Chapter (7)
Linear & Angular momentum

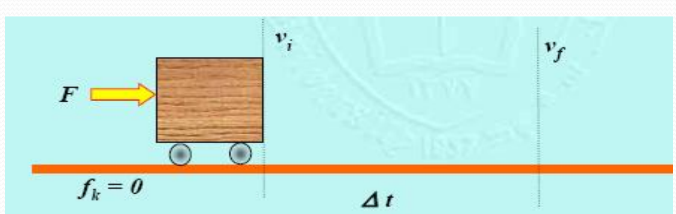
7.1 Impulse and Linear momentum

$$F\Delta t = mv_f - mv_i$$

"Impulse equals the change in momentum"

(Linear Momentum): $\vec{P} = m\vec{v}$ **Unit: Kg. m/s**

$$F\Delta t = P_f - P_i$$



$$\sum \mathbf{F} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}$$

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} \therefore F = m\vec{a}$$

An object's momentum will change if its mass and/or velocity (speed and direction) changes

Momentum is *not* the same as kinetic energy. K = ½ mv^2 is a *scalar* and has no direction. Also, K depends on the square of the speed. Because of this difference, two different objects can have the same momentum but have different kinetic energies.

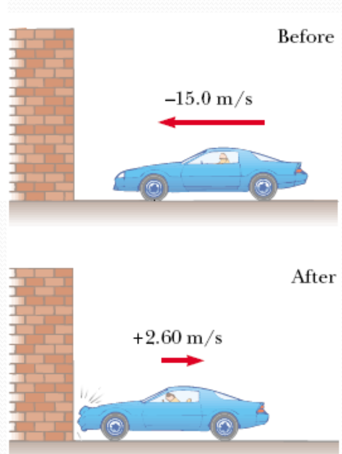
READ Examples 7.1 & 7.2

Example:

In a particular crash test, a car of mass 1 500 kg collides with a wall, as shown in Figure 9.6. The initial and final velocities of the car are $\mathbf{v}_i = -15.0\hat{\mathbf{i}}$ m/s and $\mathbf{v}_f = 2.60\hat{\mathbf{i}}$ m/s, respectively. If the collision lasts for 0.150 s, find the impulse caused by the collision and the average force exerted on the car.

$$\mathbf{p}_i = m\mathbf{v}_i = (1500 \text{ kg})(-15.0\hat{\mathbf{i}} \text{ m/s}) = -2.25 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}$$

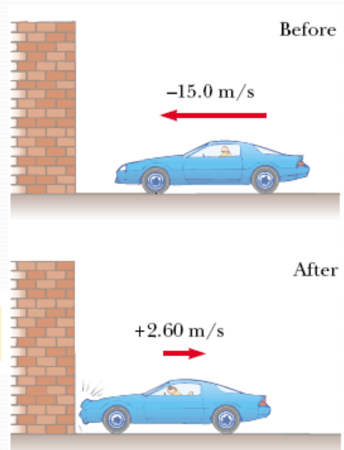
$$\mathbf{p}_f = m\mathbf{v}_f = (1500 \text{ kg})(2.60\hat{\mathbf{i}} \text{ m/s}) = 0.39 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}$$



$$\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = 0.39 \times 10^4\hat{\mathbf{i}}\ \mathrm{kg \cdot m/s} - (-2.25 \times 10^4\hat{\mathbf{i}}\ \mathrm{kg \cdot m/s})$$

$$\mathbf{I} = 2.64 \times 10^4\hat{\mathbf{i}}\ \mathrm{kg \cdot m/s}$$

$$\overline{\mathbf{F}} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{2.64 \times 10^4\hat{\mathbf{i}}\ \mathrm{kg \cdot m/s}}{0.150\ \mathrm{s}} = 1.76 \times 10^5\hat{\mathbf{i}}\ \mathrm{N}$$



Example:

1. A $3.00\,\mathrm{kg}$ particle has a velocity of $(3.0\mathrm{i} - 4.0\mathrm{j})\ \frac{\mathrm{m}}{\mathrm{s}}$. Find its x and y components of momentum and the magnitude of its total momentum.

$$\mathrm{p} = m\mathrm{v} = (3.00\,\mathrm{kg})(3.0\mathbf{i} - 4.0\mathbf{j})\ \frac{\mathrm{m}}{\mathrm{s}} = (9.0\mathbf{i} - 12.\mathbf{j})\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

$$p_x = +9.0\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}} \quad \text{and} \quad p_y = -12.\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.0)^2 + (-12.)^2}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}} = 15.\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

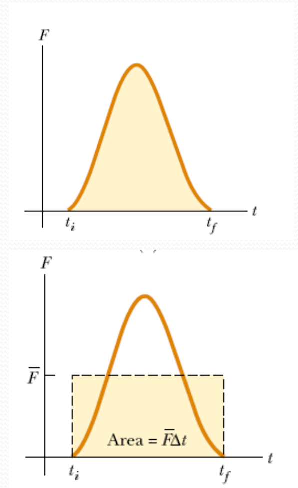
Quick Quiz Two objects have equal kinetic energies. How do the magnitudes of their momenta compare? (a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) not enough information to tell.

Impulse

$$\mathbf{I} \equiv \overline{\mathbf{F}}\,\Delta t$$

Impulse = area under $\vec{F},t$ curve

I = Σ F Δt – also a vector
I = Δp



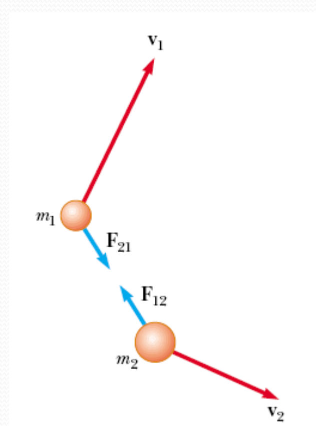
Momentum is conserved
(when ΣF =0)
$\Sigma\, mv_{initial} = \Sigma\, mv_{final}$

Conservation of Linear Momentum

$$\mathbf{p}_{tot} = \mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$$

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$$

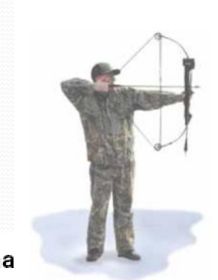
$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$



the total momentum of an isolated system at all times equals its initial momentum.

Example

A 60-kg archer stands at rest on frictionless ice and fires a 0.50-kg arrow horizontally at 50 m/s With what velocity does the archer move across the ice after firing the arrow?



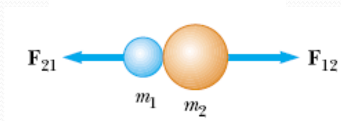
$$m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} = 0$$

$$\mathbf{v}_{1f} = -\frac{m_2}{m_1}\mathbf{v}_{2f} = -\left(\frac{0.50 \text{ kg}}{60 \text{ kg}}\right)(50\hat{\mathbf{i}} \text{ m/s}) = -0.42\hat{\mathbf{i}} \text{ m/s}$$

What If? What if the arrow were shot in a direction tha makes an angle θ with the horizontal? How will this change the recoil velocity of the archer?

$$m_1 v_{1f} + m_2 v_{2f} \cos\theta = 0$$

Collisions



Elastic collision

the total kinetic energy (as well as total momentum) of the system is the same before and after the collision.

Inelastic collision

the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).

Elastic collisions

•occur only between extremely hard objects.

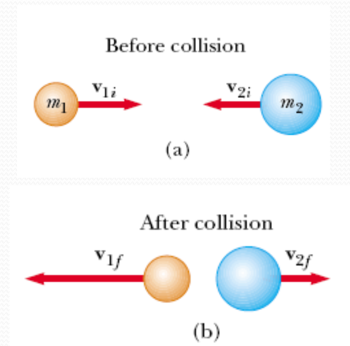
• In elastic collisions objects exchange momentum by “bouncing” from each other during the collision process (like billiard balls).

Conservation of momentum:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

Conservation of kinetic energy:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$



If v_2=0

$$K_1^{'} = \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2} K_1$$

$$K_2^{'} = \frac{4m_1m_2}{(m_1 + m_2)^2} K_1$$

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Problem A ball of mass .25 kg and speed $5m/s$ collides head-on with a ball of mass .75 kg which is at rest. If the collision is elastic, what are the final velocities?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$v_{2i} = 0.$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

→

$$v_{f1} = -2.5m/s \text{ and } v_{f2} = 2.5m/s.$$