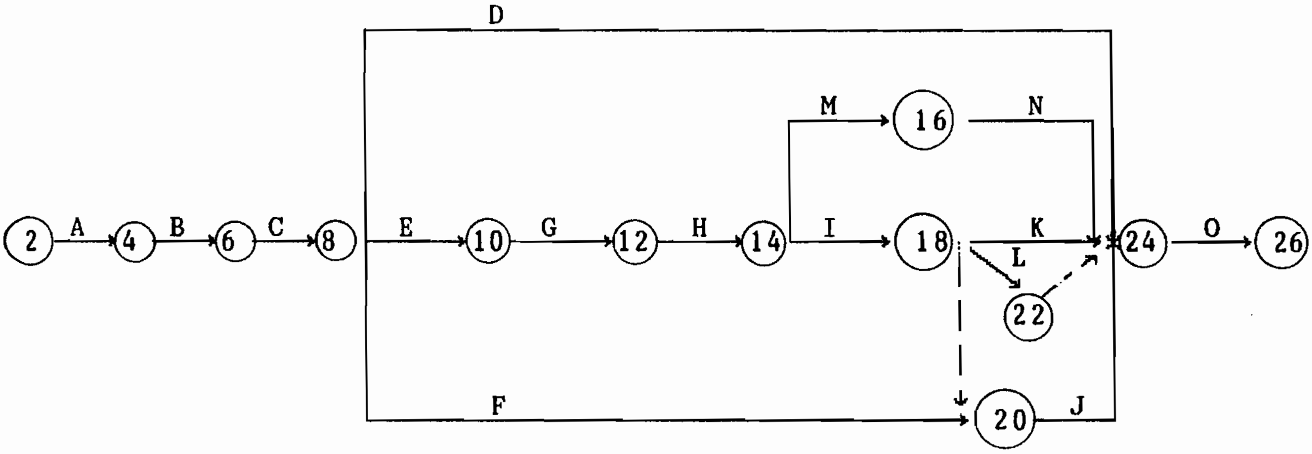


Figure 2.6 Arrow Network for Example 2.7

Early Event Time (EET): is the earliest time at which an event can occur, considering the duration of precedent activities.

Late Event Time (LET) : is the latest time at which an event can occur, if the project is to be completed on schedule.

Forward Pass

During this pass, EETs are computed. It is assumed that the end of work time zero equals the beginning of work time one. Start calculations at the left side and work to the right.

- EET for any node can be computed as soon as EETs have been computed for all nodes that are immediately precedent to it. It is necessary to compute a value for each arrow terminating at the node. The biggest of these values is the early event time. The initial node has no arrows terminating at it, hence its EET = 0.
- For a typical activity x, shown in Figure 2.7 a, with duration $D_x$

$$EET_j = EET_i + D_x \tag{2.2}$$

- The terminal node should have a computed EET that is larger than any other node in the network. The estimated project duration equals EET of the terminal node. If it is not feasible, it is necessary to reduce the duration of individual activities by the allocation of additional resources to them and to continue such an adjustment until an acceptable completion time is obtained.

Backward Pass

During this pass, LETs are computed. Start calculations at the right side and work to the left.

- LET for any node can be computed as soon as LETs have been computed for all nodes that are immediately succeeding to it. It is necessary to compute a value for each arrow originating at the node. The smallest of these values is the late event time. The terminal node has no arrows originating at it, hence its LET = its EET.
- For the typical activity x

$$LET_i = LET_j - D_x \tag{2.3}$$

- If the terminal node has LET equals the estimated project duration, then the initial node must have LET = 0. If not, you have made a computational error.

**Activity Schedule**

This is a tabulation of activity times and floats. These terms are defined with the help of Figure 2.7 b as follows:

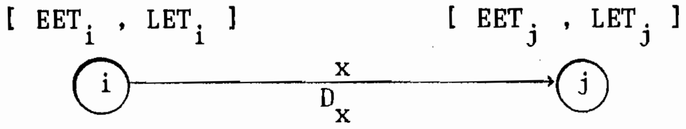


Figure 2.7 a Activity Times

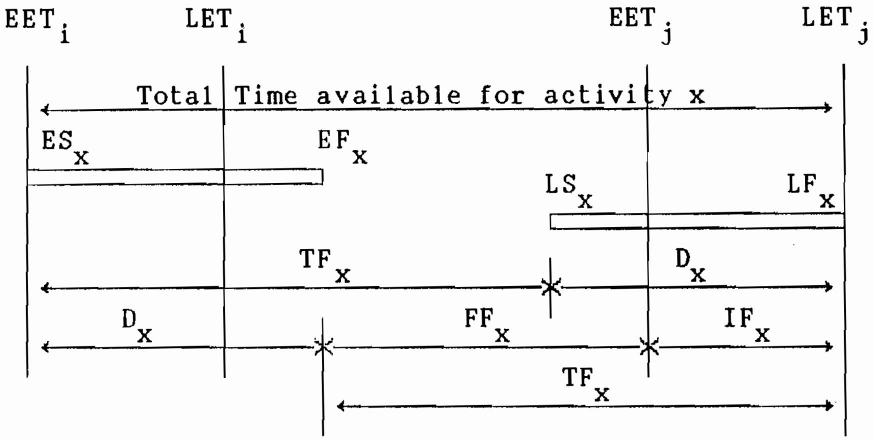


Figure 2.7 b Activity Floats

Activity Times

1. Early Start Time (ES): is the earliest time at which an activity can be started, assuming the duration of precedent activities are represented in the network. If node i is the first node of activity x, then:

$$ES_x = EET_i \tag{2.4}$$

2. Early Finish Time (EF): is the earliest time at which an activity can be completed if it is started at its ES. Then for activity x with duration $D_x$:

$$EF_x = ES_x + D_x \tag{2.5}$$

3. Late Finish Time (LF): is the latest time at which an activity can be completed without delaying project completion. If node j is the end node of activity x, then:

$$LF_x = LET_j \tag{2.6}$$

4. Late Start Time (LS): is the latest time at which an activity can be started, to keep project on schedule

$$LS_x = LF_x - D_x \tag{2.7}$$

Activity Floats

1. Total Float (TF): is the amount of time that an activity's completion may be delayed without delaying project completion. It is a measure of how noncritical an activity may be. Using Figure 2.7 b:

$$TF_x = LF_x - EF_x \tag{2.8}$$

2. Free Float (FF): is the amount of time that an activity's completion may be delayed without delaying the start of another activity beyond that activity's ES.

$$FF_x = EET_j - EET_i - D_x \tag{2.9}$$

3. Interfering Float (IF): is the difference between TF and FF. If (IF) of an activity is used, the start of some subsequent activity will be delayed beyond its ES.

Activities with zero TF are critical activities. If actual durations of these activities exceed their estimated durations, then the project actual duration will be longer than the estimated duration. Therefore, they must be started at their ES, completed within their durations, and completed by their LF. On the other hand, if the actual durations of noncritical activities exceed their estimated durations to some extent, project completion will not be delayed.

**Example 2.8**

Consider the double span bridge given in Example 2.5. The construction plan of the bridge given in Table 2.2 is reproduced in Table 2.8 where the durations of the activities have been chosen. Draw an arrow network for this contract and mark the critical path. Prepare a complete activity schedule.

If the construction of the left foundation has been delayed by one week, determine total float available for the construction of the left abutment.

Table 2.8 Data for Example 2.8

| No. | Activity | Predecessors | Duration (weeks) |
|---|---|---|---|
| 10 | Set up site | - | 2 |
| 14 | Procure reinforcement | - | 5 |
| 16 | Procure deck beams | - | 10 |
| 20 | Excavate foundation Left | 10 | 2 |
| 30 | Excavate foundation Right | 10 | 2 |
| 40 | Excavate foundation Centre | 10 | 3 |
| 50 | Construct foundation Left | 20, 14 | 2 |
| 60 | Construct foundation Right | 30, 14 | 2 |
| 70 | Construct foundation Centre | 40, 14 | 2 |
| 80 | Construct abutment Left | 50 | 4 |
| 90 | Construct abutment Right | 60 | 4 |
| 100 | Construct Centre pier | 70 | 6 |
| 110 | Place deck beams Left | 80, 100, 16 | 1 |
| 120 | Place deck beams Right | 90, 100, 16 | 1 |
| 140 | Fill embankment Left | 80 | 10 |
| 150 | Fill embankment Right | 90 | 12 |
| 155 | Construct R.C. deck slab | 110, 120 | 8 |
| 160 | Road base Left | 140 | 3 |
| 170 | Road base Right | 150 | 3 |
| 180 | Surface road | 155, 160, 170 | 2 |
| 190 | Erect bridge railing | 155 | 2 |
| 200 | Clear site | 180, 190 | 1 |

### Solution

The solution is given in Figure 2.8 and Table 2.9. The critical path is 2-6-12-18-24-32-38-40-42. If the construction of the left foundation is delayed by one week, then TF available for the construction of the left abutment = 2 - 1 = 1 week.

## 2.7 Activity-On-Node Networks ( Precedence Method )

### Network

In this method the nodes represent activities and the arrows represent logical relationships between the activities. The nodes may be boxes, circles, or ellipses that are large enough so that activity description, activity number and activity duration and the four activity times ES, EF, LS, and LF can be written within them, see figure 2.9. The left side of the node is the start side while the right side is the end side.

The logical relationships between the activities are represented by the arrows. One arrow is required for each relationship. The introduction of an

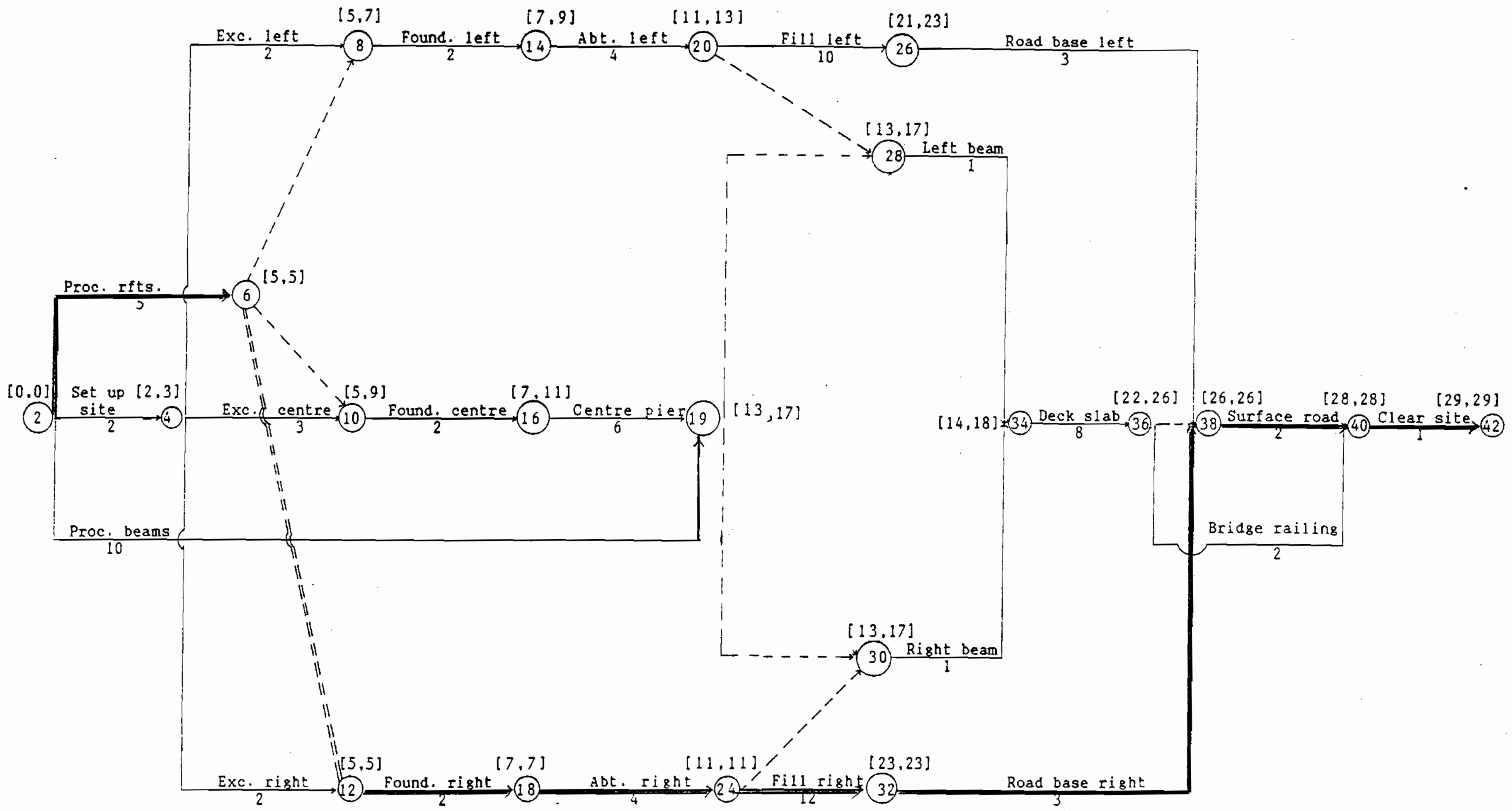


Figure 2.8 Arrow Network for the Double Span Bridge

**Table 2.9 Activity Schedule for Example 2.8.**

| Act. | i | j | D | $EET_i$ | $EET_j$ | $LET_j$ | ES | EF | LS | LF | TF | FF | IF | crt. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 2 | 4 | 2 | 0 | 2 | 3 | 0 | 2 | 1 | 3 | 1 | 0 | 1 | |
| 14 | 2 | 6 | 5 | 0 | 5 | 5 | 0 | 5 | 0 | 5 | 0 | 0 | 0 | * |
| 16 | 2 | 19 | 10 | 0 | 13 | 17 | 0 | 10 | 7 | 17 | 7 | 3 | 4 | |
| 20 | 4 | 8 | 2 | 2 | 5 | 7 | 2 | 4 | 5 | 7 | 3 | 1 | 2 | |
| 30 | 4 | 12 | 2 | 2 | 5 | 5 | 2 | 4 | 3 | 5 | 1 | 1 | 0 | |
| 40 | 4 | 10 | 3 | 2 | 5 | 9 | 2 | 5 | 6 | 9 | 4 | 0 | 4 | |
| 50 | 8 | 14 | 2 | 5 | 7 | 9 | 5 | 7 | 7 | 9 | 2 | 0 | 2 | |
| 60 | 12 | 18 | 2 | 5 | 7 | 7 | 5 | 7 | 5 | 7 | 0 | 0 | 0 | * |
| 70 | 10 | 16 | 2 | 5 | 7 | 11 | 5 | 7 | 9 | 11 | 4 | 0 | 4 | |
| 80 | 14 | 20 | 4 | 7 | 11 | 13 | 7 | 11 | 9 | 13 | 2 | 0 | 2 | |
| 90 | 18 | 24 | 4 | 7 | 11 | 11 | 7 | 11 | 7 | 11 | 0 | 0 | 0 | * |
| 100 | 16 | 19 | 6 | 7 | 13 | 17 | 7 | 13 | 11 | 17 | 4 | 0 | 4 | |
| 110 | 28 | 34 | 1 | 13 | 14 | 18 | 13 | 14 | 17 | 18 | 4 | 0 | 4 | |
| 120 | 30 | 34 | 1 | 13 | 14 | 18 | 13 | 14 | 17 | 18 | 4 | 0 | 4 | |
| 140 | 20 | 26 | 10 | 11 | 21 | 23 | 11 | 21 | 13 | 23 | 2 | 0 | 2 | |
| 150 | 24 | 32 | 12 | 11 | 23 | 23 | 11 | 23 | 11 | 23 | 0 | 0 | 0 | * |
| 155 | 34 | 36 | 8 | 14 | 22 | 26 | 14 | 22 | 18 | 26 | 4 | 0 | 4 | |
| 160 | 26 | 38 | 3 | 21 | 26 | 26 | 21 | 24 | 23 | 26 | 2 | 2 | 0 | |
| 170 | 32 | 38 | 3 | 23 | 26 | 26 | 23 | 26 | 23 | 26 | 0 | 0 | 0 | * |
| 180 | 38 | 40 | 2 | 26 | 28 | 28 | 26 | 28 | 26 | 28 | 0 | 0 | 0 | * |
| 190 | 36 | 40 | 2 | 22 | 28 | 28 | 22 | 24 | 26 | 28 | 4 | 4 | 0 | |
| 200 | 40 | 42 | 1 | 28 | 29 | 29 | 28 | 29 | 28 | 29 | 0 | 0 | 0 | * |

arrow that do not represent a constraint will result in unrealistic schedule. The arrows may be horizontal, vertical, or inclined. If an arrow originates at the end side of activity A and terminates at the start side of activity B, then A is a predecessor to B. The arrows may have no arrowheads because of the obvious left to right flow of time. A node that has no arrow terminating at its left side is a start node. A node that has no arrow originating at its right side is a finish node. The network must have one start node and one finish node.

Overlapping can be represented by entering the duration of the overlap on the dependency line.

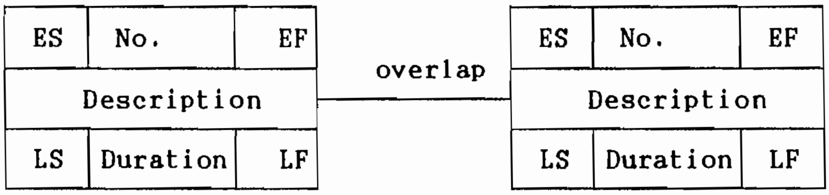


Figure 2.0 PM Notation

### Conventions

1. Logic arrows should be drawn so that they point to the right, up, or down but never to the left. In this way the scheduler can avoid loops.

2. Activity numbers, or node numbers, serve for purposes of identification only. Each node representing an activity can be given a single unique number.

3. Careful layout of the activities will minimize number of crossovers, but there are usually some that cannot be avoided.

4. Colour coding the network can be very useful. Various colours can be used to indicate different trades, work classifications and work that is subcontracted.

A comparison between precedence and arrow notation is given in Figure 2.10.

### Network Analysis

The objective of precedence network analysis is to compute for each activity in the network its four timings defined previously, i.e. ES, EF, LS and LF.

#### Forward Pass

The forward pass analysis is carried out to determine activities' ES and EF times.

1. ES of first activity is put equal to zero.
2. For any activity x with duration $D_x$,

$$EF_x = ES_x + D_x \tag{2.10}$$

3. For any activity y which is a successor to activity x with the overlap in between equals $(\text{overlap})_{xy}$,

$$ES_y = EF_x - (\text{overlap})_{xy} \tag{2.11}$$

4. In case that activity y is a successor to a group of activities; a trial value for all arrows terminating at y is calculated using Equation 2.11 and the largest one must be chosen.

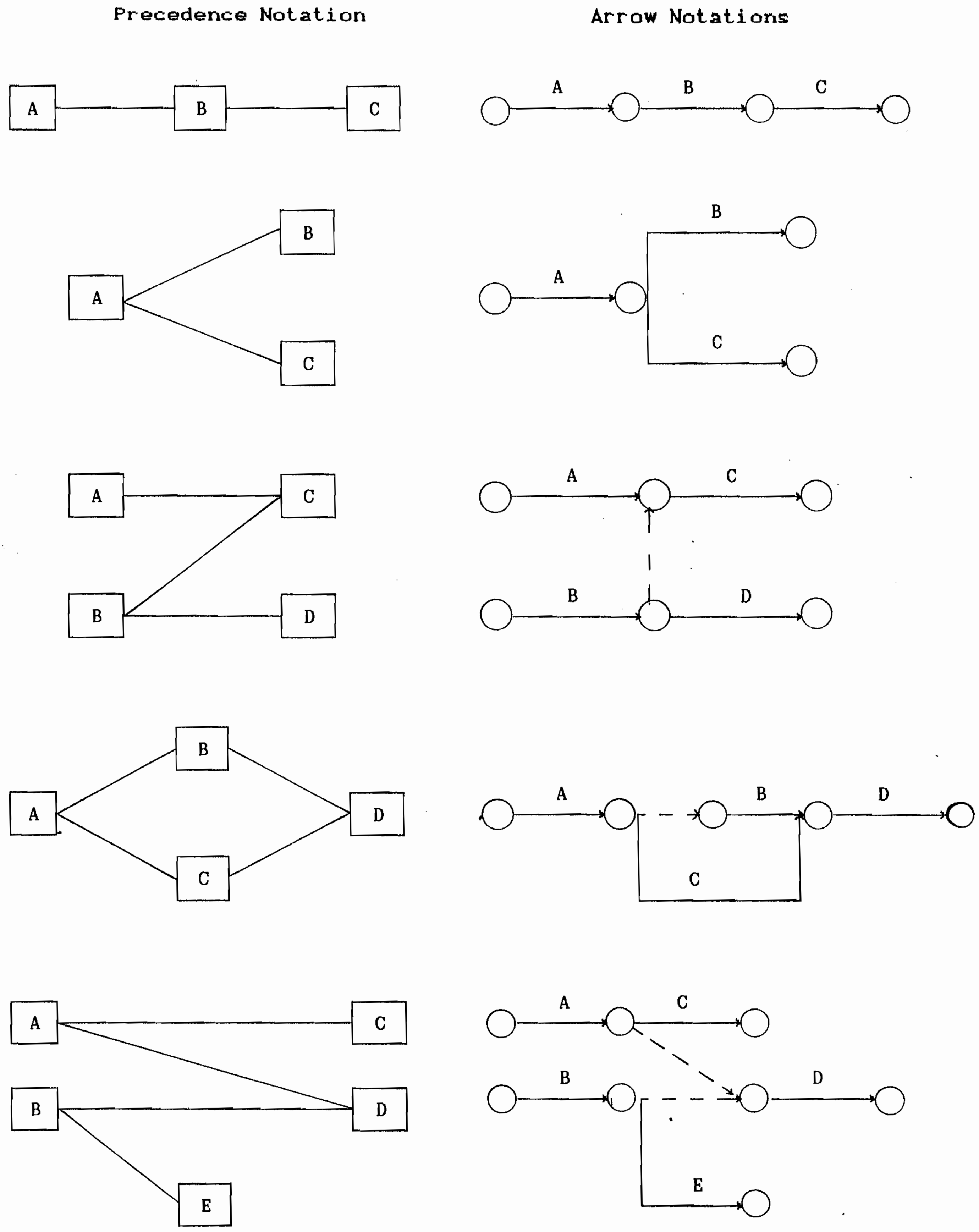


Figure 2.10 Comparing Notation of Arrow and Precedence Networks

Work from left to right. The largest value for an EF time is the project duration. If it is not feasible, it will be necessary to either increase the overlaps between activities, where feasible, or to reduce the durations of individual activities by the allocation of additional resources to them and to continue such adjustments until an acceptable completion date is obtained.

### Backward Pass

The backward pass analysis is carried out to determine activities' LS and LF times.

1. LF of last activity is put equal to project duration.
2. For any activity y with duration $D_y$,

$$LS_y = LF_y - D_y \tag{2.12}$$

3. For any activity x which is a predecessor to activity y with the overlap in between equals $(\text{overlap})_{xy}$,

$$LF_x = LS_y + (\text{overlap})_{xy} \tag{2.13}$$

4. In case that activity x is a predecessor to a group of activities; a trial value for all arrows originating from x is calculated using Equation 2.13 and the smallest one must be chosen.

Work from right to left. The initial node should have LS equal to zero.

### Activity Schedule

Activities that are fixed in the time scale are the critical activities. The sequence of critical activities is the critical path. Draw the critical path through the arrows connecting critical activities.

### Activity Floats

The total float of any activity can be calculated using Equation 2.8. Free floats can be determined by the inspection of the precedence diagram.

**Example 2.9**

Prepare a precedence diagram for the list of activities given in Table 2.10. Calculate activity times. Mark the critical path.

Table 2.10 Data for Example 2.9

| No. | Activity | Duration | Dependency | overlap |
|---|---|---|---|---|
| 10 | A | 7 | — | |
| 20 | B | 6 | A | |
| 30 | C | 7 | A | -1 |
| 40 | D | 5 | A | |
| 50 | E | 9 | B | 2 |
| 60 | F | 8 | B, C | |
| 70 | G | 6 | D | |
| 80 | H | 7 | E | 1 |
| 90 | I | 8 | F, G | 4 with G |
| 100 | J | 2 | I, H | |

## Solution

The precedence diagram is drawn in Figure 2.11. The critical path is A C F I J. Free Float of activity E = 19 + 1 - 20 = 0.

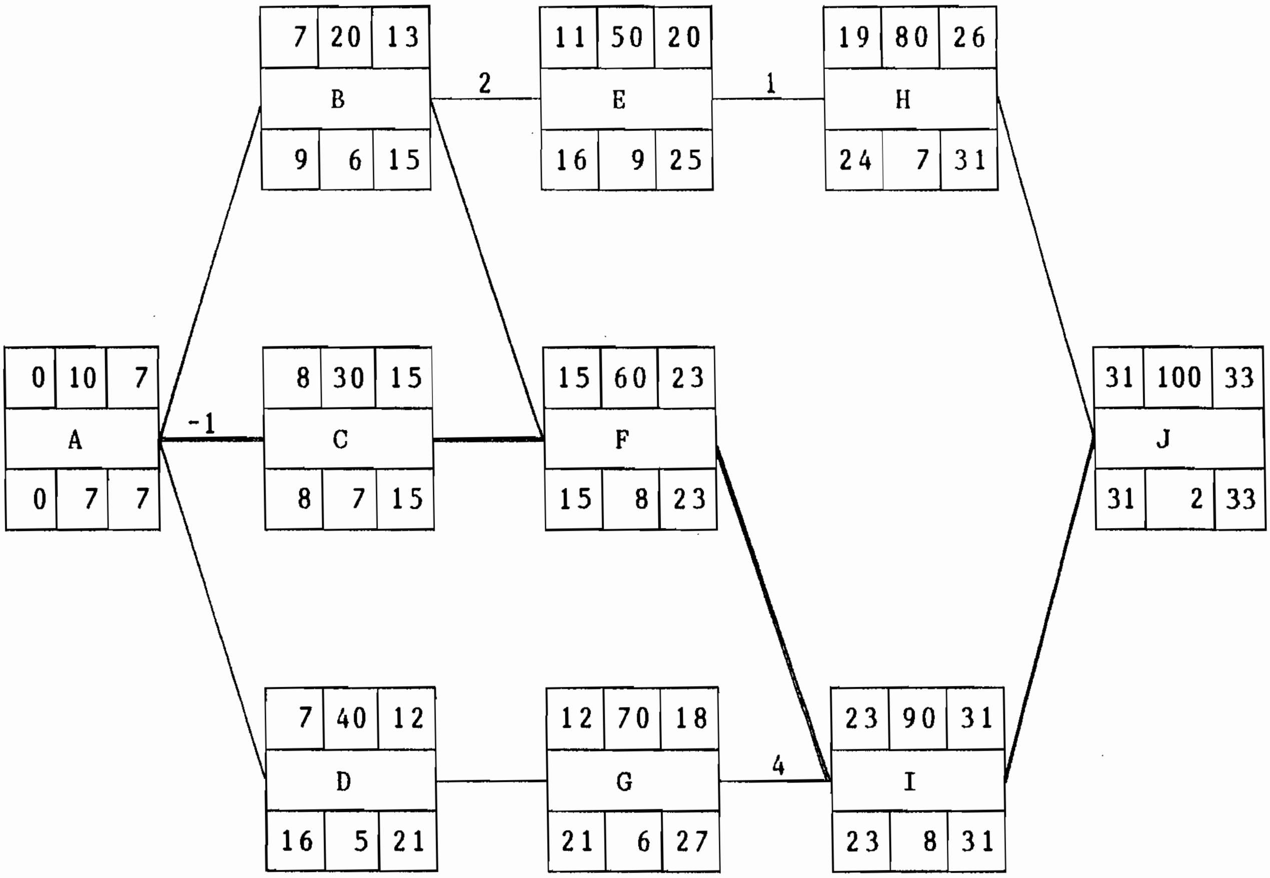


Figure 2.11 Precedence Diagram for Example 2.9

### Example 2.10

Solve example 2.8 using the precedence method.

### Solution

The activity schedule of the double span bridge using the precedence method is shown in Figure 2.12.

### Advantages of Precedence Networks over Arrow Networks

1. Absence of dummies in Precedence Networks.
2. Addition or deletion of activities and changes of logic can be accomplished easily.
3. Activity numbering system is simpler for Precedence Networks.
4. Better representation of relationships between the activities by specifying overlaps.
5. Possibility of using pre-printed node sheets which enables the engineer to devote less time to drawing the network.

## 2.8 Time-Scaled Diagrams

Time-scaled diagrams are used extensively in the construction industry. Such a diagram enables one to determine immediately which activities are scheduled to be processed at any point in time and to monitor field progress. These diagrams can also be used to determine contract weekly needs of any resource including cash money.

In a time-scaled diagram two time scales may be used : one on terms of working periods and the other as calendar dates. The length of the arrows are drawn to indicate the durations of the activities which they represent. The horizontal dashed lines represent total float for groups of activities. They also represent free floats of the activities. Vertical broken lines indicate the precedence of the activities, they do not indicate any elapsed time. The name and the duration of each activity are written above and below its arrow respectively.

The critical path of the network can easily be determined. The floats of any activity can be determined by inspection. The disadvantage of the diagram is that it needs a great effort to be modified or updated.

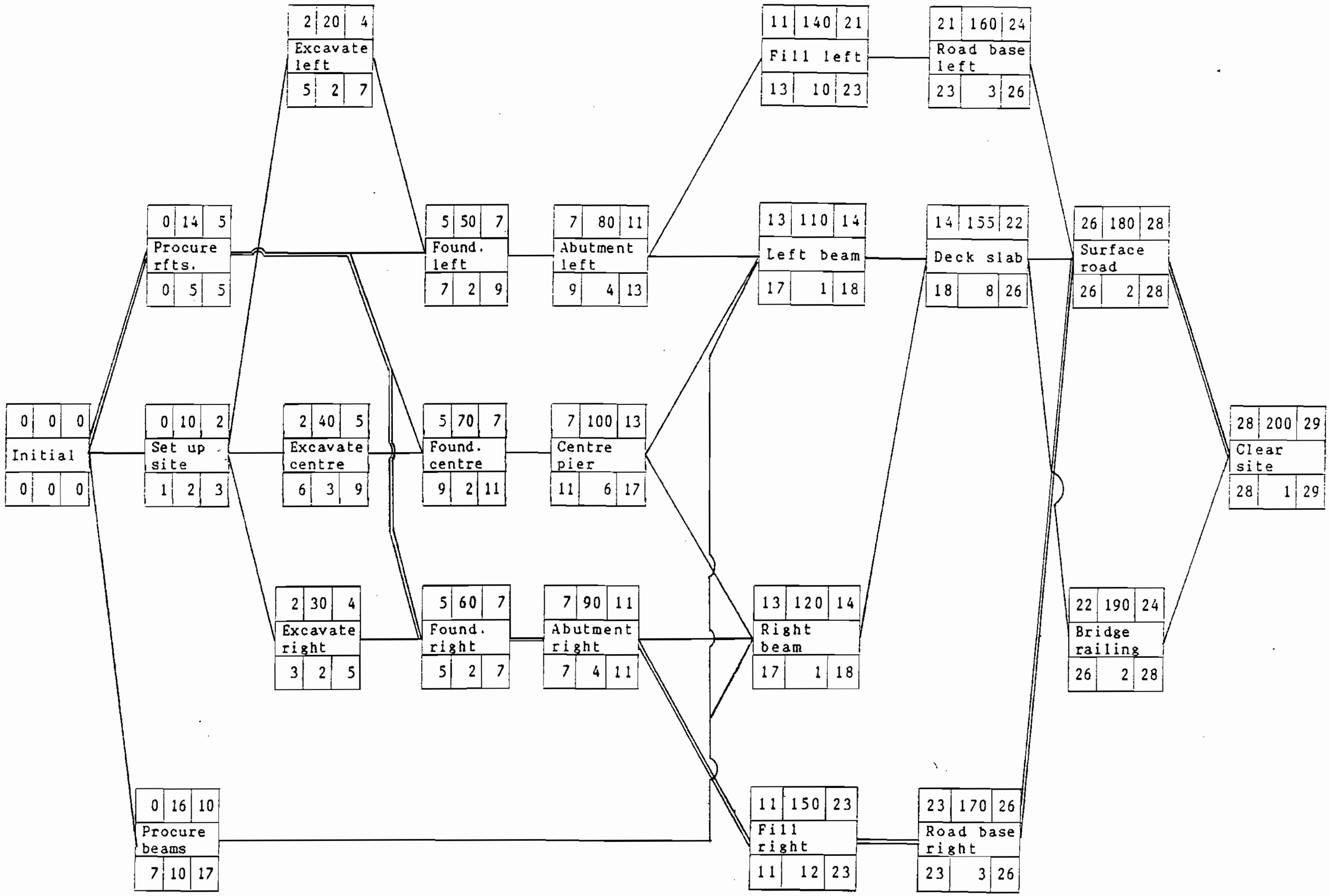


Figure 2.12 Precedence Network for the Double Span Bridge

**Example 2.11**

The activities involved in a certain contract together with their durations are given in Table 2.11. The crew employed for the foundations activities of stage 1 will be used for stage 2. Steelwork will be procured after 17 days from the start of the contract.

Draw the contract time-scaled diagram and mark the critical path. Determine contract completion time. Tabulate ES, EF, LS, LF, TF and FF of the activities.

**Table 2.11 Data for Example 2.11**

| Activity | Duration (days) |
|---|---|
| Excavate stage 1 | 4 |
| Formwork stage 1 | 4 |
| Concrete stage 1 | 8 |
| Backfill stage 1 | 3 |
| Excavate stage 2 | 5 |
| Formwork stage 2 | 2 |
| Concrete stage 2 | 8 |
| Backfill stage 2 | 5 |
| Erect steelwork | 10 |

**Solution**

The project plan is given in Table 2.12 and it is used to draw the time-scaled diagram shown in Figure 2.13. Complete activity schedule is given in Table 2.13.

## 2.9 Bar Charts ( Gantt Charts )

A bar chart is a graphical representation of the schedule for a project. It is the best form of plan for site use. Time-scaled bars are drawn to represent the scheduled start and finish dates for each activity. The time scale will be chosen to suit the user's purpose. The site manager may use weeks while the section engineer may use days or half days. The start and finish of each activity will be shown as a bar plotted to the time scale. Free and total floats of the activity may be represented as two dashed lines adjacent to the upper and lower sides of the bar respectively. Any restraint such as holidays may be marked on the diagram.

The bars may be drawn in two sections, the upper section showing the planned time, and the lower section left blank for recording progress. At the end of each time period the amount of work done in each activity is recorded by shading the lower section.

Table 2.12 Project Plan for Example 2.11

| Code | Activity | Duration | Predecessor |
|---|---|---|---|
| A | Excavate stage 1 | 4 | - |
| B | Formwork stage 1 | 4 | A |
| C | Concrete stage 1 | 8 | B |
| D | Backfill stage 1 | 3 | C |
| E | Excavate stage 2 | 5 | A |
| F | Formwork stage 2 | 2 | B,E |
| G | Concrete stage 2 | 8 | C,F |
| H | Backfill stage 2 | 5 | D,G |
| I | Procure steelwork | 17 | - |
| J | Erect steelwork | 10 | G,I |

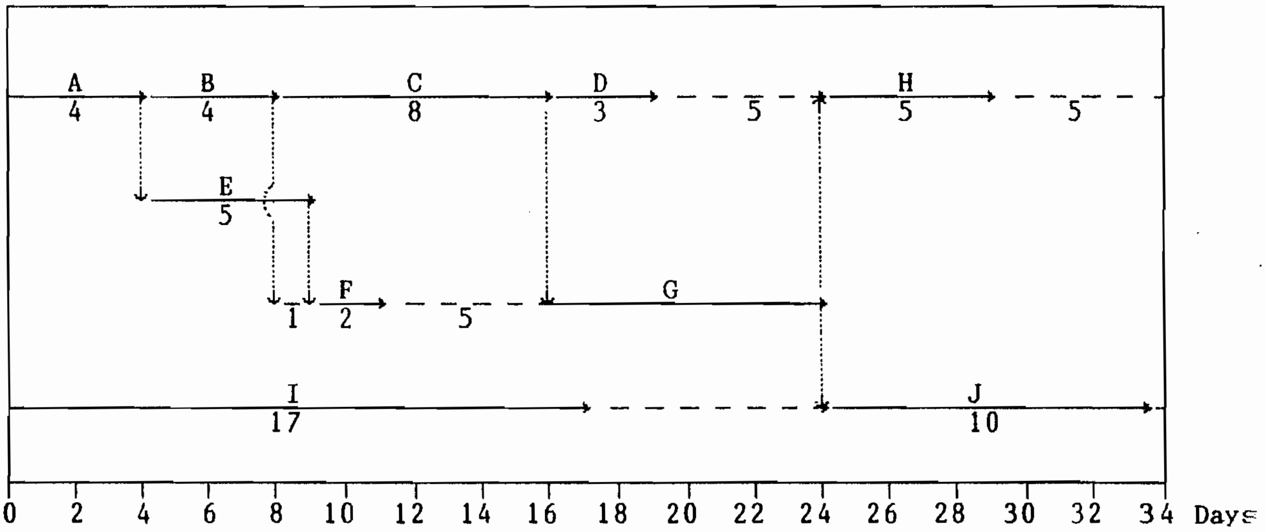


Figure 2.13 Time-Scaled Diagram for Example 2.11

Table 2.13 Timings and Floats for Activities of Example 2.11

| Activity | ES | EF | LS | LF | TF | FF |
|---|---|---|---|---|---|---|
| A | 0 | 4 | 0 | 4 | - | - |
| B | 4 | 8 | 4 | 8 | - | - |
| C | 8 | 16 | 8 | 16 | - | - |
| D | 16 | 19 | 26 | 29 | 10 | 5 |
| E | 4 | 9 | 9 | 14 | 5 | - |
| F | 9 | 11 | 14 | 16 | 5 | 5 |
| G | 16 | 24 | 16 | 24 | - | - |
| H | 24 | 29 | 29 | 34 | 5 | 5 |
| I | 0 | 17 | 7 | 24 | 7 | 7 |
| J | 24 | 34 | 24 | 34 | - | - |

The bar chart is more understandable than tables containing numerical data. Schedulers will often prepare a schedule using a network technique and then plot a bar chart using the time values computed by that method, but it will be very wide for a project that has a long duration. The chart can be used for calculating the resources required for the project at any time.

### Example 2.12

Draw a bar chart for the contract given in Example 2.8. Assume that project start date is 1 / 10 / 1988.

### Solution

The bar chart is drawn in Figure 2.14.

## 2.10 Program Evaluation and Review Technique ( PERT )

### Introduction

This method uses the activity-on-arrow notation, meaning that arrows are used to represent activities and nodes are used to represent events. The method enables the user to estimate the most probable project duration as well as the probability that any part of the project will be completed within any time period. However, the use of the method should be considered where there is considerable uncertainty concerning activity durations.

### Activity Durations

With the PERT method, three different estimates of duration are made for each activity, assuming a fixed set of resources are being used by the activity.

Optimistic duration (shorten duration) (a)

It is the estimated time required to complete the activity if everything goes well.

Pessimistic duration (prolonged duration) (b)

It is the estimated time required to complete the activity if nearly everything goes wrong. Rare events should not be included.

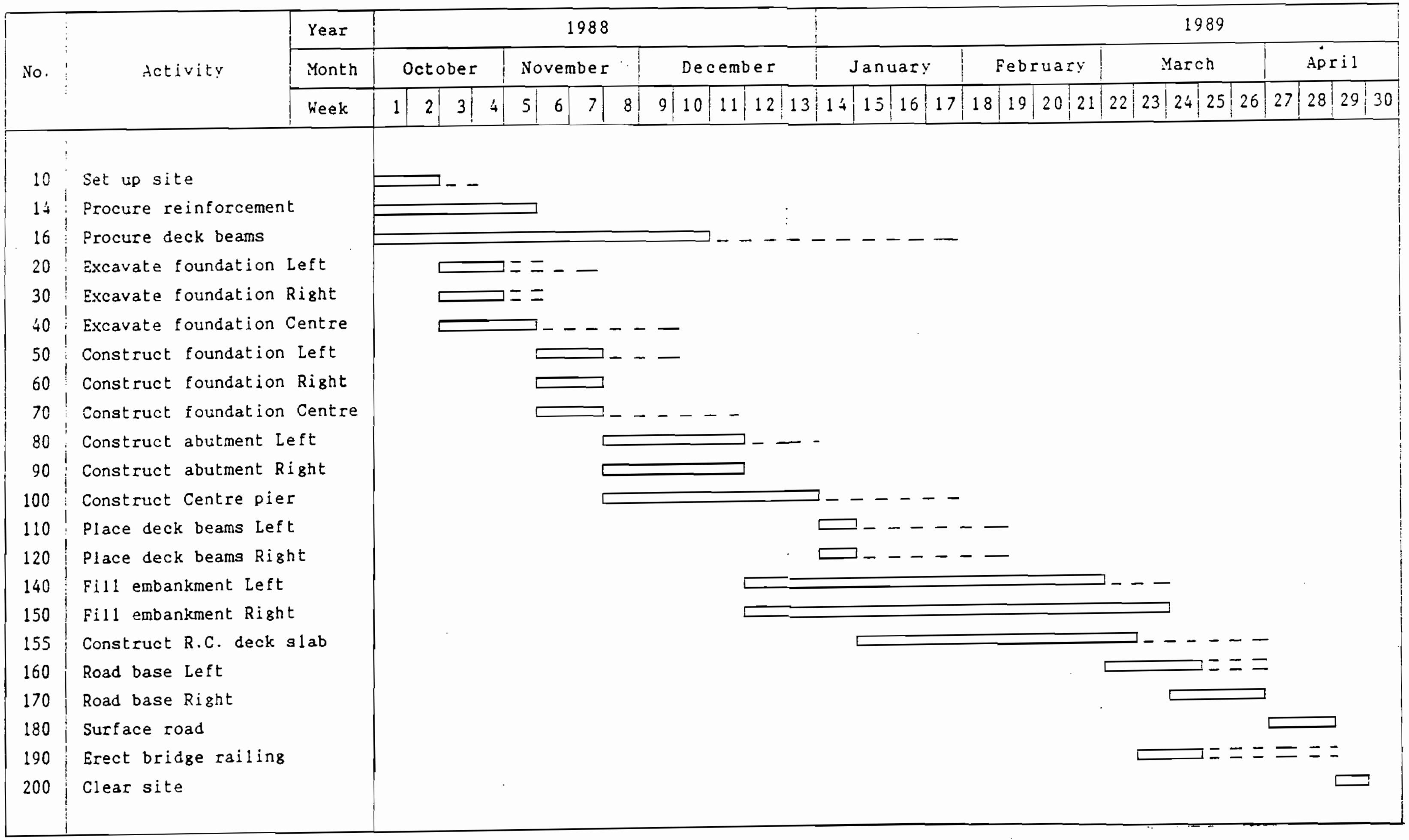


Figure 2.14 Bar Chart for Example 2.12

### Most likely duration (model time) (m)

It is the estimated time that will be required to complete the activity more often than any other time if the activity is repeated many times.

These durations can be obtained by direct enquiry of the field personnel because it is difficult to obtain sufficient time data from historical records. These time estimates contain a considerable amount of subjective evaluation and they may be in error by considerable amount.

These three estimates of activity duration are converted to the beta distribution, see Figure 2.15, to give the expected value (mean) and the standard deviation of the distribution as given in Equations 2.14 to 2.16. These values will be used in the network analysis.

Activity mean duration $$t_e = \frac{a + 4m + b}{6} \qquad (2.14)$$

Activity standard deviation $$\sigma = \frac{b - a}{6} \qquad (2.15)$$

Activity variance $$\upsilon = \sigma^2 \qquad (2.16)$$

## Probability Analysis

### Normal Distribution

The well-known normal distribution is shown in Figure 2.16. The area under the curve corresponding to a given standard deviation is given in Table 2.14. This distribution will be used to get the probability of completing the project in a specified time as will be discussed hereinafter.

### Central Limit Theorem

As applied to construction scheduling, the theorem states that: Where a series (about ten) of sequential independent activities lie on the critical path of a network, the sum of the individual activity durations will be distributed in approximately normal fashion, regardless of the way in which the individual activity durations are themselves distributed. The mean of the distribution of the sum of the activity durations will be the sum of the means of the individual activities and its variance will be the sum of the activities' variances.

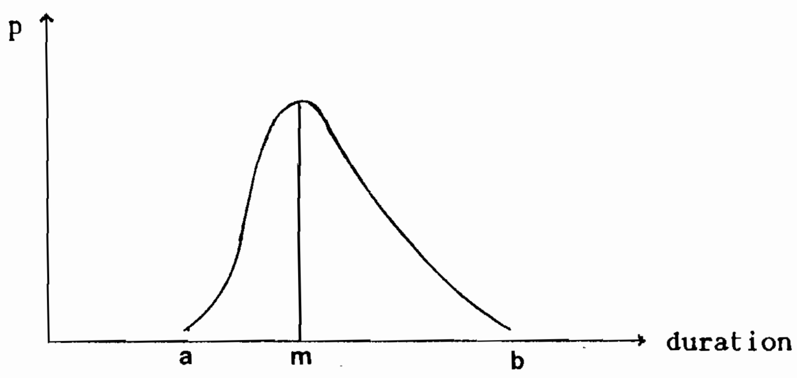


Figure 2.15 Beta Distribution

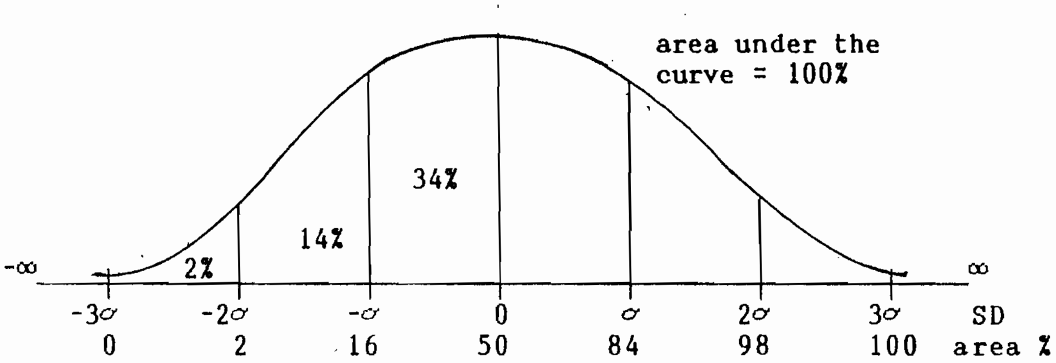


Figure 2.16 Normal Distribution

Table 2.14 Area under the Normal Curve

| SD | area % | SD | area % |
|---|---|---|---|
| 0.1 | 4.0 | 1.6 | 44.5 |
| 0.2 | 7.9 | 1.7 | 45.5 |
| 0.3 | 11.8 | 1.8 | 46.4 |
| 0.4 | 15.5 | 1.9 | 47.1 |
| 0.5 | 19.2 | 2.0 | 47.7 |
| 0.6 | 22.6 | 2.1 | 48.2 |
| 0.7 | 25.8 | 2.2 | 48.6 |
| 0.8 | 28.8 | 2.3 | 48.9 |
| 0.9 | 31.6 | 2.4 | 49.2 |
| 1.0 | 34.1 | 2.5 | 49.4 |
| 1.1 | 36.4 | 2.6 | 49.5 |
| 1.2 | 38.5 | 2.7 | 49.6 |
| 1.3 | 40.3 | 2.8 | 49.7 |
| 1.4 | 41.9 | 2.9 | 49.98 |
| 1.5 | 43.3 | 3.0 | 49.99 |

## PERT Assumptions

1. Any PERT path must contain enough activities to make the central limit theorem valid.

2. The mean of the distribution of the path with the greatest duration, from the initial node to a given node, is given by the maximum mean of the duration distribution of the paths entering the node.

3. PERT critical path is long enough in time so as to be longer than any other path in the network.

Therefore

An event variance $$V = \Sigma \upsilon_c \qquad (2.17)$$

An event standard deviation $$SD = \sqrt{V} \qquad (2.18)$$

Where $\upsilon_c$ are variances for critical activities that are precedent to the event ( starting from the initial node. )

## Probability of Meeting a Scheduled Date

The following steps will be adopted to get the probability of meeting a scheduled date T for a critical event i with an early event time = $EET_i$:

1. Find the event standard deviation $SD_i$ using Equation 2.18.

2. The probability of meeting $EET_i$ equals 50%. Calculate the difference Z between T and $EET_i$ expressed in standard deviation units as follows:

$$Z = \frac{T - EET_i}{SD_i} \qquad (2.19)$$

3. Find the area under the normal curve corresponding to Z to represent the required probability.

## PERT Computations

1. Compute for each activity the expected (mean) duration and standard deviation. The values of a, b and m will not be used in subsequent calculations.

2. Construct the project network and carry out forward and backward passes to get EET and LET, exactly as in the CPM.

3. Find the probability of meeting a scheduled date as given previously.

### Example 2.13

Consider the project given in Table 2.15. Find the probability that the project will be completed within 70 and 80 days.

**Table 2.15 Data for Example 2.13**

| Activity | Durations (days) | | | Dependencies |
|---|---|---|---|---|
| | a | m | b | |
| A | 10 | 16 | 22 | - |
| B | 24 | 32 | 42 | - |
| C | 22 | 32 | 40 | A |
| D | 12 | 16 | 21 | B |
| E | 20 | 25 | 35 | C, D |
| F | 13 | 16 | 19 | A, B |

### Solution

The project contains six activities only. Therefore PERT results will be approximate. The activities' mean durations and standard deviations are given in Table 2.16. The PERT network with the computed EET s and LET s is shown in Figure 2.17.

**Table 2.16 Activities' Mean Durations and Standard Deviations**

| Activities | $t_e$ | $\sigma$ |
|---|---|---|
| A | 16.00 | 2 |
| B | 32.33 | 3 |
| C | 31.66 | 3 |
| D | 16.16 | 1.5 |
| E | 25.83 | 2.5 |
| F | 16.00 | 1 |

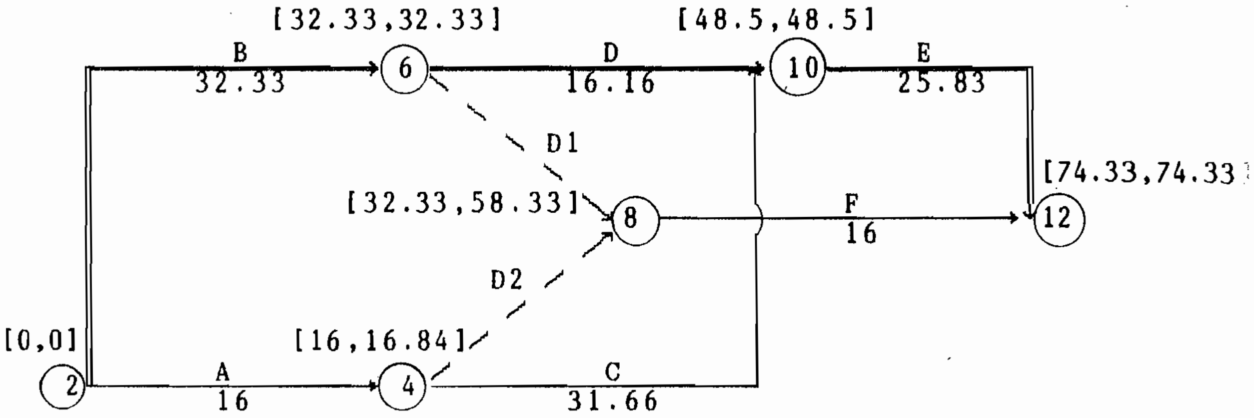


**Figure 2.17 PERT Network for Example 2.13**

For node 12 There is 50% probability that the project duration ≤ 74.33 days.

$$V_{12} = 3^2 + 1.5^2 + 2.5^2 = 17.5$$

$$SD_{12} = \sqrt{17.5} = 4.18$$

$$Z_1 = \frac{70 - 74.33}{4.18} = -1.04$$

$\therefore$ P for completing the project within 70 days = 50 - 35.2 = 14.8 %

$$Z_2 = \frac{80 - 74.33}{4.18} = 1.35$$

$\therefore$ P for completing the project within 80 days = 50 + 40.7 = 90.7 %

### Advantages and Disadvantages of PERT

The advantage of the method is that it permits calculation of the probability of completing the project at specified times. The disadvantage of the method is the assumption that the critical path is longer than any other path in the network which gives wrong results if the network contains several concurrent critical or nearly critical paths.

## 2.11 Allocation of Resources

In the time analysis of networks made so far it was assumed that all resources required to carry out the job are available when required. Consideration must be given now to the utilization of resources in planning a project.

### Scheduling Procedure

The process of review of the plan is demonstrated in Figure 2.18. If the determined early start completion time of the project (the completion time assuming that every activity is to start as soon as it is possible to do so) is not acceptable, it can be shortened by revising the overlap between the activities and by allocating more resources to individual activities in order to reduce their durations.

Having accepted the earliest completion time of the project, the planner should check the demand for resources in order not to exceed the planned availability of resources or to produce a fluctuating pattern for their use.

When smoothing the demand for the resources, the float available on non-critical activities is utilized to adjust the timing of all activities requiring a common resource, so that the best possible pattern of use is achieved for that particular resource within the previously calculated completion time of the project. Resource smoothing is therefore a process of smoothing the peaks and troughs in the total-demand histogram. It gives the answer of the question: how much of the resource will be needed to accomplish the project within the stated time interval.

Resource scheduling is needed when there are real limits on the resources. Float is here used to adjust the timing of activities so that the imposed resource limits are not exceeded. In some cases, it will not be possible to satisfy these constraints together with the previously calculated earliest completion time; then the duration of the project is extended. If the resulting completion time is not acceptable, the plan should be revised. The process of refinement of the plan can involve adjustments of a number of variables: logic, overlaps, demand for resources, output of resources and the level of resources provided.

When resource scheduling and resource smoothing have produced a satisfactory solution, the start and finish times for each activity are said to be their "scheduled" values. It is probable that few scheduled activities will still offer float.

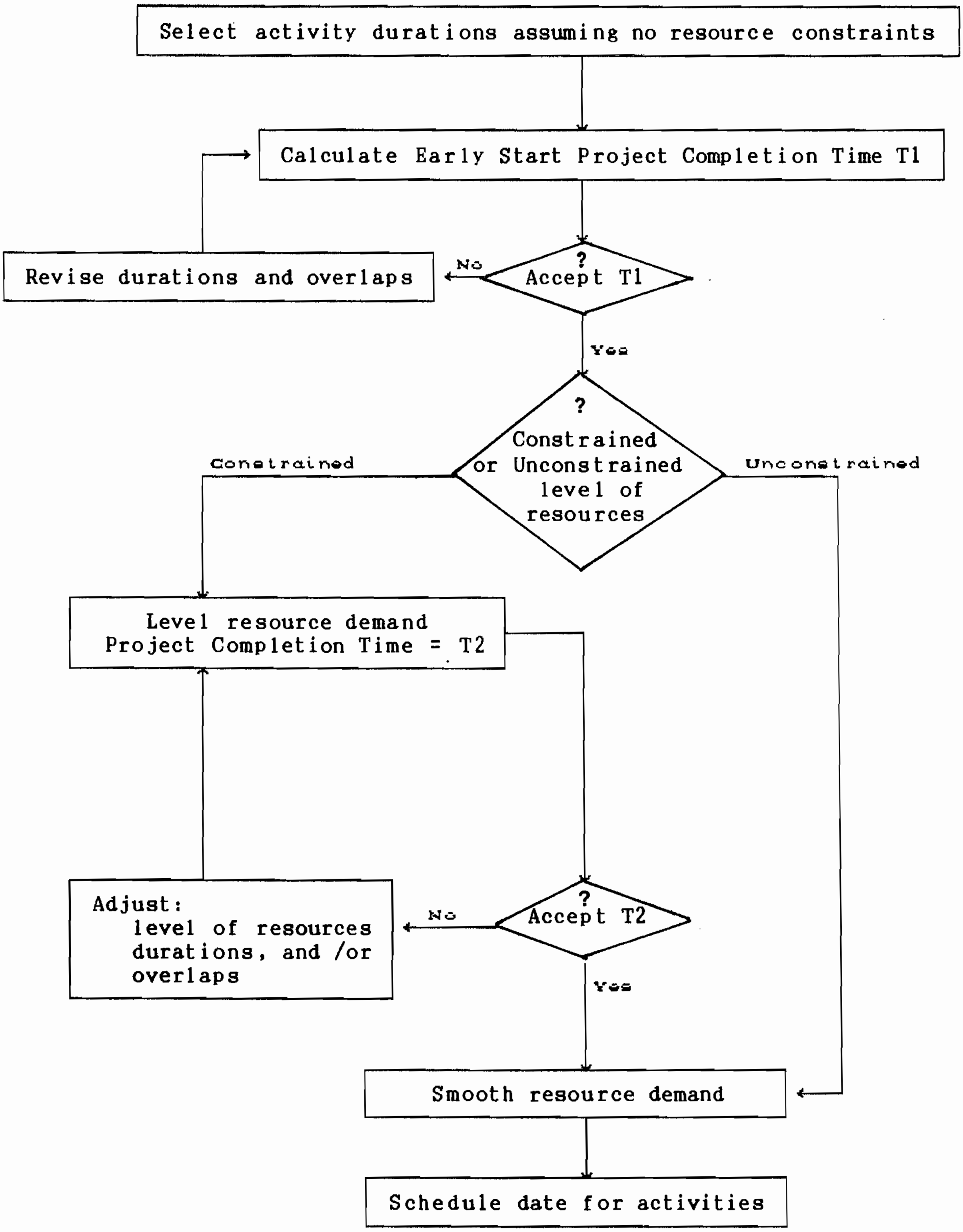


Figure 2.18 Flow Chart of the Scheduling Procedure

### Types of Construction Resources

The following classification of resources is usually used in the construction industry:

1. Key resources: These are resources which will be considered in the resource scheduling or resource smoothing process.
2. Secondary resources: Where there is no constraints on the availability of a resource, it is considered as a secondary resource. It will not be included in the resource smoothing or resource scheduling process.
3. General resources: These resources can be used by all or most of the activities of the contract such as site overheads. Each of the general resources is provided as a single unit. It will not be included in the resource smoothing or resource scheduling process.

## 2.12 Unconstrained Resource Allocation ( Resource Smoothing )

### Objectives

The main objectives of resource smoothing are:

1. To avoid the period-to-period fluctuations in key resource demands because the hiring and releasing of workers on a short-term basis is inefficient and costly. New employees need time to learn their tasks and even previously employed persons need time to readjust the working conditions of a particular job.
2. To maintain an even flow of application for the resource because the emphasis must be the continuous application of a high-cost resource once it is assigned to the work.

### Procedure for Smoothing a Single Resource

The following procedure can be used for smoothing a single resouce:

1. Prepare a complete activity schedule.
2. Draw a bar chart of the project using early start timing of the activities. Each activity is represented by an open bar. Above each bar put the name of the activity together with the resource rate. Critical activities are to be drawn first so that noncritical activities may be shifted quickly. Beside each bar draw the activity free float as a dashed line adjacent to the upper side of the bar and the activity total float as a dashed line adjacent to the lower side of the bar. Beneath the bar chart are shown the adjusting steps taken during the smoothing process.

3. Determine resource sum in each time period and calculate the average of resource usage. It will be considered a lower bound on resource demand. Obviously there may be one or more activities which have resource rate bigger than this average level. If so, this rate will be the lower bound. However, any schedule for which peak usage is close to this lower bound will be a nearly optimum schedule.

4. Test the demand for the resource allover project period. Shift noncritical activities within their free floats in order to optimize the use of the resource, i.e. to lower peak demand and to raise trough demand. Having shifted a noncritical activity, you should revise free float of its immediately preceding activity(ies). Show this on the bar chart. Obviously, an activity may be shifted a maximum number of periods that is equal to its total float. Therefore an activity shifted within its free float may be reshifted once its free float has been adjusted.

5. The shifting of an activity is shown by the subtraction of its resource rate from resource sum on old positions and the addition of the rate to resource sum on the new positions of the activity.

6. The final position for an activity after smoothing of the resource has been accomplished is shown by the crosshatched bars.

### Example 2.14

A certain contract consists of the activities given in Table 2.17. The key resources R1 and R2 will be used during the course of the contract. For the purpose of this example assume that only resource R1 will be used by the activities. Determine minimum level of the resource required to complete the contract in its early start timing.

**Table 2.17 Data for Example 2.14**

| No. | Act. | D(Weeks) | Predecessor | R1 | R2 |
|---|---|---|---|---|---|
| 1 | A | 0 | - | 0 | 0 |
| 2 | B | 2 | 1 | 0 | 2 |
| 3 | C | 5 | 1 | 2 | 3 |
| 4 | D | 3 | 1 | 2 | 0 |
| 5 | E | 2 | 2 | 1 | 4 |
| 6 | F | 6 | 2 | 2 | 5 |
| 7 | G | 6 | 3 | 3 | 2 |
| 8 | H | 6 | 4 | 1 | 0 |
| 9 | I | 4 | 4 | 0 | 5 |
| 10 | J | 2 | 5,6 | 4 | 5 |
| 11 | K | 7 | 6,7 | 2 | 0 |
| 12 | L | 3 | 2,8 | 2 | 8 |
| 13 | M | 2 | 2,8,9 | 4 | 3 |
| 14 | N | 2 | 10,11,12,13 | 0 | 0 |

Solution

The contract early start completion time is calculated using the precedence diagram, Figure 2.19.

Timings and floats of the activities are given in Table 2.18. These timings and floats are used to draw the bar chart shown in Figure 2.20 a. Beneath the bar chart are found resource sums on line Σ 0. The corresponding histogram of the resource is also shown in the same figure.

Average usage of the resource = 90/18 = 5. Peak usage = 13. The schedule for which peak usage is close to 5 will be a nearly optimum schedule.

On weeks 9, 10 and 11 the demand for the resource is high while on weeks 13 through 18 the demand is low. The shifting process will try to transfer the peak usage into the trough usage. This is demonstrated using Figure 2.20 b as follows:

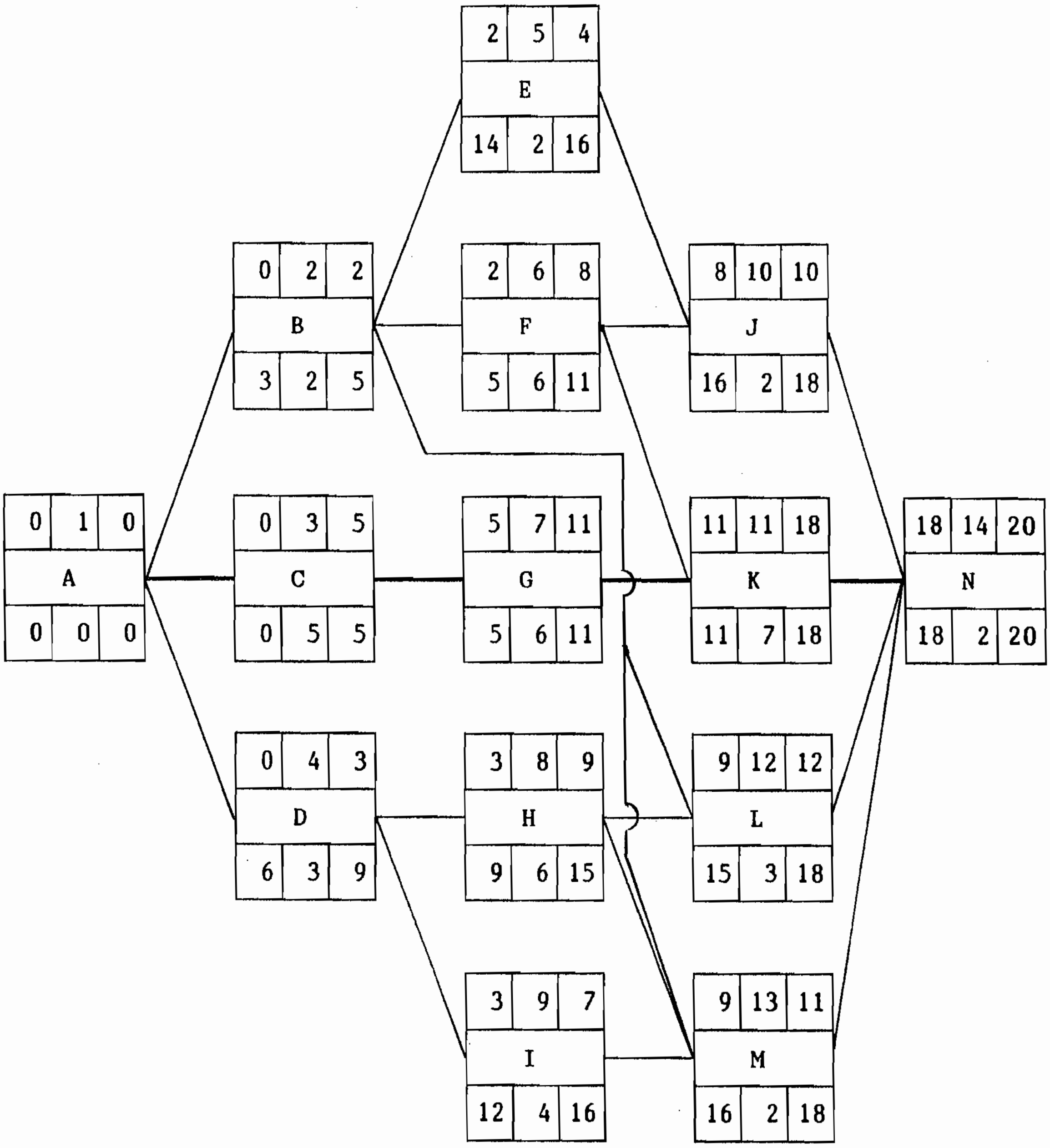


Figure 2.19 Precedence Diagram for Example 2.14

**Table 2.18 Timings and Floats of Activities for Example 2.14**

| Activity | ES | EF | FF | TF |
|---|---|---|---|---|
| A | 0 | 0 | 0 | 0 |
| B | 0 | 2 | 0 | 3 |
| C | 0 | 5 | 0 | 0 |
| D | 0 | 3 | 0 | 6 |
| E | 2 | 4 | 4 | 12 |
| F | 2 | 8 | 0 | 3 |
| G | 5 | 11 | 0 | 0 |
| H | 3 | 9 | 0 | 6 |
| I | 3 | 7 | 2 | 9 |
| J | 8 | 10 | 8 | 8 |
| K | 11 | 18 | 0 | 0 |
| L | 9 | 12 | 6 | 6 |
| M | 9 | 11 | 7 | 7 |
| N | 18 | 20 | 0 | 0 |

1. Activity M has a free float of 7 weeks. The shifting of M by 7 weeks will reduce peak usage on weeks 11 and 12 and give a chance for preceding activities to be shifted. New resource sums are given on line $\Sigma$ 1. Now free float of activity I is increased by 7 weeks. However, activity I will not be shifted because it has zero resource rate.

2. Activity J has a free float of 8 weeks. The shifting of J by 6 weeks will optimize the use of the resource. New resource sums are given on line $\Sigma$ 2. Now free float of activity E is increased by 6 weeks and that of F by 3 weeks.

3. Activity L has a free float of 6 weeks. The shifting of L by 2 weeks will optimize the use of the resource. New resource sums are given on line $\Sigma$ 3. Now activity H will have a free float of 2 weeks.

4. Activity E has a free float of 10 weeks. Shifting E by 10 weeks will improve the use of the resource. New resource sums are given on line $\Sigma$ 4. Note that there is no change in free float of activity B.

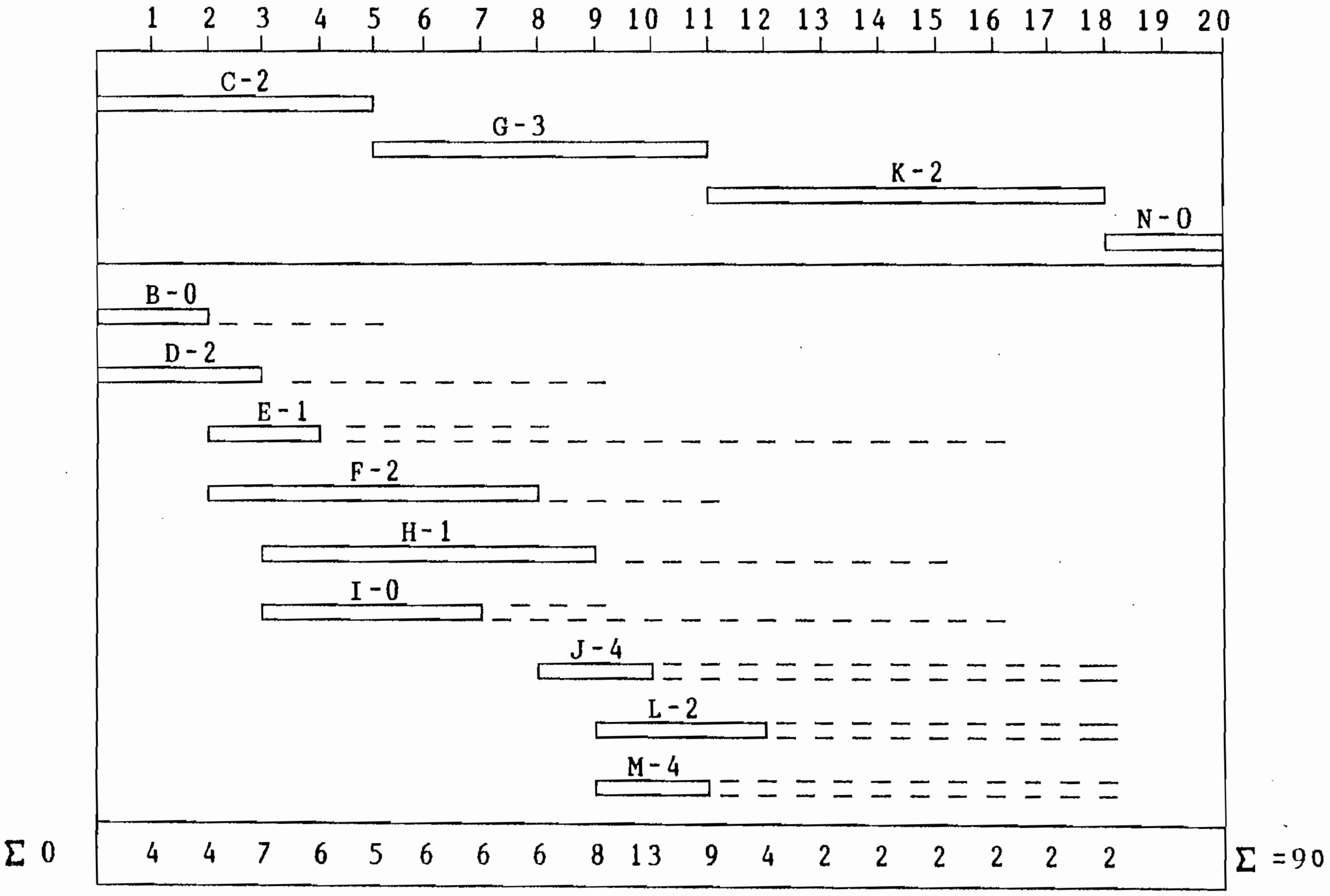
5. Activity H has a free float of 2 weeks. The shifting of H by 2 weeks will improve resource sums on weeks 10 and 11. New resource sums are given on line $\Sigma$ 5.

6. Activity F can be shifted by one week. This will improve the use of the resource. Now any other shifting will not improve resource usage, therefore we stop at this point. The final position of the activities are shown by the crosshatched bars.

7. Draw resource histogram and compare it with that shown in Fig. 2.20 a.

### Example 2.15

The activities given in Table 2.19 represent a section of work being undertaken by a subcontractor. The activities' predecessor(s) and labour requirement and early start, early finish and late finish timings are also listed. The labour histogram made up of the preferred limits chosen by the subcontractor is shown in Figure 2.21. What are the scheduled timings of the activities that satisfy the preferred histogram.



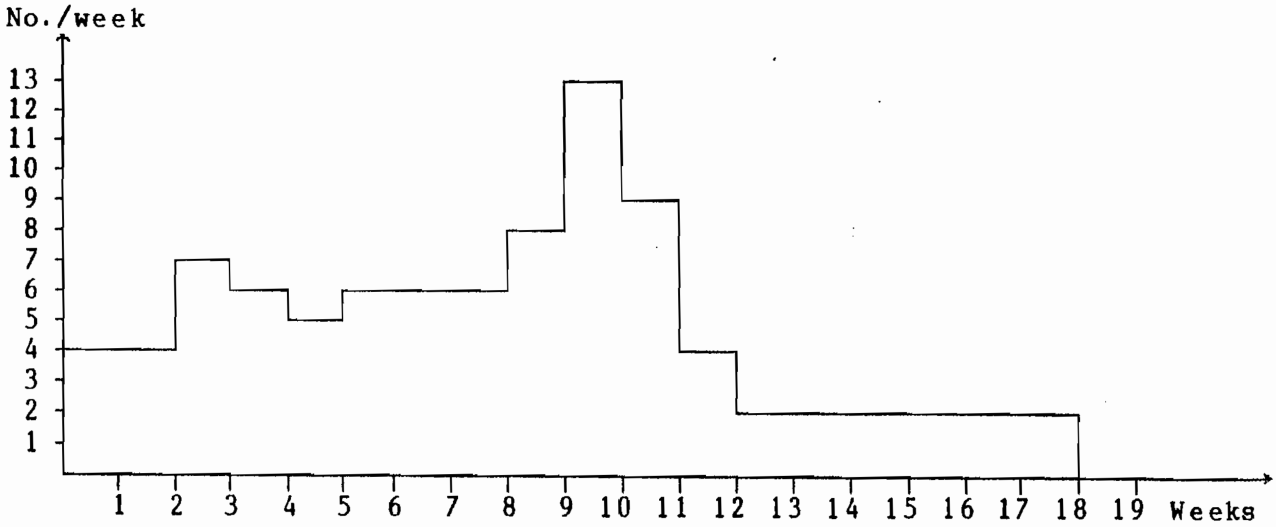
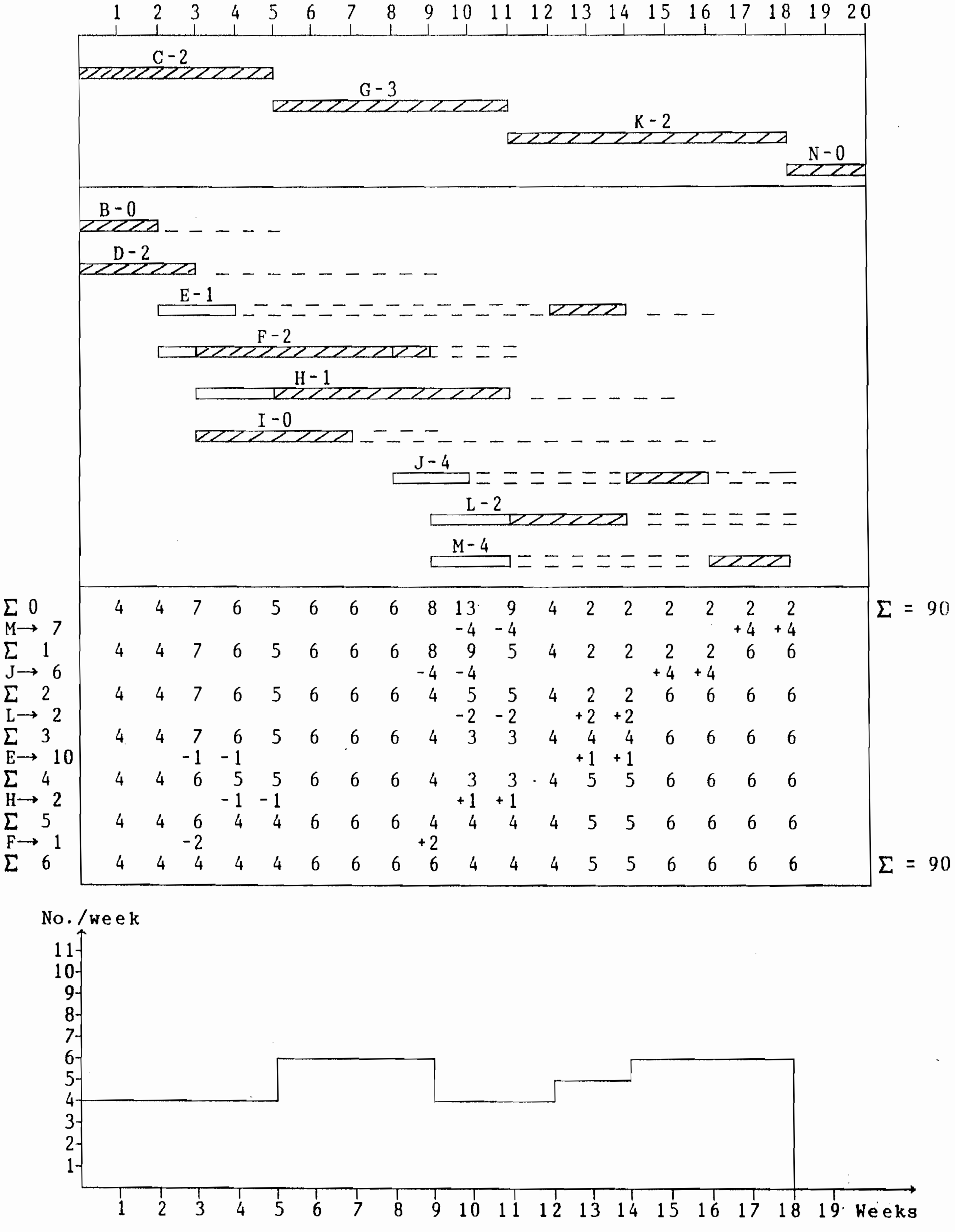


Figure 2.20 a Histogram of Resource R1 (before smoothing)



| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Σ 0 | 4 | 4 | 7 | 6 | 5 | 6 | 6 | 6 | 8 | 13 | 9 | 4 | 2 | 2 | 2 | 2 | 2 | 2 | Σ = 90 |
| M→ 7 | | | | | | | | | | -4 | -4 | | | | | | +4 | +4 | |
| Σ 1 | 4 | 4 | 7 | 6 | 5 | 6 | 6 | 6 | 8 | 9 | 5 | 4 | 2 | 2 | 2 | 2 | 6 | 6 | |
| J→ 6 | | | | | | | | | -4 | -4 | | | | | +4 | +4 | | | |
| Σ 2 | 4 | 4 | 7 | 6 | 5 | 6 | 6 | 6 | 4 | 5 | 5 | 4 | 2 | 2 | 6 | 6 | 6 | 6 | |
| L→ 2 | | | | | | | | | | -2 | -2 | | +2 | +2 | | | | | |
| Σ 3 | 4 | 4 | 7 | 6 | 5 | 6 | 6 | 6 | 4 | 3 | 3 | 4 | 4 | 4 | 6 | 6 | 6 | 6 | |
| E→ 10 | | | -1 | -1 | | | | | | | | | +1 | +1 | | | | | |
| Σ 4 | 4 | 4 | 6 | 5 | 5 | 6 | 6 | 6 | 4 | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | |
| H→ 2 | | | | -1 | -1 | | | | | +1 | +1 | | | | | | | | |
| Σ 5 | 4 | 4 | 6 | 4 | 4 | 6 | 6 | 6 | 4 | 4 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | |
| F→ 1 | | | -2 | | | | | | +2 | | | | | | | | | | |
| Σ 6 | 4 | 4 | 4 | 4 | 4 | 6 | 6 | 6 | 6 | 4 | 4 | 4 | 5 | 5 | 6 | 6 | 6 | 6 | Σ = 90 |

Figure 2.20 b Histogram of Resource R1 (after smoothing)

**Table 2.19 Data for Example 2.15**

| Activity | Predecessor | No. of labour | ES | EF | LF |
|---|---|---|---|---|---|
| A | — | 2 | 0 | 4 | 4 |
| B | A | 3 | 4 | 6 | 25 |
| C | A | 4 | 4 | 10 | 19 |
| D | A | 4 | 4 | 13 | 13 |
| E | D | 4 | 13 | 23 | 23 |
| F | C | 3 | 10 | 13 | 25 |
| G | C | 4 | 10 | 18 | 27 |
| H | B, F | 2 | 13 | 15 | 27 |
| I | E | 2 | 23 | 27 | 27 |
| J | G, H, I | 1 | 27 | 29 | 29 |

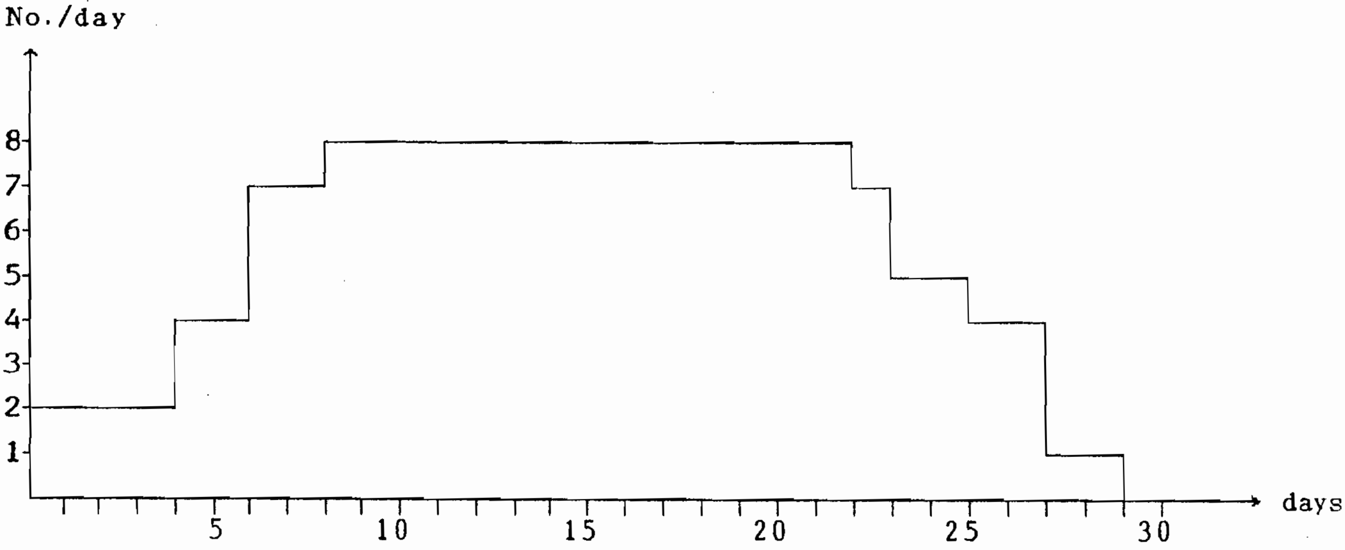


**Figure 2.21 Preferred Resource Histogram for Example 2.15**

## Solution

The information given in Table 2.19 are used to extract total and free floats of the activities, which are listed in Table 2.20. The bar chart shown in Figure 2.22 is then drawn in the manner discussed before.

The resource sums given in line $\Sigma$ 0 are compared with the values of the histogram, Figure 2.21, and the solution steps are chosen in order to make these resource sums identical with the values of the preferred histogram. The scheduled timings of the activities are shown by the crosshatched bars in Figure 2.22 and are listed in Table 2.21.

**Table 2.20 Floats for Activities of Example 2.15**

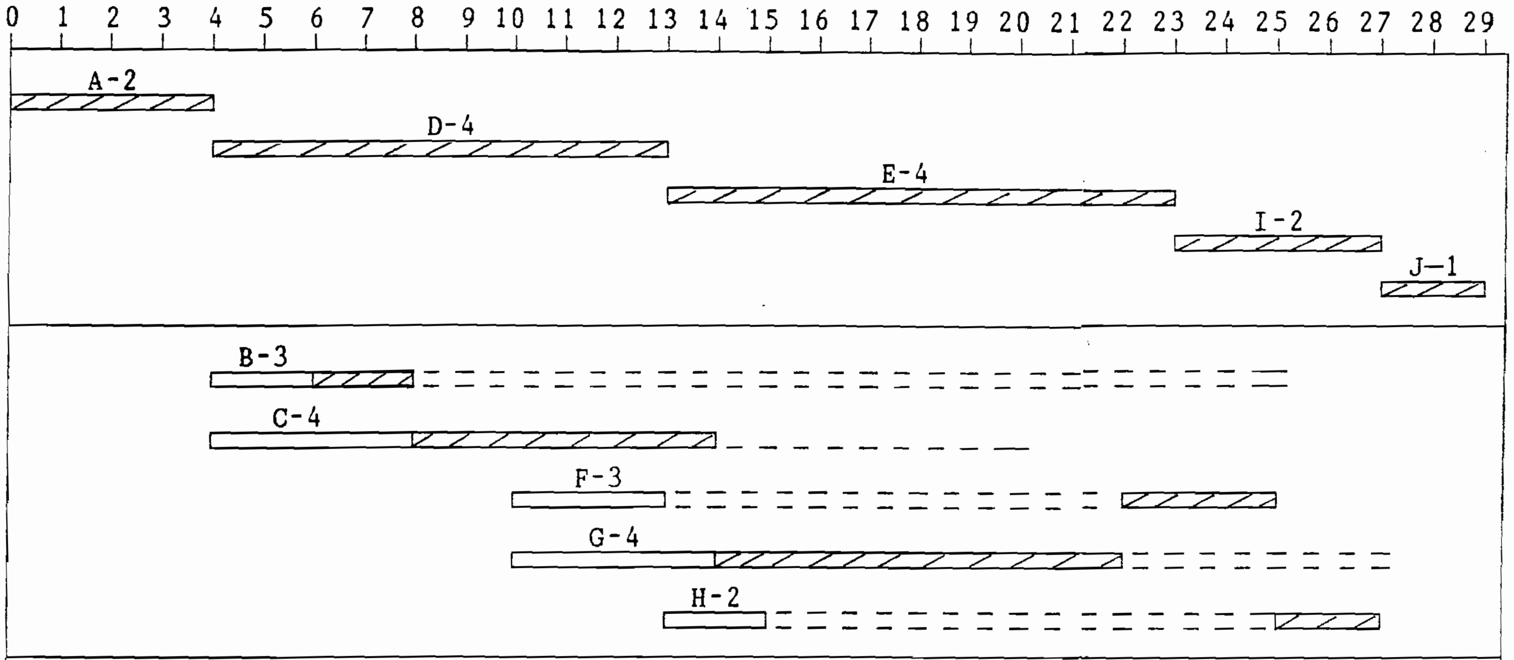
| Activity | Duration | TF | FF |
|---|---|---|---|
| A | 4 | - | - |
| B | 2 | 19 | 7 |
| C | 6 | 9 | - |
| D | 9 | - | - |
| E | 10 | - | - |
| F | 3 | 12 | - |
| G | 8 | 9 | 9 |
| H | 2 | 12 | 12 |
| I | 4 | - | - |
| J | 2 | - | - |

**Table 2.21 Scheduled Timings for Activities of Example 2.15**

| Activity | SS | SF |
|---|---|---|
| A | 0 | 4 |
| B | 6 | 8 |
| C | 8 | 14 |
| D | 4 | 13 |
| E | 13 | 23 |
| F | 22 | 25 |
| G | 14 | 22 |
| H | 25 | 27 |
| I | 23 | 27 |
| J | 27 | 29 |

## 2.13 Smoothing Combined Resources

If the activities of a project use more than one resource, the smoothing process can be accomplished by one of the following methods. With the activities positioned as they were after the smoothing of the first priority resource, the same procedure can be adopted to smooth the second resource. Smoothing resources one priority at a time will cause some activities to be shifted from a previous position established by the smoothing of a higher priority resource. Hence, when smoothing resources in series it should not be expected that the resource requirements for any one of the resources would be minimal. The resource which is smoothed last may have the controlling values.



| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Σ 0 | 2 | 2 | 2 | 2 | 11 | 11 | 8 | 8 | 8 | 8 | 11 | 11 | 11 | 10 | 10 | 8 | 8 | 8 | 4 | 4 | 4 | 4 | 4 | 2 | 2 | 2 | 2 | 1 | 1 |
| H→ 12 | | | | | | | | | | | | | | -2 | -2 | | | | | | | | | | | 2 | 2 | | |
| Σ 1 | 2 | 2 | 2 | 2 | 11 | 11 | 8 | 8 | 8 | 8 | 11 | 11 | 11 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 | 4 | 2 | 2 | 4 | 4 | 1 | 1 |
| F→ 12 | | | | | | | | | | | -3 | -3 | -3 | | | | | | | | | | 3 | 3 | 3 | | | | |
| Σ 2 | 2 | 2 | 2 | 2 | 11 | 11 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 | 7 | 5 | 5 | 4 | 4 | 1 | 1 |
| G→ 4 | | | | | | | | | | | -4 | -4 | -4 | -4 | | | | | 4 | 4 | 4 | 4 | | | | | | | |
| Σ 3 | 2 | 2 | 2 | 2 | 11 | 11 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 7 | 5 | 5 | 4 | 4 | 1 | 1 |
| C→ 4 | | | | | -4 | -4 | -4 | -4 | | | 4 | 4 | 4 | 4 | | | | | | | | | | | | | | | |
| Σ 4 | 2 | 2 | 2 | 2 | 7 | 7 | 4 | 4 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 7 | 5 | 5 | 4 | 4 | 1 | 1 |
| B→ 2 | | | | | -3 | -3 | 3 | 3 | | | | | | | | | | | | | | | | | | | | | |
| Σ 5 | 2 | 2 | 2 | 2 | 4 | 4 | 7 | 7 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 7 | 5 | 5 | 4 | 4 | 1 | 1 |

Figure 2.22 Solution of Example 2.15

Another effective, but approximate, method is to smooth combination of resources by adding the resource rates together, smoothing the resulting resource sums, and separating the resource sums into their respective histogram values at the end of the progress. The method can be illustrated by the following example.

### Example 2.16

Consider example 2.14. Assume that resources R1 and R2 will now be used. Smooth the demand for resources R1 and R2 simultaneously.

### Solution

Construct Table 2.22. ERU is the equivalent resource usage of the two resources and can be obtained by adding the resource rates of the two resources simultaneously. The demand for ERU by the contract activities is given in Table 2.23.

Smooth the demand for ERU as given in Example 2.14. The steps of the smoothing process are L → 6, J → 5, M → 2, F → 3, and H → 1. The scheduled start and finish timings of each activity are given in Table 2.24. These scheduled timings give the histograms of the resources after the smoothing process has been accomplished. These histograms are shown in Figure 2.23.

Table 2.22 Calculations for Smoothings R1 and R2 simultaneously

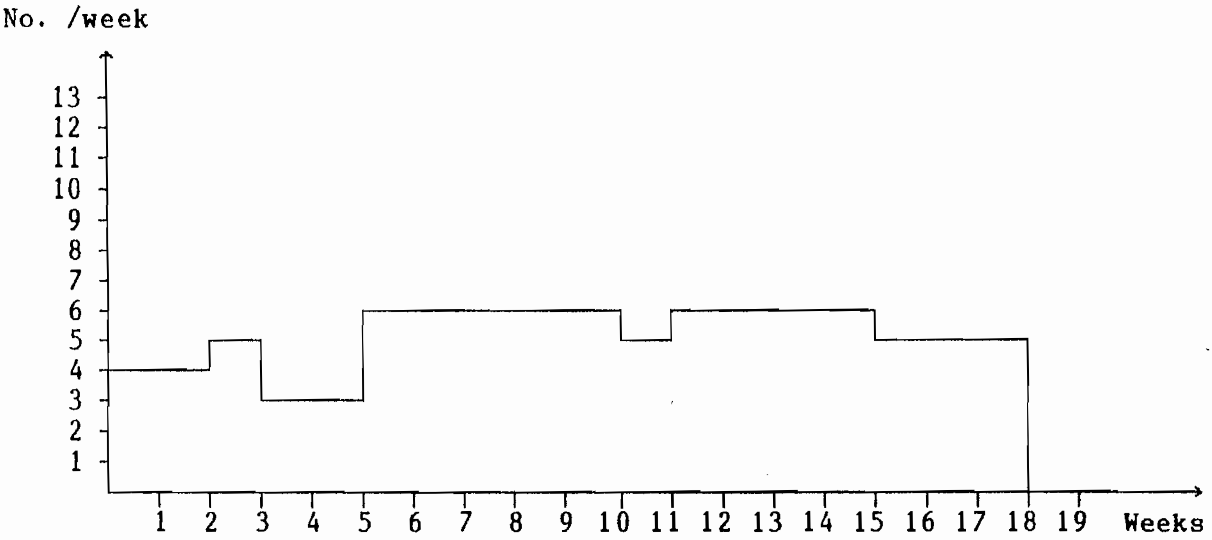
| Week no. | R1 Histogram | R2 Histogram | ERU=R1+R2 |
|---|---|---|---|
| 1 | 4 | 5 | 9 |
| 2 | 4 | 5 | 9 |
| 3 | 7 | 12 | 19 |
| 4 | 6 | 17 | 23 |
| 5 | 5 | 13 | 18 |
| 6 | 6 | 12 | 18 |
| 7 | 6 | 12 | 18 |
| 8 | 6 | 7 | 13 |
| 9 | 8 | 7 | 15 |
| 10 | 13 | 18 | 31 |
| 11 | 9 | 13 | 22 |
| 12 | 4 | 8 | 12 |
| 13 | 2 | 0 | 2 |
| 14 | 2 | 0 | 2 |
| 15 | 2 | 0 | 2 |
| 16 | 2 | 0 | 2 |
| 17 | 2 | 0 | 2 |
| 18 | 2 | 0 | 2 |

Table 2.23 Activities' ERU

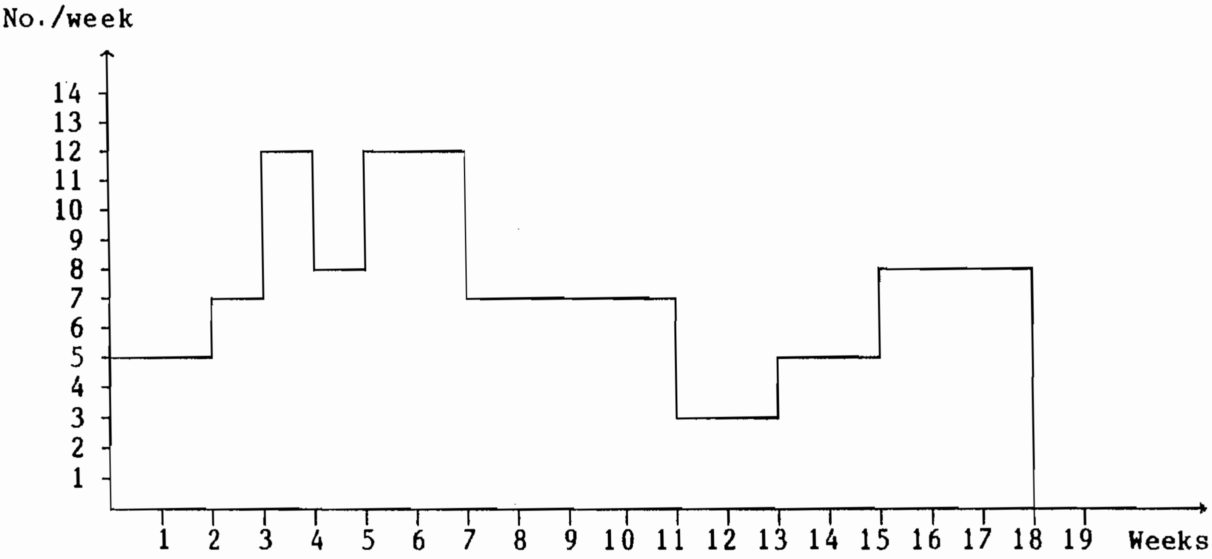
| Activity | R1 | R2 | ERU |
|---|---|---|---|
| B | 0 | 2 | 2 |
| C | 2 | 3 | 5 |
| D | 2 | 0 | 2 |
| E | 1 | 4 | 5 |
| F | 2 | 5 | 7 |
| G | 3 | 2 | 5 |
| H | 1 | 0 | 1 |
| I | 0 | 5 | 5 |
| J | 4 | 5 | 9 |
| K | 2 | 0 | 2 |
| L | 2 | 8 | 10 |
| M | 4 | 3 | 7 |

Table 2.24 Scheduled Timings of Activities of Example 2.16

| Activity | Scheduled start | Scheduled finish |
|---|---|---|
| B | 0 | 2 |
| C | 0 | 5 |
| D | 0 | 3 |
| E | 2 | 4 |
| F | 5 | 11 |
| G | 5 | 11 |
| H | 4 | 10 |
| I | 3 | 7 |
| J | 13 | 15 |
| K | 11 | 18 |
| L | 15 | 18 |
| M | 11 | 13 |
| N | 18 | 20 |



(a)



(b)

Figure 2.23 Smoothing Resources R1 and R2 Simultaneously
(a) R1 Histogram (b) R2 Histogram

## 2.14 Constrained Resource Allocation ( Resource Scheduling )

### Objective

It is not uncommon to find a limited number of resources available for a certain project. The objective of the levelling process is to reduce the demand for the resources so that it falls within the prescribed limits.

### Procedure for Resources Scheduling

The resource scheduling problem can be solved using the following procedure:

1. Prepare a complete activity schedule.

2. Draw a bar chart of the project using early start timing of the activities. Each activity is represented by an open bar. Above each bar put the name of the activity together with the resource(s) rate(s). Beside each activity draw its free float as a dashed line adjacent to the upper side of the bar and its total float as a dashed line adjacent to the lower side of the bar. The steps taken to adjust the use of the resources are shown beneath the bar chart.

3. Determine resource sum in each time period. Test the demand for the resource period by period. If the demand of any resource exceeds its limit, choose an activity to be shifted. Primarily, noncritical activities should be shifted within their total floats and critical activities should be shifted the least number of time periods. Subsequently, an activity may be reshifted. The problem ultimate goal is to satisfy resource(s) constraint(s) with the minimum possible extension of project completion time.

4. Having shifted any activity, you should revise the values of free and total floats of all subsequent activities and show this on the figure:

- an activity shifted within its free float will not change floats of subsequent activities.
- an activity shifted within its total float may change floats of its immediately subsequent activities.
- an activity shifted beyond its total float will change floats of its subsequent activities together with project completion time.

5. The shifting of an activity is shown by the subtraction of its resource rate from resource sums on the old positions and the addition of the rate to resource sums on the new positions of the activity.

6. The shifting process is continued in order not to violate any resource constraint. The final position for an activity after levelling of the resource(s) has been accomplished is shown by crosshatching its bar.

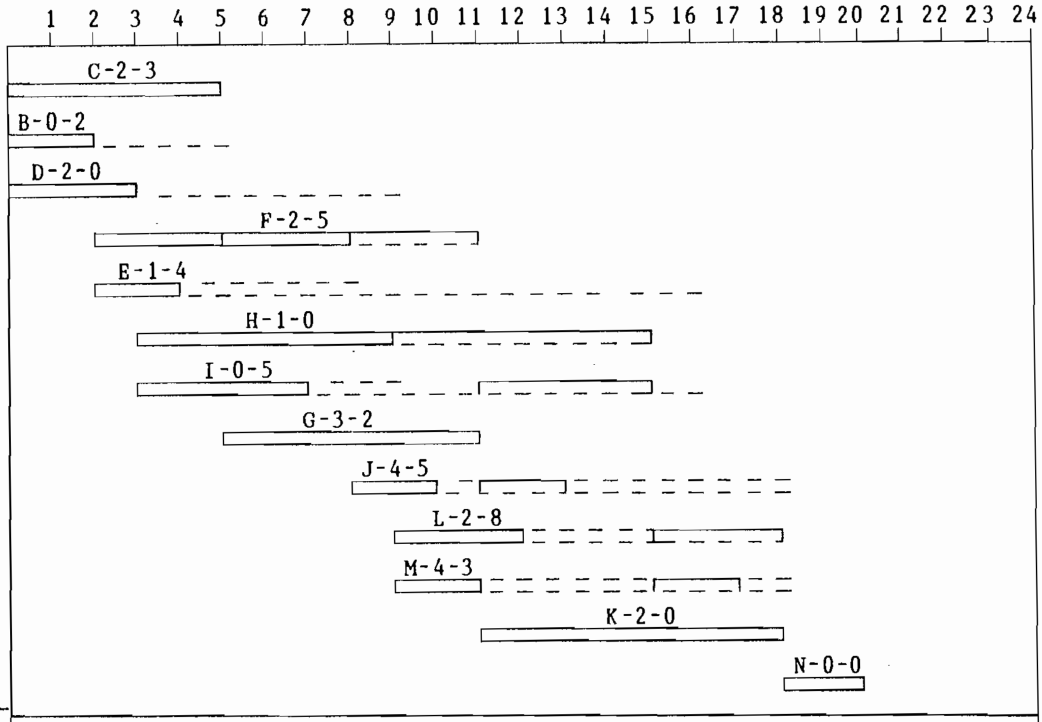
### Example 2.17

Consider the contract given in example 2.14. Assume that resources R1 and R2 are limited to 5 and 8 units respectively. Determine contract completion time that satisfies these constraints.

### Solution

See figure 2.24 for explanation:

1. The activities are listed according to their early start timings.
2. Beneath the bar chart are shown sums of resources R1 and R2.
3. The demand for R1 and R2 on the third week exceeds the limits. Activity E may be shifted. However, shifting of E is not enough to satisfy R1 constraint and F will subsequently be shifted. Therefore the scheduler may decide to shift F instead of E and F. A decision is taken to shift F within its total float. Shifting of F by 3 weeks affects J which must also be shifted by 3 weeks. New resource sums are then calculated.
4. Now the demand for R1 through weeks 6 to 13 exceeds the limit. Activity H is chosen to be shifted by 6 weeks, i.e. within its total float. Shifting of H will affect L and M. They must be shifted by 6 weeks also. Having shifted H, L and M, calculate new resource sums.
5. The demand for R2 on the fourth week exceeds the limit. Total float of activity I equals 9 weeks. It is decided to shift I by 8 weeks only in order not to disturb M. Resource sums show now that the first violation of resource constraints is on week 10.
6. Activity H should be reshifted. It has no float and therefore it should be shifted the least number of weeks. However, shifting of H affects L and M and this will subsequently affects N. Activity H uses one unit of resource R1 and shifting of H will satisfy the limit on R1 for weeks 10 and 11. On weeks 12 and 13, the shifting of H is inadequate and the scheduler may decide to shift K instead. Therefore a decision is taken to reshift H by 2 weeks only. L and M must be shifted now by 2 weeks. However, putting L and M to start on the same week is not fair: L should precede M. Now float of I must be adjusted. Activity N is now delayed by 4 weeks. This will give activity K a float of 4 weeks.
7. Activity K is shifted by 2 weeks.
8. The demand for resource R2 on weeks 12 and 13 exceeds the limit and activity I should be reshifted by two weeks. This will not affect any other activity. New resource sums show that the two resource constraints have been satisfied and the project completion time equals 24 weeks.



| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Σ 0 | R1 ≤ 5 | 4 | 4 | 7 | 6 | 5 | 6 | 6 | 6 | 8 | 13 | 9 | 4 | 2 | 2 | 2 | 2 | 2 | 2 |
| | R2 ≤ 8 | 5 | 5 | 12 | 17 | 13 | 12 | 12 | 7 | 7 | 18 | 13 | 8 | 0 | 0 | 0 | 0 | 0 | 0 |
| Σ 1 | F → 3 | | | -2 | -2 | -2 | | | | +2 | +2 | +2 | | | | | | | |
| | J → 3 | | | | | | | | | -4 | -4 | | +4 | +4 | | | | | |
| | R1 | 4 | 4 | 5 | 4 | 3 | 6 | 6 | 6 | 6 | 11 | 11 | 8 | 6 | 2 | 2 | 2 | 2 | 2 |
| | F → 3 | | | -5 | -5 | -5 | | | | +5 | +5 | +5 | | | | | | | |
| | J → 3 | | | | | | | | | -5 | -5 | | +5 | +5 | | | | | |
| | R2 | 5 | 5 | 7 | 12 | 8 | 12 | 12 | 7 | 7 | 18 | 18 | 13 | 5 | 0 | 0 | 0 | 0 | 0 |
| Σ 2 | H → 6 | | | | -1 | -1 | -1 | -1 | -1 | -1 | +1 | +1 | +1 | +1 | +1 | +1 | | | |
| | L → 6 | | | | | | | | | | -2 | -2 | -2 | | | | +2 | +2 | +2 |
| | M → 6 | | | | | | | | | | -4 | -4 | | | | | +4 | +4 | |
| | R1 | 4 | 4 | 5 | 3 | 2 | 5 | 5 | 5 | 5 | 6 | 6 | 7 | 7 | 3 | 3 | 8 | 8 | 4 |
| | H → 6 | | | | | | | | | | | | | | | | | | |
| | L → 6 | | | | | | | | | | -8 | -8 | -8 | | | | +8 | +8 | +8 |
| | M → 6 | | | | | | | | | | -3 | -3 | | | | | +3 | +3 | |
| | R2 | 5 | 5 | 7 | 12 | 8 | 12 | 12 | 7 | 7 | 7 | 7 | 5 | 5 | 0 | 0 | 11 | 11 | 8 |
| Σ 3 | I → 8 | | | | | | | | | | | | | | | | | | |
| | R1 | 4 | 4 | 5 | 3 | 2 | 5 | 5 | 5 | 5 | 6 | 6 | 7 | 7 | 3 | 3 | 8 | 8 | 4 |
| | I → 8 | | | | -5 | -5 | -5 | -5 | | | | | +5 | +5 | +5 | +5 | | | |
| | R2 | 5 | 5 | 7 | 7 | 3 | 7 | 7 | 7 | 7 | 7 | 7 | 10 | 10 | 5 | 5 | 11 | 11 | 8 |

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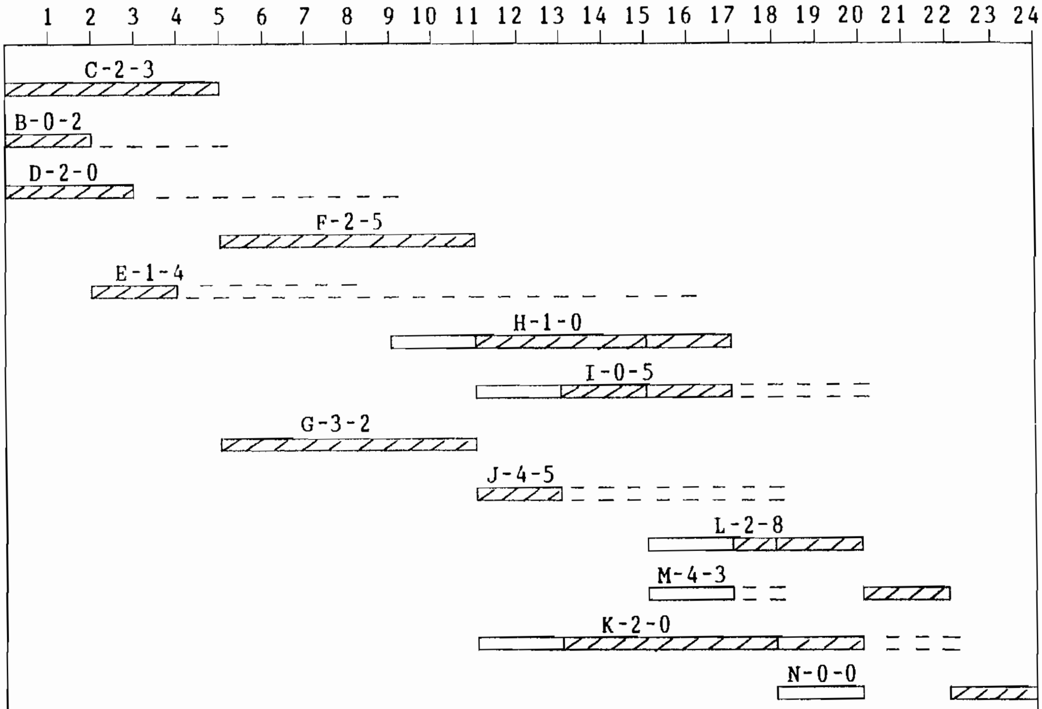


| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Σ 3 | R1 ≤ 5 | 4 | 4 | 5 | 3 | 2 | 5 | 5 | 5 | 5 | 6 | 6 | 7 | 7 | 3 | 3 | 8 | 8 | 4 | | | | |
| | R2 ≤ 8 | 5 | 5 | 7 | 7 | 3 | 7 | 7 | 7 | 7 | 7 | 7 | 10 | 10 | 5 | 5 | 11 | 11 | 8 | | | | |
| Σ 4 | H → 2 | | | | | | | | | | -1 | -1 | | | | | +1 | +1 | | | | | |
| | L → 2 | | | | | | | | | | | | | | | | -2 | -2 | | +2 | +2 | | |
| | M → 5 | | | | | | | | | | | | | | | | -4 | -4 | | | | +4 | +4 |
| | N → 4 | | | | | | | | | | | | | | | | | | | | | | |
| | R1 | 4 | 4 | 5 | 3 | 2 | 5 | 5 | 5 | 5 | 5 | 5 | 7 | 7 | 3 | 3 | 3 | 3 | 4 | 2 | 2 | 4 | 4 |
| | H → 2 | | | | | | | | | | | | | | | | | | | | | | |
| | L → 2 | | | | | | | | | | | | | | | | -8 | -8 | | +8 | +8 | | |
| | M → 5 | | | | | | | | | | | | | | | | -3 | -3 | | | | +3 | +3 |
| | N → 4 | | | | | | | | | | | | | | | | | | | | | | |
| | R2 | 5 | 5 | 7 | 7 | 3 | 7 | 7 | 7 | 7 | 7 | 7 | 10 | 10 | 5 | 5 | 0 | 0 | 8 | 8 | 8 | 3 | 3 |
| Σ 5 | K → 2 | | | | | | | | | | | | -2 | -2 | | | | | | +2 | +2 | | |
| | R1 | 4 | 4 | 5 | 3 | 2 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 |
| | K → 2 | | | | | | | | | | | | | | | | | | | | | | |
| | R2 | 5 | 5 | 7 | 7 | 3 | 7 | 7 | 7 | 7 | 7 | 7 | 10 | 10 | 5 | 5 | 0 | 0 | 8 | 8 | 8 | 3 | 3 |
| Σ 6 | I → 2 | | | | | | | | | | | | | | | | | | | | | | |
| | R1 | 4 | 4 | 5 | 3 | 2 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 |
| | I → 2 | | | | | | | | | | | | -5 | -5 | | | +5 | +5 | | | | | |
| | R2 | 5 | 5 | 7 | 7 | 3 | 7 | 7 | 7 | 7 | 7 | 7 | 5 | 5 | 5 | 5 | 5 | 5 | 8 | 8 | 8 | 3 | 3 |

Figure 2.24 Levelling Resources R1 and R2 to 5 and 8 units

## 2.15 Resource Scheduling by the Current Float Technique

This is an approximate method for resource scheduling. The current float is defined as the finish float available to the activity with respect to the latest finish time of that activity as calculated in the original CPM network. The current time; CT, is the time when the resource conflict has arisen. Then, the current float will be calculated as

$$CF = LF - CT - D \qquad (2.20)$$

where, CF = current float, CT = current time, D = activity duration, and
LF = activity late finish time.

### Priority Rules

The resource scheduling using the current float technique uses the following priority rules:

1. allocate resources to the activity that has the least CF
2. in case of a tie, allocate resources to the activity that has the least ES (as calculated by CPM analysis)
3. in case of a tie, allocate resources to the activity with largest D
4. in case of a tie, allocate resources to the activity with the smallest activity number.

### Physical Significance of the Technique

At a particular time of resource conflict, the following interpretation can be made:

1. If no delay has occurred up to that time, this will be reflected by the values of CF of the activities considered; they will be positive or zero.
2. The most negative value of the current float will be an indicator of the extension that has been caused to the project duration up to that point. The extension of the project completion time will be the absolute value of the most negative CF.

### Allocation Procedure

At one particular current time, the activities that can be started are considered in conjunction with those that are continuing. The priorities are assigned to those activities that are waiting to start. This is done on the basis of the priority rules given above. The smallest finish time of the activities that have been started becomes the next current time for the technique.

### Example 2.18

Resolve the previous example using the current float technique.

### Solution

CPM analysis of this project, given in example 2.14, is used to calculate required information as shown in Table 2.25. This table is self explanatory. Resource scheduling using the current float technique results in the project duration being extended by 8 weeks to 28 weeks. A study of this table reveals the following:

1. No delay is encountered up to the time CT = 5.
2. The first delay occurs due to lack of resources to start activity 7 at CT = 5.
3. Another delay of two weeks has occurred at CT = 18, which increases to six weeks at CT = 22 and to eight weeks at CT = 24.
4. The technique gives an approximate solution as compared with the solution obtained in the previous example.

## 2.16 Summary Diagrams

Linear projects form an important part of construction projects. These projects contain repetitive construction works. Examples include long bridges and retaining walls, highways and pipelines. One method for preparing schedules for such projects is the summary diagram. This diagram can be constructed as follows:

1. The repetitive work of one trade is represented by a single activity. The duration of this activity is the summation of the durations for individual activities within the trade.
2. In order to maintain the logic of the network of such a project, relationships between the start of each activity and the start of its successor and between the finish of each activity and the finish of its successor are introduced as follows:

start to start relationship = duration of individual predecessor
finish to finish relationship = duration of individual successor

Table 2.25 Computations Performed by the Current Float Technique

| CT | Activities considered | Resources requirements | | D | LF | CF | Decision | SF |
|---|---|---|---|---|---|---|---|---|
| | | R1 | R2 | | | | | |
| 0 | 3 | 2 | 3 | 5 | 5 | 0 | start | 5 |
| | 2 | 0 | 2 | 2 | 5 | 3 | start | 2* |
| | 4 | 2 | 0 | 3 | 9 | 6 | start | 3 |
| 2 | 3 | 2 | 3 | 5 | - | - | continue | 5 |
| | 4 | 2 | 0 | 3 | - | - | continue | 3* |
| | 6 | 2 | 5 | 6 | 11 | 3 | cannot start | - |
| | 5 | 1 | 4 | 2 | 16 | 12 | start | 4 |
| 3 | 3 | 2 | 3 | 5 | - | - | continue | 5 |
| | 5 | 1 | 4 | 2 | - | - | continue | 4* |
| | 6 | 2 | 5 | 6 | 11 | 2 | cannot start | - |
| | 8 | 1 | 0 | 6 | 15 | 6 | start | 9 |
| | 9 | 0 | 5 | 4 | 16 | 9 | cannot start | - |
| 4 | 3 | 2 | 3 | 5 | - | - | continue | 5* |
| | 8 | 1 | 0 | 6 | - | - | continue | 9 |
| | 6 | 2 | 5 | 6 | 11 | 1 | start | 10 |
| | 9 | 0 | 5 | 4 | 16 | 8 | cannot start | - |
| 5 | 8 | 1 | 0 | 6 | - | - | continue | 9* |
| | 6 | 2 | 5 | 6 | - | - | continue | 10 |
| | 7 | 3 | 2 | 6 | 11 | 0 | cannot start | - |
| | 9 | 0 | 5 | 4 | 16 | 7 | cannot start | - |
| 9 | 6 | 2 | 5 | 6 | - | - | continue | 10* |
| | 7 | 3 | 2 | 6 | 11 | -4 | start | 15 |
| | 9 | 0 | 5 | 4 | 16 | 3 | cannot start | - |
| | 12 | 2 | 8 | 3 | 18 | 6 | cannot start | - |
| 10 | 7 | 3 | 2 | 6 | - | - | continue | 15 |
| | 9 | 0 | 5 | 4 | 16 | 2 | start | 14* |
| | 12 | 2 | 8 | 3 | 18 | 5 | cannot start | - |
| | 10 | 4 | 5 | 2 | 18 | 6 | cannot start | - |
| 14 | 7 | 3 | 2 | 6 | - | - | continue | 15* |
| | 12 | 2 | 8 | 3 | 18 | 1 | cannot start | - |
| | 10 | 4 | 5 | 2 | 18 | 2 | cannot start | - |
| | 13 | 4 | 3 | 2 | 18 | 2 | cannot start | - |
| 15 | 11 | 2 | 0 | 7 | 18 | -4 | start | 22 |
| | 12 | 2 | 8 | 3 | 18 | 0 | start | 18* |
| | 10 | 4 | 5 | 2 | 18 | 1 | cannot start | - |
| | 13 | 4 | 3 | 2 | 18 | 1 | cannot start | - |
| 18 | 11 | 2 | 0 | 7 | - | - | continue | 22* |
| | 10 | 4 | 5 | 2 | 18 | -2 | cannot start | - |
| | 13 | 4 | 3 | 2 | 18 | -2 | cannot start | - |
| 22 | 10 | 4 | 5 | 2 | 18 | -6 | start | 24* |
| | 13 | 4 | 3 | 2 | 18 | -6 | cannot start | - |
| 24 | 13 | 4 | 3 | 2 | 18 | -8 | start | 26* |
| 26 | 14 | - | - | 2 | 20 | -8 | start | 28 |

* Next current time

3. The timings of each activity can be calculated as follows:

ES of each activity = ES of its predecessor + lag between them

EF of each activity = EF of its predecessor + lag between them
or = ES of the activity + its duration
(whichever is the longer)

LF of each activity = LF of its successor - lag between them

LS of each activity = LS of its successor - lag between them
or = LF of the activity - its duration
(whichever is the shorter)

4. The early and late start timings of each activity represent timings of the first individual activity of each trade. On the other hand, the early and late finish timings of each activity represent timings of the last individual activity of the trade. Therefore, contract completion time equals finish time of the last activity in the summary diagram.

**Example 2.19**

The construction of a jetty is represented by three sequential activities: Pile driving, Pile cap and Deck construction. This sequence has to be repeated four times to finish the jetty. Table 2.26 gives durations of the activities for one part of the jetty. Construct the contract summary diagram using the precedence notation and compare it with the complete precedence diagram.

**Table 2.26 Data for Example 2.19**

| Activity | Code | Duration (weeks) |
|---|---|---|
| Pile driving | P | 3 |
| Pile cap | C | 2 |
| Deck construction | D | 2 |

**Solution**

The contract precedence diagram is shown in Figure 2.25 and the summary diagram is shown in Figure 2.26.

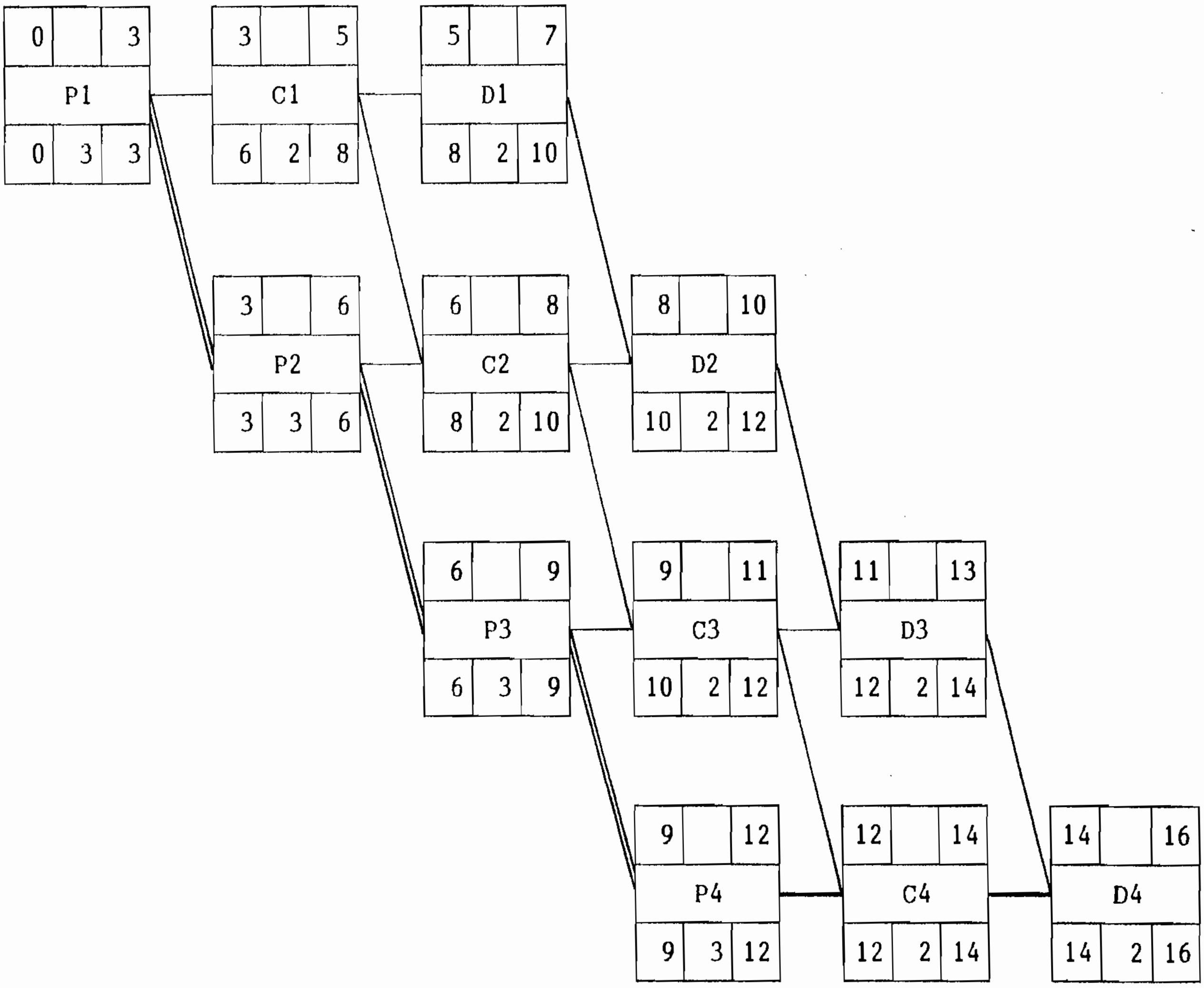


Figure 2.25 Precedence Diagram for Example 2.19

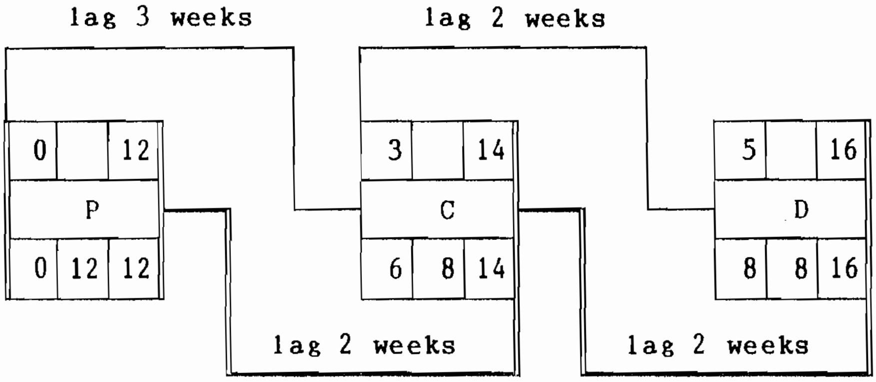


Figure 2.26 Summary Diagram for Example 2.19

From the summary diagram it is obvious that the critical activities are: all sections of pile driving, then last section of pile cap and then last section of deck construction. The first section of both pile cap and deck construction have a total float of three weeks each.

The results of the summary diagram are compatible with the results of the precedence diagram on the assumption that durations of all activities will not be changed. If the duration of one of the non-critical activities is changed, it will be possible for the critical path to run through other activities and this will not be shown by the summary diagram.

## 2.17 The Line-Of-Balance (LOB) Method for Scheduling Repetitive Construction Works.

The process of scheduling repetitive construction works aims at meeting a programmed rate of completed units and ensuring that a balanced mix of resources is kept fully employed. The LOB method achieves these objectives.

The basis of the LOB technique is to find the required resources for each activity so that the following activities are not interfered with and the target completion rate can be achieved. Therefore, the difference between the LOB schedule and a networking technique is that the former is based on chosen set of resources to achieve a construction rate while the second is based on separating logic and resource allocation.

The basic assumption of the technique is that each crew of labour should work at their natural rate. Figure 2.27 is used to compare between an activity LOB using one and two crews of labour. It is obvious that the inclination of the LOB will increase with the increase of the number of crews used.

### Preparing an LOB Schedule

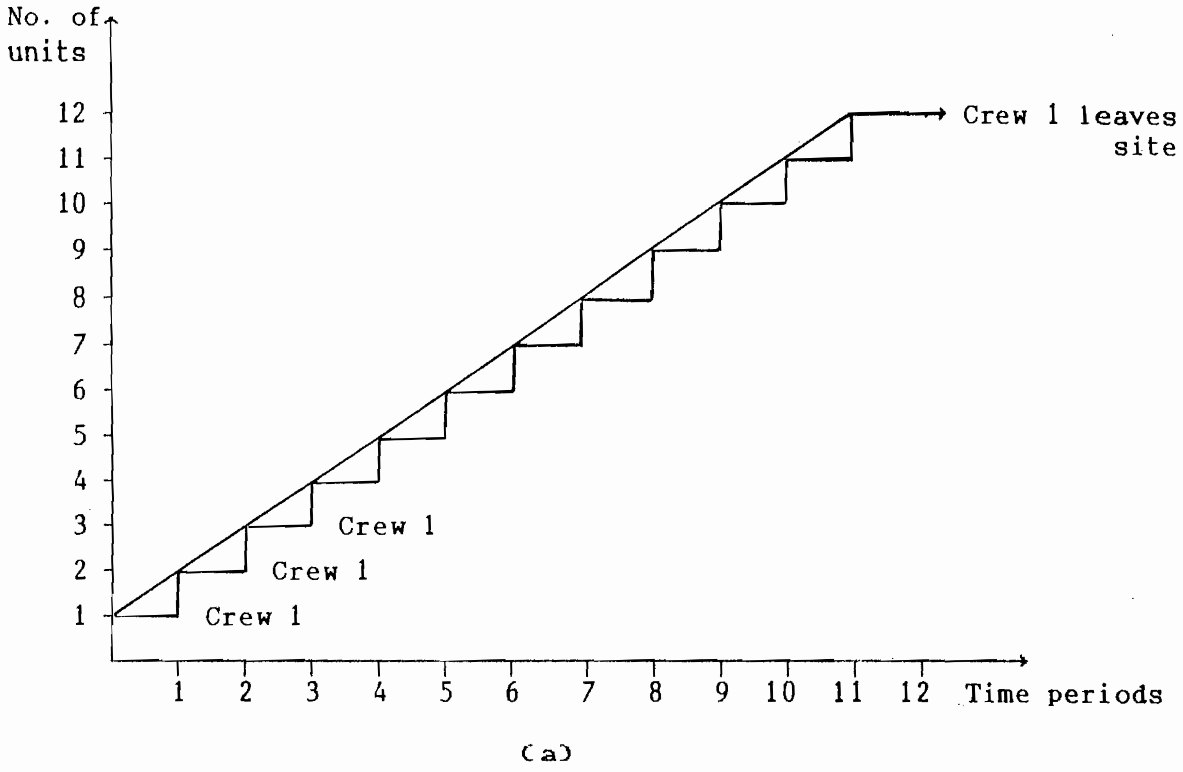
The following steps demonstrate how an LOB schedule can be prepared for a contract:

1. The manhours necessary, as well as the optimum crew size are estimated for each activity. This information may be obtained from field personnel who have many years of site experience or from previous records.

2. Choose buffer times between the activities in order to absorb unforeseen events and errors. It can be assumed from knowledge of the likely variability of the preceding activity.

3. Prepare a logic diagram for the activities of one of the many units to be constructed. Take into consideration the buffer times.

4. Calculate the required target output in order to meet a given contract completion date. Figure 2.28 demonstrates how this rate can be calculated. For



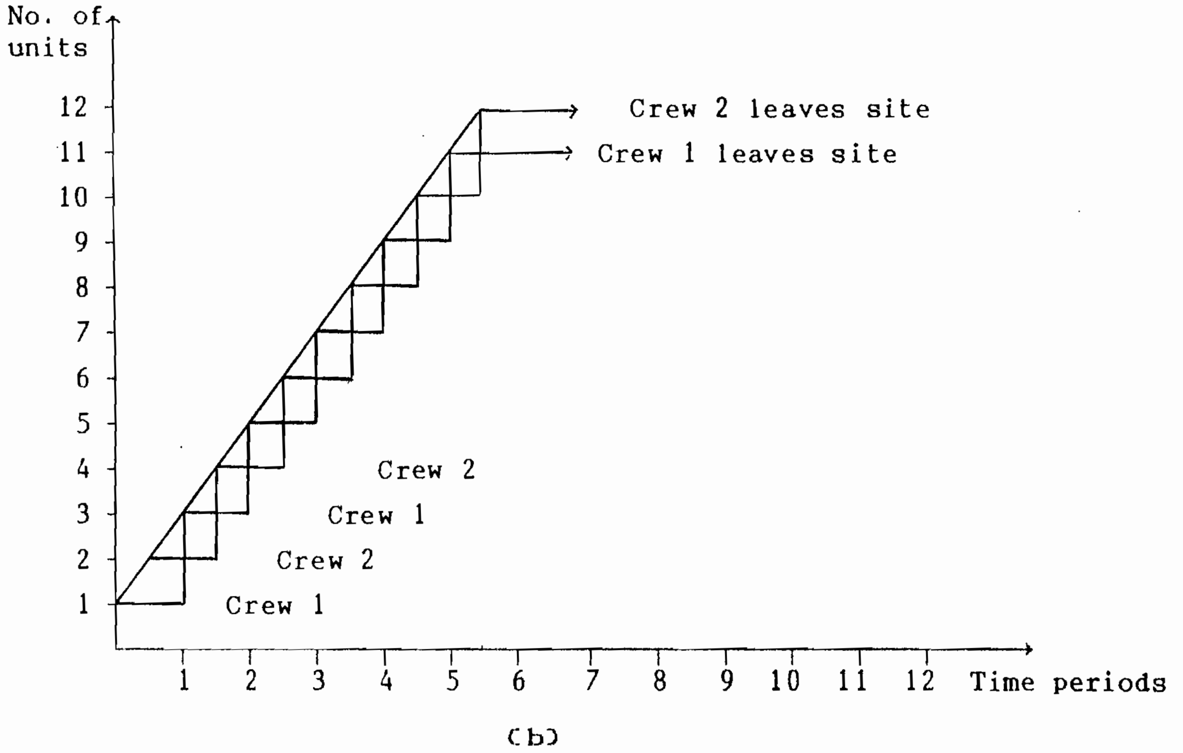


Figure 2.27 An Activity LOB
(a) Using Single Crew (b) Using Two Crews

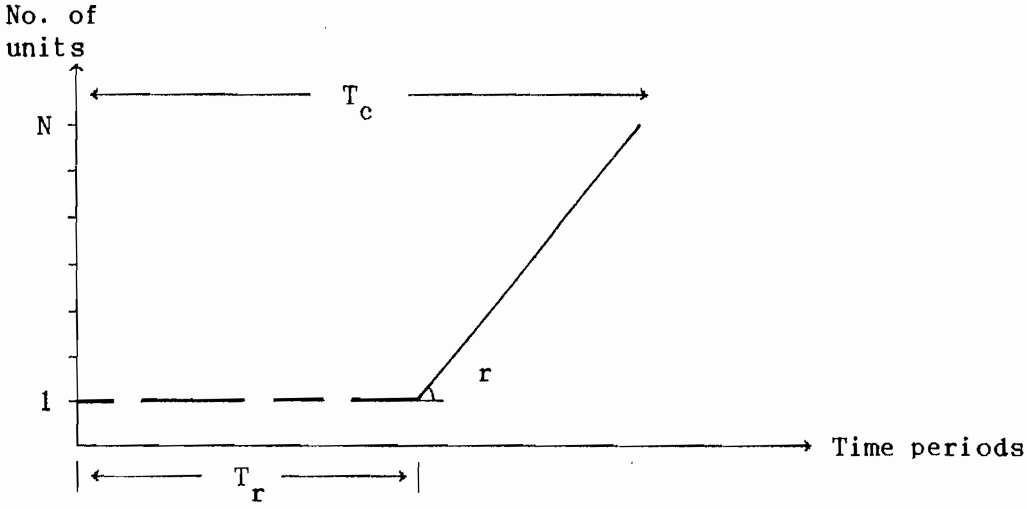


Figure 2.28 Determination of Target Rate of Build

a contract with several activities, calculate the time required to complete all the activities in the first unit assuming all resources will be available. This time, $T_r$, should include buffer times between the activities. Then the target rate of build; r, is given by:

$$r = \frac{N - 1}{T_c - T_r} \qquad (2.21)$$

where N = no. of units to be constructed
$T_c$ = chosen contract completion time.

The target rate of build should be expressed as a number of units per working period. The chosen working period must be used in all succeeding calculations. The number of crews necessary for the contract must be arranged in order that the rate of output is as close to the target rate of build as possible.

5. Assume the following notation:

m = actual rate of output = slope of LOB,
M = required manhours per unit,
D = activity duration,
n = number of hours in a working period,
$S_o$ = optimum number of men/unit,
$S_t$ = theoretical gang size required to maintain r,

$S_a$ = actual gang size, a number which would be a multiple of $S_o$; if $S_a > S_t$ then $m > r$,

$ST_1$ = start time of activity in first unit which can be calculated from the logic diagram, and

$ST_i$ = start time of activity in the ith unit.

Then we have :

$$S_t = r \cdot M / n \tag{2.22}$$

$$m = r \cdot S_a / S_t \tag{2.23}$$

$$D = M / (S_o \cdot n) \tag{2.24}$$

$$ST_i = ST_1 + (i-1) / m , \; i \leq N \tag{2.25}$$

6. Draw the LOB from the information obtained. The number of units to be constructed is plotted against time. Two parallel lines, whose slope is equal to the actual rate of output, will denote the start and finish times respectively of each activity in all the units, from the first to the last, the distance between the two lines represents the duration of the activity.

7. Examine the schedule and assess possible alternatives to bring about a more balanced schedule.

**Example 2.20**

The construction of a long reinforced concrete retaining wall involves the following three sequential activities: steelfixing, formwork erection and concreting. The wall is divided into 15 equal sections. The manhours required and the team size for each section are given in Table 2.27.

**Table 2.27 Data for Example 2.20**

| Activity | Manhours/section | Team size/section |
|---|---|---|
| Steelfixing (S) | 576 | 4 |
| Formwork erection (F) | 1440 | 6 |
| Concreting (C) | 576 | 6 |

Prepare a line of balance schedule for the contract using a target rate of build of one section per week and each team working at their natural rate. Assume a minimum buffer time of three days between activities and six 8-hour days per week.

What is the overall duration of the contract and when will the first team of carpenters leave the site?

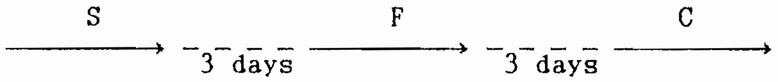


Figure 2.29 Logic Diagram for One Section

Table 2.28 LOB Calculations for Example 2.20

| Activity | M | $S_o$ | $S_t$ | $S_a$ | m | D | $ST_N - ST_1$ |
|---|---|---|---|---|---|---|---|
| Steelfixing | 576 | 4 | 12 | 12 | 1/6 | 18 | 84 |
| Formwork | 1440 | 6 | 30 | 30 | 1/6 | 30 | 84 |
| Concreting | 576 | 6 | 12 | 12 | 1/6 | 12 | 84 |

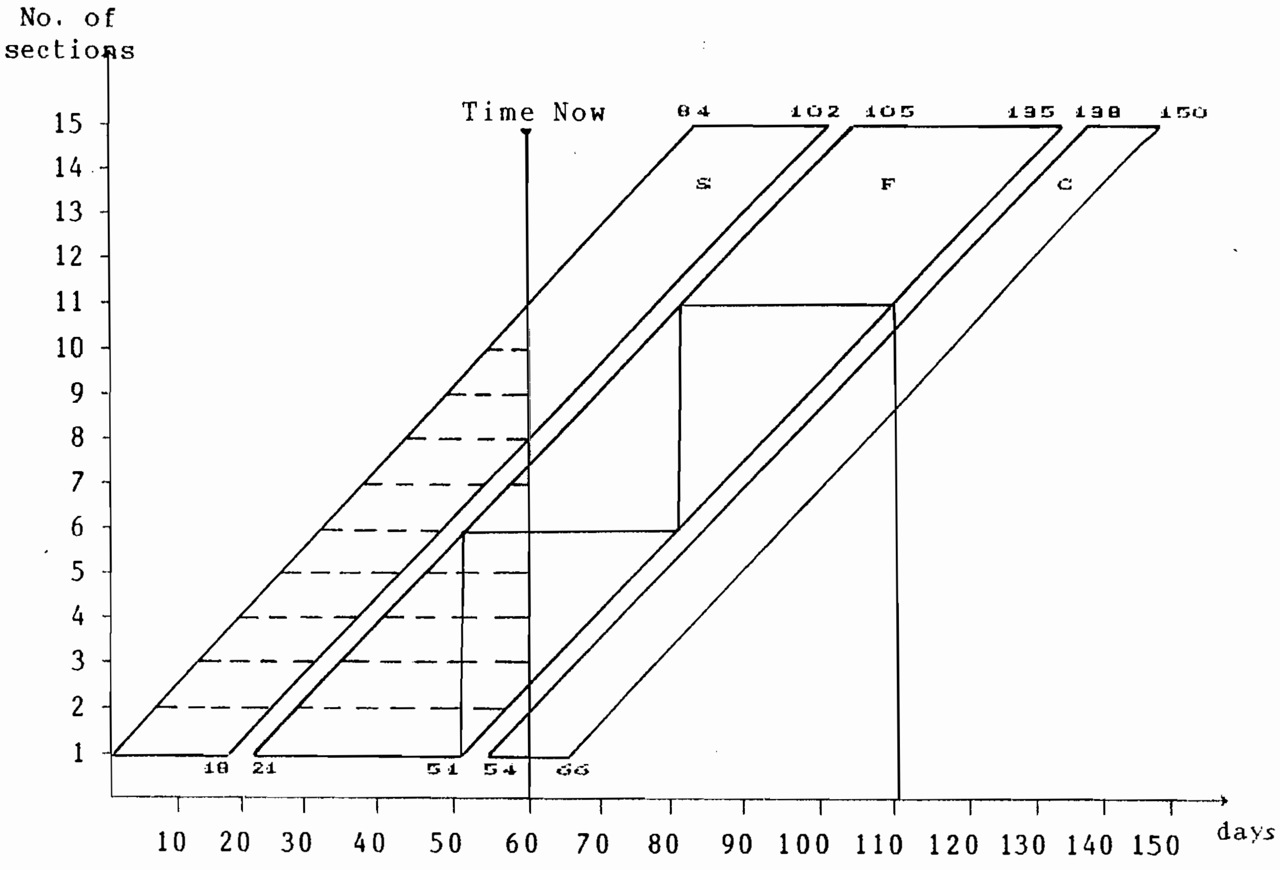


Figure 2.30 LOB for Example 2.20

### Solution

Target rate of build; r = 1/6 section per day

The solution is given in Table 2.28 and Figures 2.29 and 2.30. The contract duration = 150 days. The first team of carpenters will leave the site on day 111.

### Using the Schedule

The schedule can be used to monitor progress. The horizontal dashed lines shown in Figure 2.30 represent work that should be completed by end of day 60. If some of the activities are lagging behind schedule, additional resources will be allocated to them to improve production rate. On the other hand, if a successor activity is moving ahead too quickly and may therefore interfere with a less efficient predecessor activity, the scheduler should either to speed up the predecessor activity or to slow down the successor one.

### Example 2.21

The construction of a bridge involves the following five sequential activities: excavation, foundation, columns, beams and slabs. The bridge is divided into 5 equal bays, i.e. there will be 6 columns, 6 beams and 5 slabs. The manhours required and the team size for one unit are given in Table 2.29.

Prepare an LOB schedule for this contract using a target rate of build of one bay every 14 days and each team working at their natural rate. Assume a minimum buffer time of 6 days between activities and a working day of 8 hours.

What is the overall duration of the contract. Determine total floats available for beams construction.

**Table 2.29 Data for Example 2.21**

| Activity | Manhours / unit | Team size / unit |
|---|---|---|
| Excavation | 672 | 6 |
| Foundation | 880 | 10 |
| Columns | 1456 | 14 |
| Beams | 528 | 6 |
| Slabs | 1120 | 10 |

### Solution

The target rate of build is 1/14 unit per day.

The solution is given in Table 2.30 and Figure 2.31. The contract duration = 182 days. The floats available for beams construction are given in Table 2.31.

Table 2.30 LOB Calculations for Example 2.21

| Activity | M | $S_o$ | $S_t$ | $S_a$ | m | D | N | $ST_N - ST_1 = (N-1)/m$ |
|---|---|---|---|---|---|---|---|---|
| Excavation (E) | 672 | 6 | 6 | 6 | 1/14 | 14 | 6 | 70 |
| Foundation (F) | 880 | 10 | 7.8 | 10 | 1/11 | 11 | 6 | 55 |
| Columns (C) | 1456 | 14 | 13 | 14 | 1/13 | 13 | 6 | 65 |
| Beams (B) | 528 | 6 | 4.7 | 6 | 1/11 | 11 | 6 | 55 |
| Slabs (S) | 1120 | 10 | 10 | 10 | 1/14 | 14 | 5 | 56 |

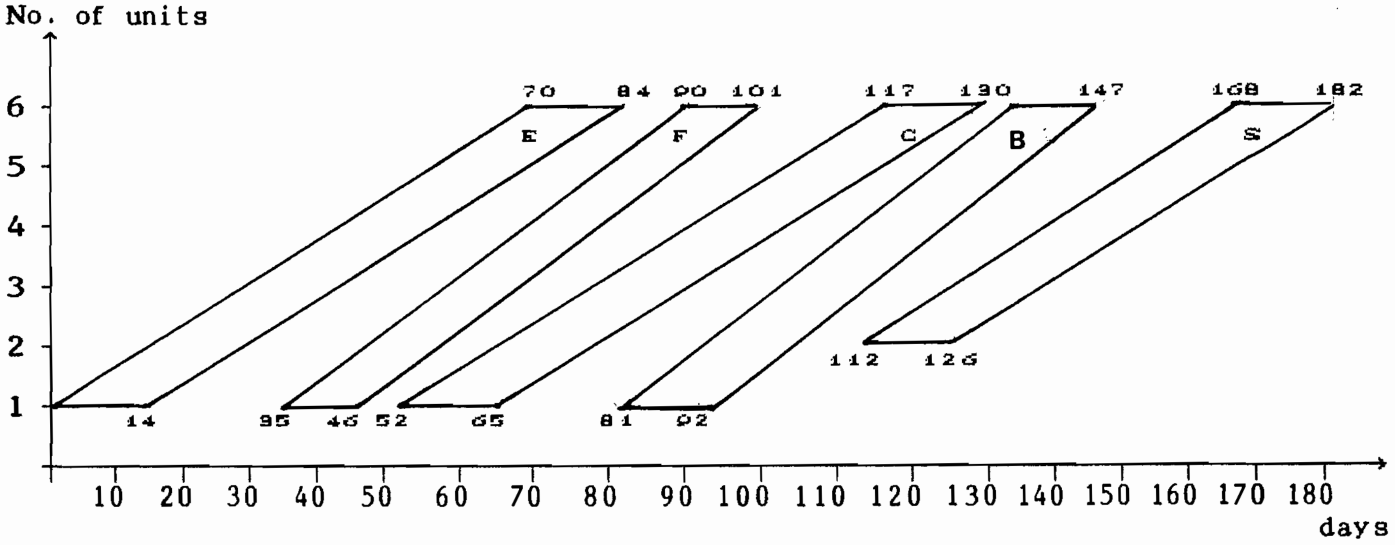


Figure 2.31 LOB for Example 2.21

Table 2.31 Floats for Construction of Beams

| Beam no. | Total float (days) |
|---|---|
| 1 | 6 |
| 2 | 9 |
| 3 | 12 |
| 4 | 15 |
| 5 | 18 |
| 6 | 21 |

### Example 2.22

Table 2.32 gives data required to schedule a small contract of 6 similar units. A minimum buffer of 6 days should be assumed.

a) Draw the contract LOB. State the overall duration of the contract.

b) Give a suggestion as to how the overall duration of the contract could be reduced.

c) Assume that the materials required for activity X ( units 4 : 6 ) have not been delivered to site until day 50. What is the effect of this delay on contract completion time.

Table 2.32 Data for Example 2.22

| Sequence of activities | Unit duration (days) | Slope of LOB |
|---|---|---|
| X | 11 | 1/11 |
| Y | 13 | 1/13 |
| Z | 11 | 1/11 |

### Solution

a) The LOB is drawn in Figure 2.32. The contract duration is 112 days.

b) The overall duration of the contract could be reduced by increasing slope of activity Y to be 1/11 by using extra resources. Thus it will be a parallel scheduling.

c) The buffer time may be used to absorb this delay without extending contract duration as shown in Figure 2.33.

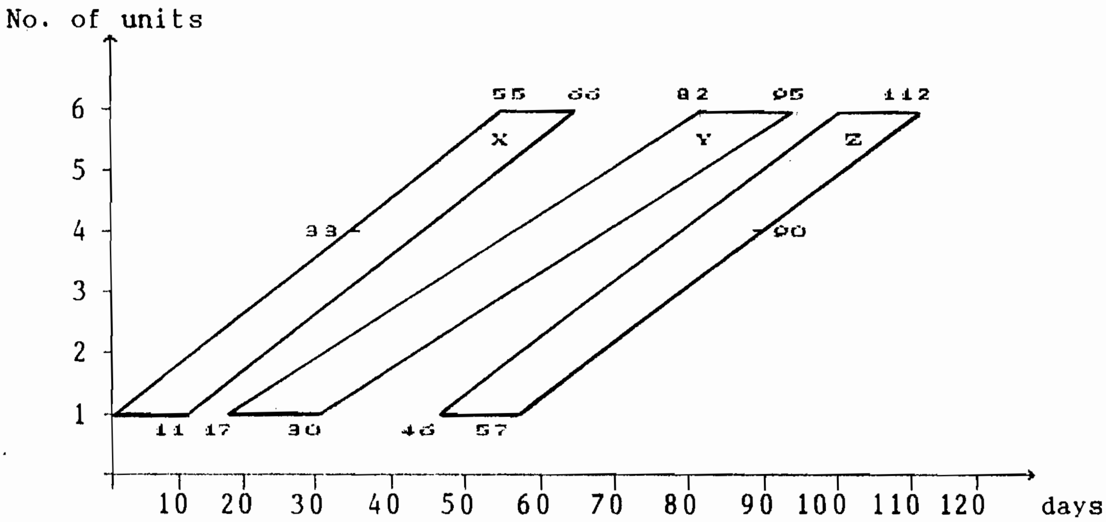


Figure 2.32 LOB for Example 2.22

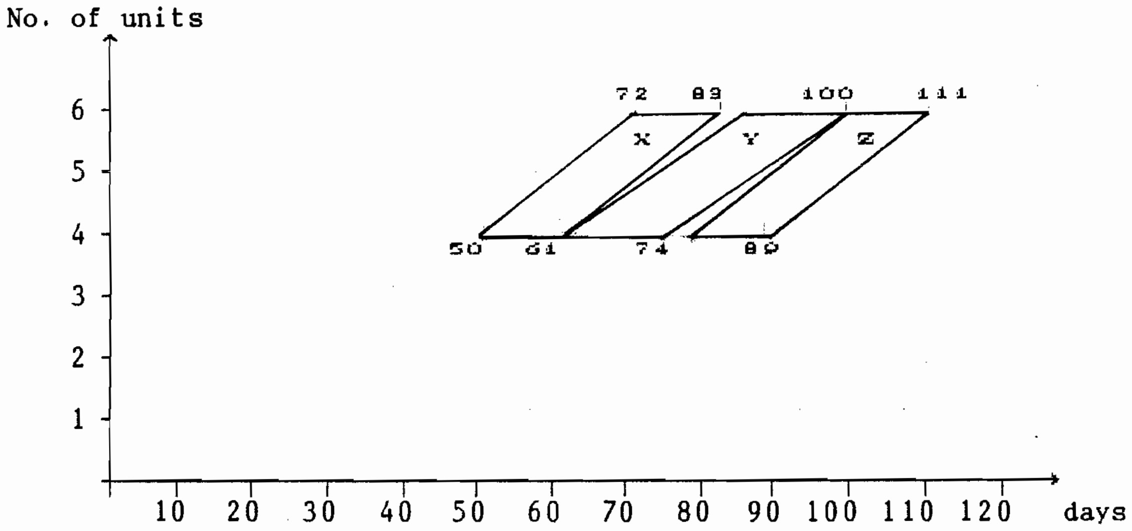


Figure 2.33 Effect of Delay on LOB Schedule

### Integrated CPM / LOB

The same procedure described above can be used to draw a line of balance based on a CPM network (without buffer times). In this case, some activities may have float that can be utilized to relax the demand on the resources. Consider Figure 2.34 where activities A, B and D are critical activities and activity C is a noncritical activity. Activity C may progress in any rate of completion which is anywhere between situation I and situation II. If total float (TF) of activity C is totally utilized (situation II), the work will be spread over a longer period and reduce the demand on the number of resources needed. In this case, the target rate of build given by Equation 2.21 must be modified to be:

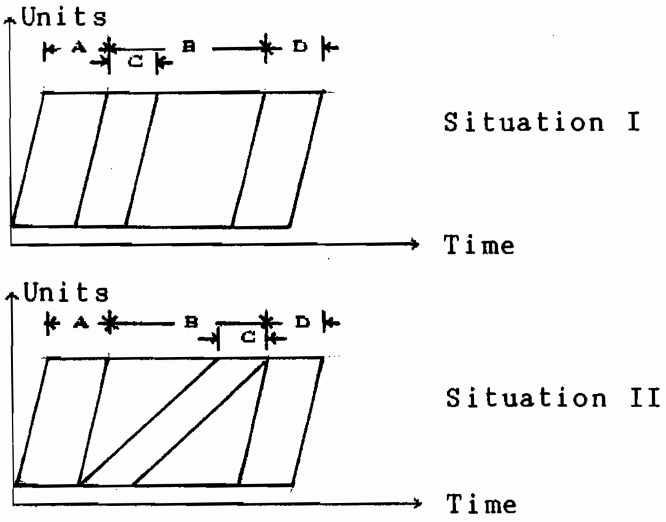


Figure 2.34 Use of Float in LOB

$$r = \frac{N - 1}{(T_c - T_r) + TF} \qquad (2.26)$$