King Saud University
College of Engineering in Alkharj

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Solution

Second Exam
Semester II (1428/429)

Math 105

Date: 05-14-1429 **Max. MARKS: 15**
Time : 90min **Number of Pages :4**

NAME : ____

REG.NO : ______________________________________

GROUP :____

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Exercise	Marks
1	
2	
3	
4	
5	
Total	

Exercise1 (3 degrees):
Solve each equation for x.

a) $2\ln x^2 = 1$	**b)** $e^{2x-3} - 7 = 0$
$2Lnx^2 = 1 \Rightarrow Lnx^2 = \frac{1}{2}$ $\Rightarrow e^{Lnx2} = e\frac{1}{2}$ $\Rightarrow x^2 = e^{\frac{1}{2}} \Rightarrow \lvert x \rvert = \sqrt{e^{1/2}}$ $\Rightarrow x = \pm\sqrt{e^{1/2}}$	$\Rightarrow e^{2x-3} = 7$ $\Rightarrow Ln(e^{2x-3}) = Ln7$ $\Rightarrow 2x - 3 = Ln7 \Rightarrow x = \frac{3 + Ln7}{2}$
c) $e^{2x} - 2e^x + 1 = 0$	**d)** $\ln x + \ln(x-1) = 1$
$e^{2x} - 2e^x + 1 = 0$ $(e^x)^2 - 2e^x + 1 = 0$ $(e^x - 1)^2 = 0$ $\Rightarrow e^x - 1 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$	$Lnx + Ln(x-1) = 1$ $\Rightarrow Ln[x(x-1)] = 1$ $\Rightarrow x(x-1) = e^1 = e$ $\Rightarrow x^2 - x - e = 0 \Rightarrow \begin{cases} x_1 = \frac{1+\sqrt{1+4e}}{2} \\ x_2 = \frac{1-\sqrt{1+4e}}{2} \end{cases}$
e) $e^{2x+1} - e^x = 0$	**f)** $\ln(5-2x) = -3$
$e^{2x+1} - e^x = 0$ $\Rightarrow e^{2x+1} = e^x$ $\Rightarrow Ln(e^{2x+1}) = Lne^x$ $\Rightarrow 2x + 1 = x \Rightarrow x = -1$	$Ln(5-2x) = -3$ $\Rightarrow e^{[Ln(5-2x)]} = e^{-3}$ $\Rightarrow 5 - 2x = e^{-3}$ $\Rightarrow x = \frac{5 - e^{-3}}{2}$

Exercise2 (3 degrees):

- Prove that $\sin(\tan^{-1} x) = \dfrac{x}{\sqrt{1+x^2}}$ where **tan⁻¹= arctan** is the inverse function of **tan**

suppose that : $\tan^{-1} x = y \Rightarrow x = \tan y = \dfrac{\sin y}{\cos y}$

We know that $\sin^2 y = 1 - \cos^2 y \;\Rightarrow \sin^2 y = 1 - \dfrac{\sin^2 y}{\tan^2 y}$

$$\Rightarrow \sin^2 y + \frac{\sin^2 y}{\tan^2 y} = 1$$

$$\Rightarrow \sin^2 y\left(1 + \frac{1}{\tan^2 y}\right) = 1 \sin(\tan^{-1} x)$$

$$\Rightarrow \sin^2 y\left(\frac{\tan^2 y + 1}{\tan^2 y}\right) = 1$$

$$\Rightarrow \sin^2 y = \frac{1}{\dfrac{\tan^2 y + 1}{\tan^2 y}} = \frac{\tan^2 y}{\tan^2 y + 1}$$

$$\Rightarrow \sin y = \sqrt{\frac{\tan^2 y}{\tan^2 y + 1}}$$

$$\Rightarrow \sin(\tan^{-1} x) = \sqrt{\frac{x^2}{x^2 + 1}} = \frac{x}{\sqrt{1+x^2}}$$

- Prove that $\sin(\cos^{-1} x) = \sqrt{1-x^2}$ where **cos⁻¹= arccos** is the inverse function of **cos**.

suppose that $\cos^{-1} x = y \Rightarrow x = \cos y$

We know that $\sin^2 y + \cos^2 y = 1 \quad \Rightarrow \quad \sin y = \sqrt{1-\cos^2 y} \Rightarrow \sin(\cos^{-1} x) = \sqrt{1-x^2}$

Exercise3 (3 degrees):

Determine whether f is even, odd, or neither even nor odd.

- $f(x) = -3x^2 + 2$

$$f(-x) = -3(-x)^2 + 2 = -3x^2 + 2 = f(x) \Rightarrow f \textit{ is even}$$

- $f(x) = \dfrac{x^3 - x^7}{x^2 + 2}$

$$f(-x) = \frac{(-x)^3 - (-x)^7}{(-x)^2 + 2} = \frac{-x^3 + x^7}{x^2 + 2} = -\frac{(x^3 - x^7)}{x^2 + 2} = -f(x) \Rightarrow f \textit{ is odd}$$

- $f(x) = \cos x . \sin x$

$f(-x)=\cos(-x).\sin(-x)=\cos x\cdot(-\sin x)=-\cos x\cdot\sin x=-f(x)\Rightarrow f(x)\ is\ odd\cdot$

Exercise4 (2 degrees):

Determine if f is a increasing function or a decreasing function.

a) $f(x)=x\ \ +2$

$Let\ x_1\langle x_2\Rightarrow x_1+2\langle x_2+2\Rightarrow f(x1)\langle f(x2)$

$\Rightarrow f$ is an increasing function

b) $f(x)=x^2+2$

$x\in\ (0,+\infty)$

$x1\ \langle\ x2\Rightarrow x_1{}^2\langle\ x_2{}^2\ \Rightarrow x_1{}^2+2\langle x_2{}^2+2\Rightarrow f(x_1)\langle f(x_2)\Rightarrow$ Increasing

$x\in(-\infty,0)$

$x_1\langle x_2\Rightarrow x^2{}_1\rangle x_2{}^2\Rightarrow x^2{}_1+2\rangle x_2{}^2+2\Rightarrow f(x_1)\rangle f(x_2)\Rightarrow$ decreasing.

c) $f(x)=|x+2|$

$x\langle -2$

$x_1\ \langle\ x_2\langle 0^{\supset}\Rightarrow x_1+2\langle x_2+2\ \langle\ 0\quad\Rightarrow|x_1+2|\rangle|x_2+2|\Rightarrow f$ is a Decreasing function

$x\rangle-2$

$x_1\langle x_2\Rightarrow 0<x_1+2\langle x_2+2\Rightarrow|x_1+2|\langle|x_2+2|\Rightarrow f$ is an Increasing function

Exercise 5 (4 degrees):

Find the following limits:

a) $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+2}}{3x-6}$	**b)** $\lim\limits_{x\to-\infty}\dfrac{x^4-x}{x^3-5x^2+2}$
$\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+2}}{3x-6}=\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2\left(1+\frac{2}{x^2}\right)}}{x\left(3-\frac{2}{x}\right)}$ $=\lim\limits_{x\to-\infty}\dfrac{\lvert x\rvert\sqrt{\left(1+\frac{2}{x^2}\right)}}{x\left(3-\frac{2}{x}\right)}$ $=\lim\limits_{x\to-\infty}\dfrac{\sqrt{\left(1+\frac{2}{x^2}\right)}}{\left(3-\frac{2}{x}\right)}=\dfrac{1}{3}$	$\lim\limits_{x\to-\infty}\dfrac{x^4-x}{x^3-5x^2+2}=\lim\limits_{x\to-\infty}\dfrac{x^4}{x^3}=\lim\limits_{x\to-\infty}x=-\infty$
c) $\lim\limits_{x\to+\infty}\dfrac{x}{\lvert x\rvert}$	**d)** $\lim\limits_{x\to0^+}\dfrac{x+2}{e^x-1}$
$\lim\limits_{x\to+\infty}\dfrac{x}{\lvert x\rvert}=\lim\limits_{x\to+\infty}\dfrac{x}{x}=\lim\limits_{x\to+\infty}1=1$	$\lim\limits_{x\to0^+}\dfrac{x+2}{e^x-1}=\dfrac{2}{1^+-1}=+\infty$